Assignment

Subject Code: MAT-111

Course Title: Basic Mathematics

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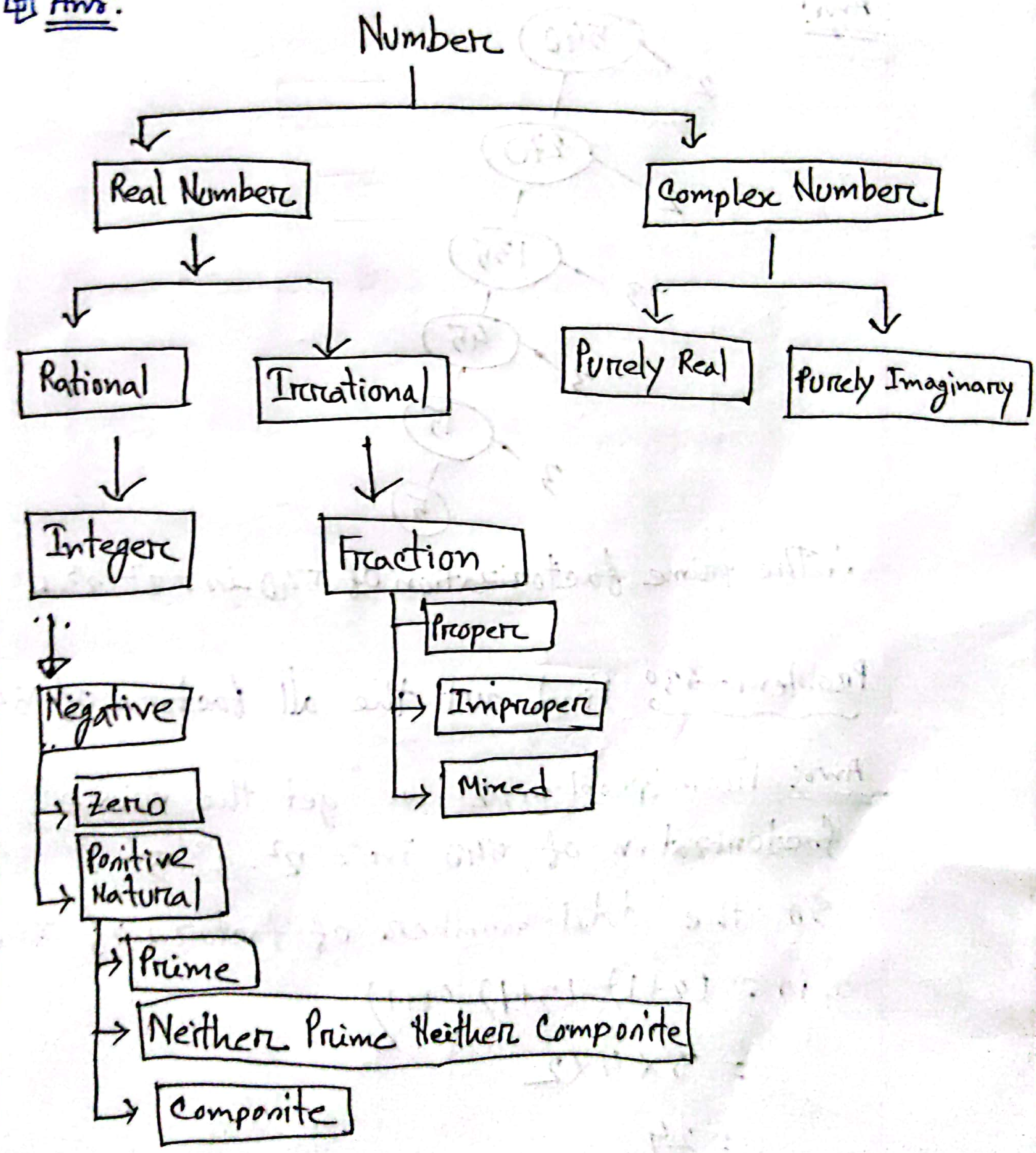
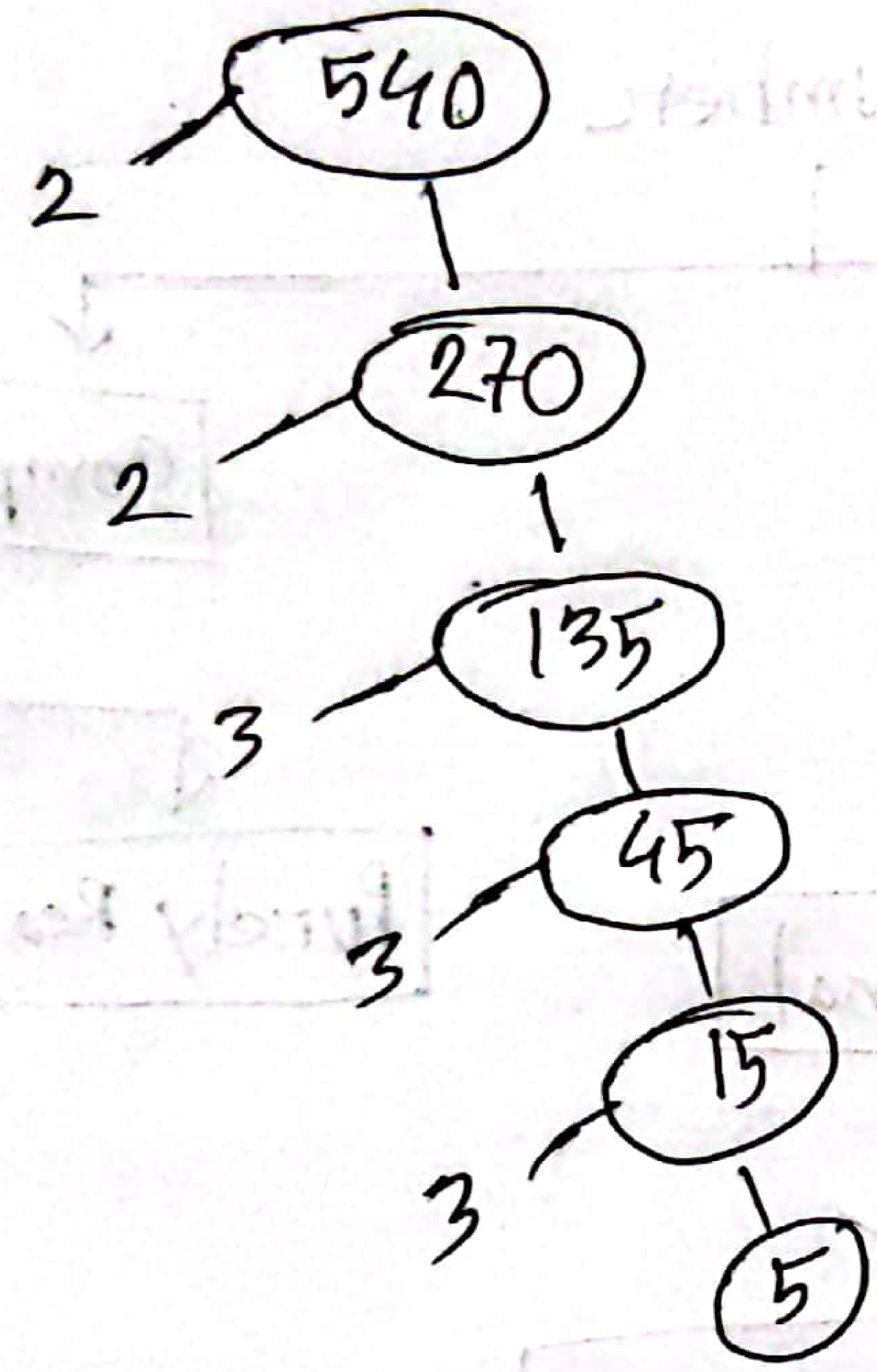
* **Problem-01:** Write down the classification of number system.

