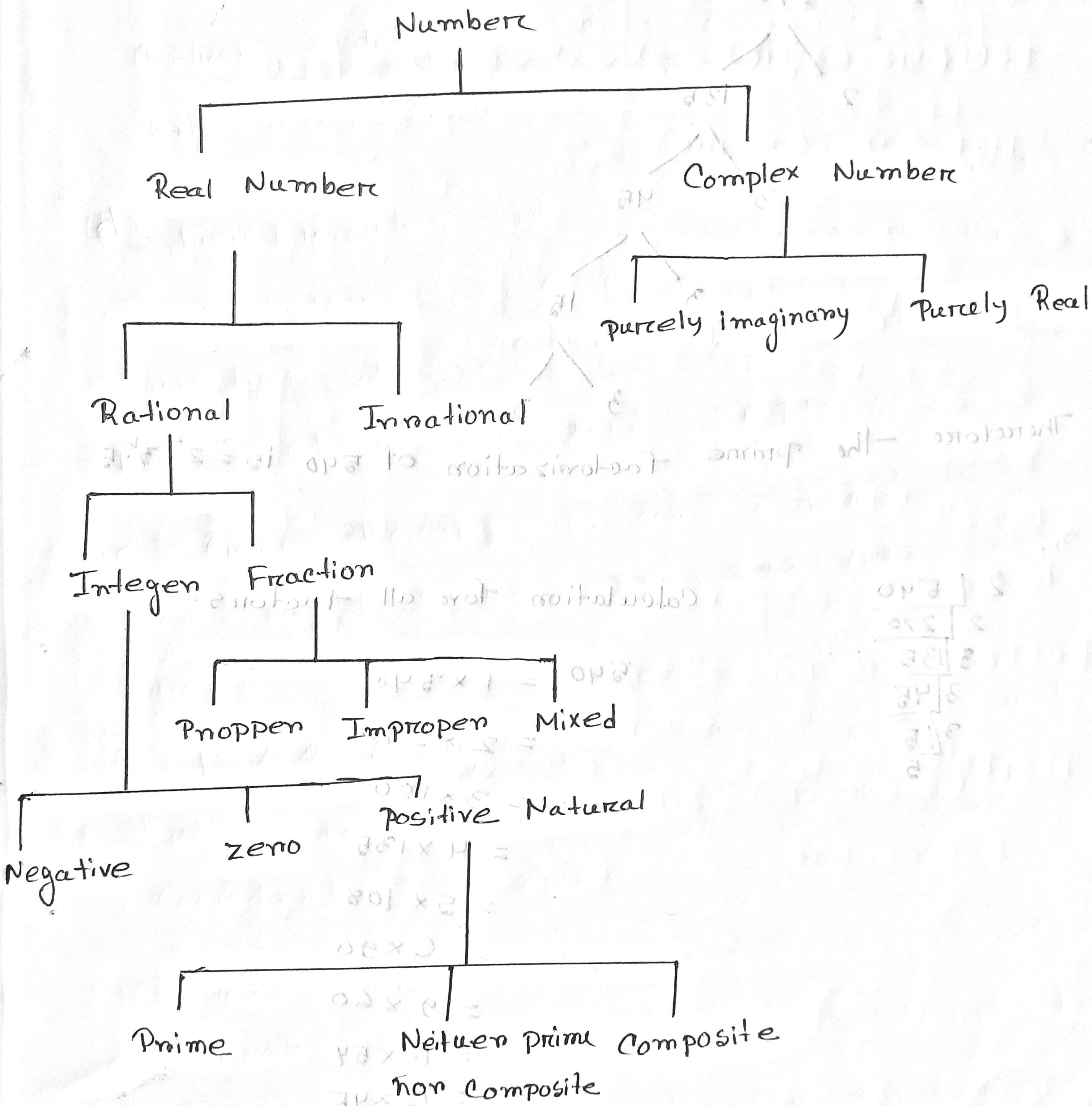
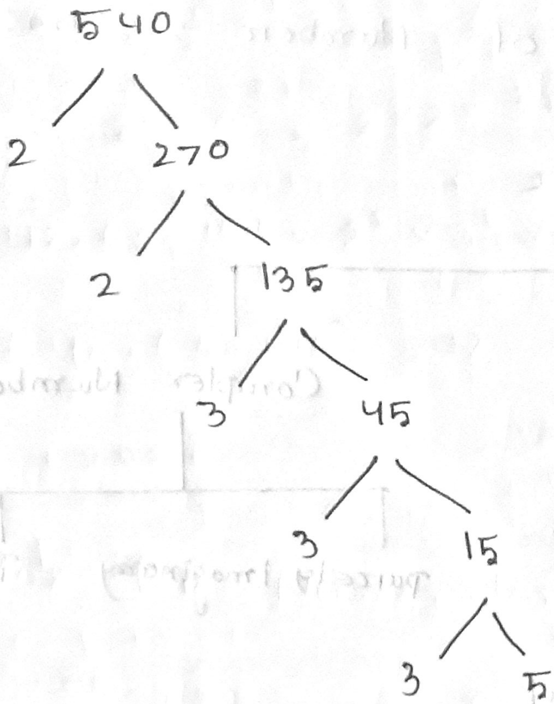
□ Complex Number

