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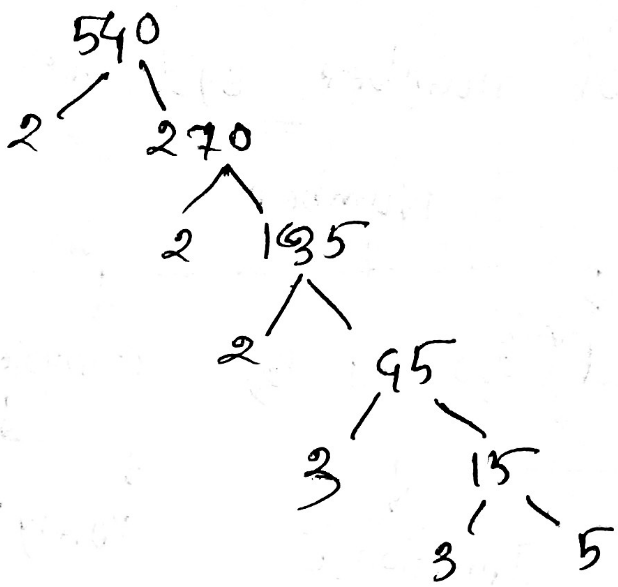
1. Classification of number system:

Number
- Real $(-2, 0, 2, 4, \pi, \frac{1}{2})$
  - ~~Integer~~ Rational $(0.5, 4, \frac{1}{4}, .. -\frac{2}{5})$
    - Integer
      - Negative $(-2, -5)$
      - Zero
      - Positive/Neutral $(1, 2, 3)$
        - Prime
        - Neither Prime
        - Composit
    - Fractional $(\frac{1}{2}, 0.53)$
      - Proper
      - Improper
      - mixed
  - Irrational $(\sqrt{2}, \pi, \sqrt{.4343})$
- Complex
  - Purely Real
  - imaginary $(\sqrt{2}i, 3i)$

2. Tree diagram



Prime factorization = $2^3 \cdot 3^2 \cdot 5^1$

3.

$$\begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 2 & 135 \\ 3 & 45 \\ 3 & 15 \\ \cancel{3} & 5 \\ & \cancel{5} \end{array}$$

$\therefore 540 = 2^3 \cdot 3^2 \cdot 5^1$ [only prime]

$\therefore$ All factorization $540 = (3+1) \cdot (2+1) + (1+1)$

$= 4 \cdot 3 \cdot 2$

$= 24$

3

Here, $540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

All factors of 540 = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540 a

4.

$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array} \qquad \begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

Prime factorization of $240 = 2^4 . 3^1 . 5^1$

$540 = 2^2 . 3^3 . 5^1$

$\therefore$ GCD $= 2^2 . 3^1 . 5^1 = 60$

LCM $= 2^4 . 3^3 . 5^1 = 2160$

5.

$$\begin{array}{r|l} 2 & 42 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ 3 & 21 \\ & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ 2 & 70 \\ 5 & 35 \\ & 7 \end{array}$$

$42 = 2 \times 3 \times 7$

$63 = 2 \times 3 \times 7$

$140 = \cancel{2 \times 2 \times 3 \times 7}\ 2 \times 2 \times 5 \times 7$

$\therefore$ HCF $= 7$

LCM $= 2 \times 2 \times 3 \times 3 \times \cancel{5}\, 7 = 252$

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c. $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$

factorization of numerators:

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 . 5$

H.CF $= 2$

LCM $= 2^4 . 5 = 80$

Factorization of Denominators:

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

H.CF $= 3$

LCM $= 81$

$\therefore$ H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{HCF}(2,8,16,10)}{\text{LCM}(3,9,81,27)}$

$= \frac{2}{81}$

$\therefore$ L.CM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{\text{Lcm}.(2,8,16,10)}{\text{H.CF}(3,9,81,27)}$

$= \frac{80}{3}$

7 We have,

$$\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{(1+\sqrt{3}i)(1-\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1+\sqrt{3}i)^2}{i^2-(\sqrt{3})^2} = \frac{i^2+2.\sqrt{3}.i+3i^2}{1+3}$$

$$= \frac{-2+2\sqrt{3}i}{1+3} = \frac{2(-1+\sqrt{3}i)}{4} = \frac{-1+\sqrt{3}i}{9} = \frac{1}{2} + \frac{\sqrt{3}i}{2}$$

$\therefore x+iy$ [where $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$].

Now, $r = \sqrt{x^2+y^2}$

$$= \sqrt{1/4 + 3/4} = 1$$

and $\theta = \pi - \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right)$

$$= \pi - \tan^{-1}\sqrt{3}$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

So, $z = r(\cos\theta + i\sin\theta)$

$$= 1\left(\cos\frac{2\pi}{3} + \sin\frac{2\pi}{3}\right)$$

$$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

and exponential from is $z = e^{i \cdot \frac{2\pi}{3}}$

7

8. $\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16i^2} \times \sqrt{4i^2} = \sqrt{4^2 i^2} \times \sqrt{2^2 \cdot i^2} = 4i \times 2i = 8i^2$

$= -8$

again, $\frac{\sqrt{-16}}{\sqrt{-4}} = \frac{\sqrt{16i^2}}{\sqrt{4i^2}} = \frac{-4i}{2i} = 2$

9. $8z - z^2$

$= 8(2+i) - (2+i)^2$

$= 16 + 8i - 4 - 4i + 1$

$= 12 + 4i + 1$

$= 13 + 4i$

$= x + iy \quad (x = 13, \; y = 4)$

modulus, $r = \sqrt{13^2 + 4^2} = \sqrt{185}$

arg, $\theta = \tan^{-1} \frac{4}{13} = 17.102$

8/

10. $1+i\sqrt{3}$

Let, $z = 1+i\sqrt{3}$

$z = x+iy$

$r = \sqrt{1^2+(\sqrt{3})^2} = 2$ $\quad \therefore r = 2$

~~Ag~~ Arg of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}.\tan\frac{\pi}{3}$

$= \pi/3$

$\therefore r(\cos\theta + i\sin\theta)$ from is

$= 2(\cos\theta + i\sin\theta)$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(Ans)