Ans:



Problem-02: Find the prime factorization of 540 using tree.

Ans:



$\therefore$ The prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

Problem-03: Find out the all factors of 540

Ans: From problem, 2 we get the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5^1$

So the total number of factors of 540

is $= (2+1)(3+1)(1+1)$

$= 3 \times 4 \times 2$

$= 24$

Now, $540 = 1 \times 540$
$= 2 \times 270$
$= 3 \times 180$
$= 4 \times 135$
$= 5 \times 108$
$= 6 \times 90$
$= 9 \times 60$
$= 10 \times 54$
$= 12 \times 45$
$= 15 \times 36$
$= 18 \times 30$
$= 20 \times 27$

$\therefore$ The all factors of 540 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 54, 60, 90, 108, 135, 180, 270, 540

Problem: 04 What is the GCD and LCM of 240 & 540.

Ans:

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$240 = 2^4 \cdot 3 \cdot 5$ $\qquad$ $540 = 2^3 \cdot 3^3 \cdot 5$

LCM of 240 & 540 $= 2^4 \cdot 3^3 \cdot 5 = 2160$

GCD of 240 and 540 $= 2^2 \cdot 3 \cdot 5 = 60$

Problem-5: Find the H.C.F. and LCM of 42, 63 and 140

Ans:

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$\therefore 42 = 2\cdot3\cdot7$ $\qquad 63 = 3^2\cdot7$ $\qquad 140 = 2^2\cdot5\cdot7$

$\therefore$ L.C.M of $(42, 63, 140) = 2^2\cdot3^2\cdot7 = 1260$

$\therefore$ H.C.F of $(42, 63, 140) = 7$

Problem-6: Find the H.C.F and L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Ans:

Calculation of Numerator

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2\cdot5$

Calculation of Denumerator

$3 = 3^1$

$9 = 3\ 3$

$81 = 3^4$

$27 = 3^3$

$\therefore$ L.C.M of Numerator $= 2^4 . 5$
$= 80$

$\therefore$ H.C.F of Numerator $= 2$

$\therefore$ L.C.M of Denumerator
$= 3^4$
$= 81$

$\therefore$ Hcf of Denumerator
$= 3$

$\therefore$ L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27} = \frac{80}{3}$

$\therefore$ HCF of $(\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27} = \frac{2}{81}$

Problem-7: Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and also its polar, exponential form.

Ans: Given that,

$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+3i)(1+\sqrt{3}i)}{1^2+3}$$

$$= \frac{1+\sqrt{3}i+\sqrt{3}i+3i^2}{4}$$

$$= \frac{2\sqrt{3}i-2}{4}$$

$$= \frac{\sqrt{3}}{2}i - \frac{1}{2}$$

So polar form $= \frac{1}{2} - \frac{\sqrt{3}}{2}i$

Now, $z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\therefore |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$

So, modulus of $z = 1$

And Argument of z,

$\theta = \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)$

$= \pi - \tan^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right)$

$= \pi - \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\therefore$ exponential form of $z = r \cdot e^{i\theta}$

$= 1 \cdot e^{\frac{2\pi}{3} i}$

$= e^{\frac{2\pi}{3} i}$

Problem-8: Evaluate $\sqrt{-16} \times \sqrt{-4}$ & $\frac{\sqrt{-16}}{\sqrt{-4}}$

Ans: $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{(4i)^2} \times \sqrt{(2i)^2}$

$= 4i \times 2i$

$= 8i^2$

$= -8$

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{\sqrt{(4i)^2}}{\sqrt{(2i)^2}}$

$= 2$

Problem - 09: Evalute Modulus & Argument of $8z - z^2$ by replacing $z = 2+i$

Ans: Here given that,

$$z = 2i$$

Now, $8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - 4 - 4i - i^2$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Modulus, $r = \sqrt{x^2+y^2} = \sqrt{(13)^2+(4)^2} = \sqrt{185}$

Argument, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{4}{13}\right) = 17.10^\circ$

Problem - 10: Express $1+\sqrt{3}i$ in the form of $r(\cos\theta + i\sin\theta)$

Ans: Here, $z = 1+\sqrt{3}i$

Modulus, $r = \sqrt{1^2+(\sqrt{3})^2} = 2$

Argument, $\theta = \tan^{-1}\frac{\sqrt{3}}{1}$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

$\therefore$ $r(\cos\theta + i\sin\theta)$ form is $= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$