1. classification of Number system:



2. Find the prime factorization of 540 ~~with~~ using tree



Therefore the prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

3.

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Calculation for all factors -

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 12 \times 30$ $= 20 \times 27$

Therefore, the prime factorization of 540 is

$= 2^2 \cdot 3^3 \cdot 5$

So the total Number of factors of 540 is

$= (2+1)(3+1)(1+1)$

$= 3 \cdot 4 \cdot 2$

$= 24$

The factors of 540 are : 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4. $240 = 2 \times 120 = 2 \times 2 \times 60 = 2 \times 2 \times 2 \times 2 \times 15 = 2^4 \times 3 \times 5$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2 \times 2 \times 3 \times 45 = 2 \times 2 \times 3 \times 3 \times 15$

$= 2^2 \times 3^3 \times 5$

$\therefore$ LCM $(240, 540) = 2^4 \cdot 3^3 \cdot 5 = 2160$

HCF or GCD $(240, 540) = 2^2 \cdot 3 \cdot 5 = 60$

5.

$42 = 2\times21 = 2\times3\times7$

$63 = 3\times21 = 3\times3\times7 = 3^2\times7$

$140 = 2\times70 = 2\times2\times35 = 2^2\times5\times7$

$\therefore$ LCM $(42, 63, 140) = 2^2\times3^2\times5\times7 = 1260$

HCF $(42, 63, 140) = 7$

6.

Calculation for Numerators

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1\times5^1$

LCM $(2, 8, 16, 10) = 2^4\times5$

$= 80$

HCF $(2, 8, 16, 10) = 2$

Calculation for denominators

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$\text{LCM}(3, 9, 81, 27) = 3^4$
$= 81$

$\text{HCF}(3, 9, 81, 27) = 3$

$$\text{HCF of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} = \frac{\text{HCF}(2, 8, 16, 10)}{\text{LCM}(3, 9, 81, 27)}$$

$$= \frac{2}{81}$$

$$\text{LCM of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} = \frac{\text{LCM}(2, 8, 16, 10)}{\text{HCF}(3, 9, 81, 27)}$$

$$= \frac{80}{3}$$

(7)

We have $\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{1+2\sqrt{3}i-3}{1^2-(\sqrt{3}i)^2}$

$= \frac{-2+2\sqrt{3}i}{1+3}$

$= \frac{2(-1+\sqrt{3}i)}{4}$

$= \frac{-1+\sqrt{3}i}{2}$

$= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

let $z = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$|z| = \sqrt{(-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$r = \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$

Modulus of z is $= 1$

and Argument of z −

$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$

$= \pi - \tan^{-1}(\sqrt{3})^2$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

polar form $= -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

Exponential form is $z = re^{i\theta}$

$= 1.e^{i\frac{2\pi}{3}}$

$= e^{\frac{2\pi}{3}i}$

8. we have,

$\sqrt{-16} \times \sqrt{-4}$

$= \sqrt{16}i \times \sqrt{4}i$

$= 4i \times 2i$

$= 8i^2$

$= 8$

And

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$

9. we have, z

$z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16+8i-(4+4i+i^2)$

$= 16+8i-4-4i+1$

$= 13+4i$

Modulus, $r = \sqrt{(13)^2+(4)^2}$

$= \sqrt{169+16} = \sqrt{185}$

$\theta = \tan^{-1}\frac{4}{13}$

$= 17.102$

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let, $z = 1 + i\sqrt{3}$, $z = x + iy$

$|z| = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

Modulus of $z = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$

Argument of $z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\tan\frac{\pi}{3}$

$= \frac{\pi}{3}$

Therefore, $r(\cos\theta + i\sin\theta)$ from is

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$