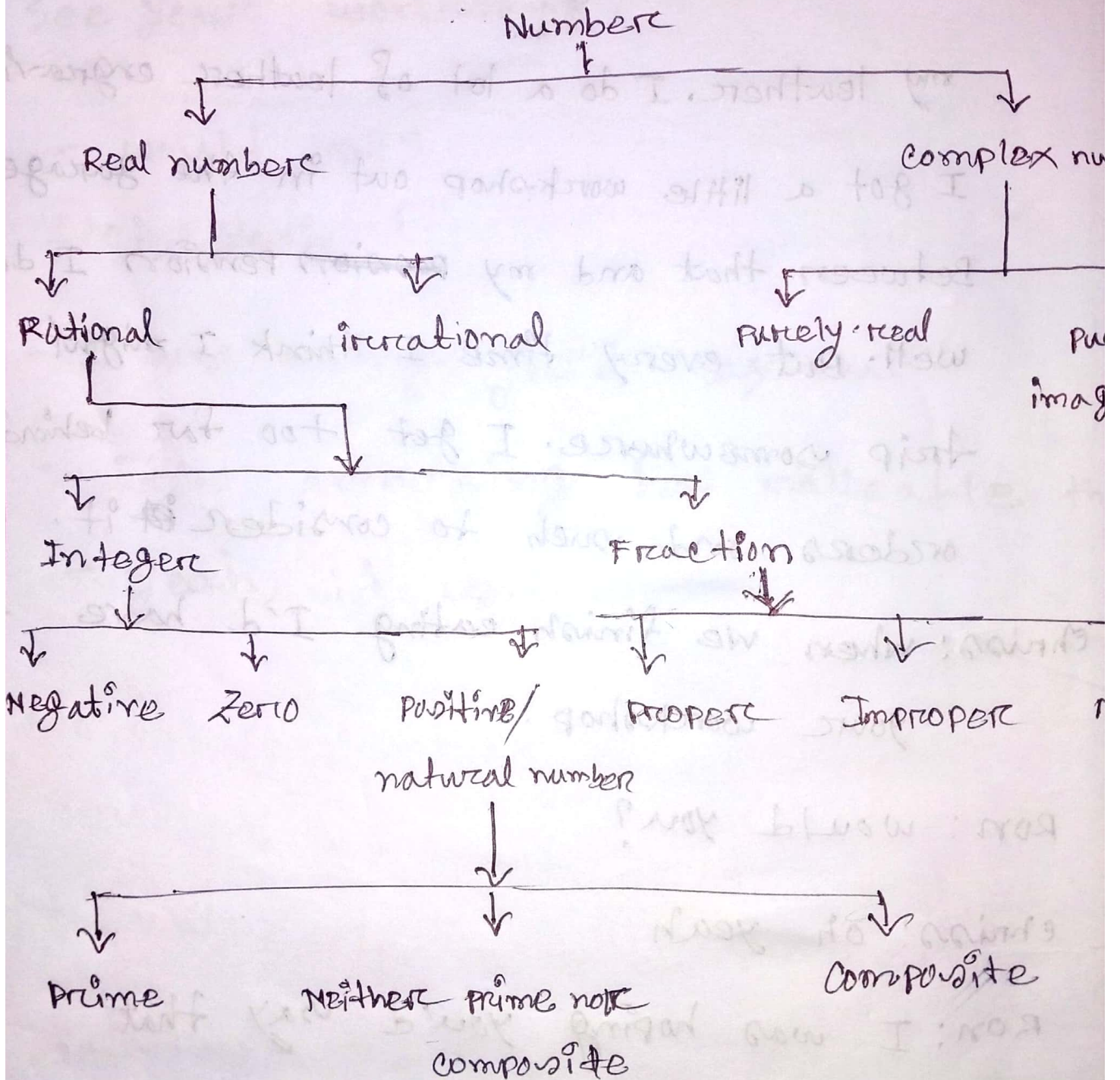
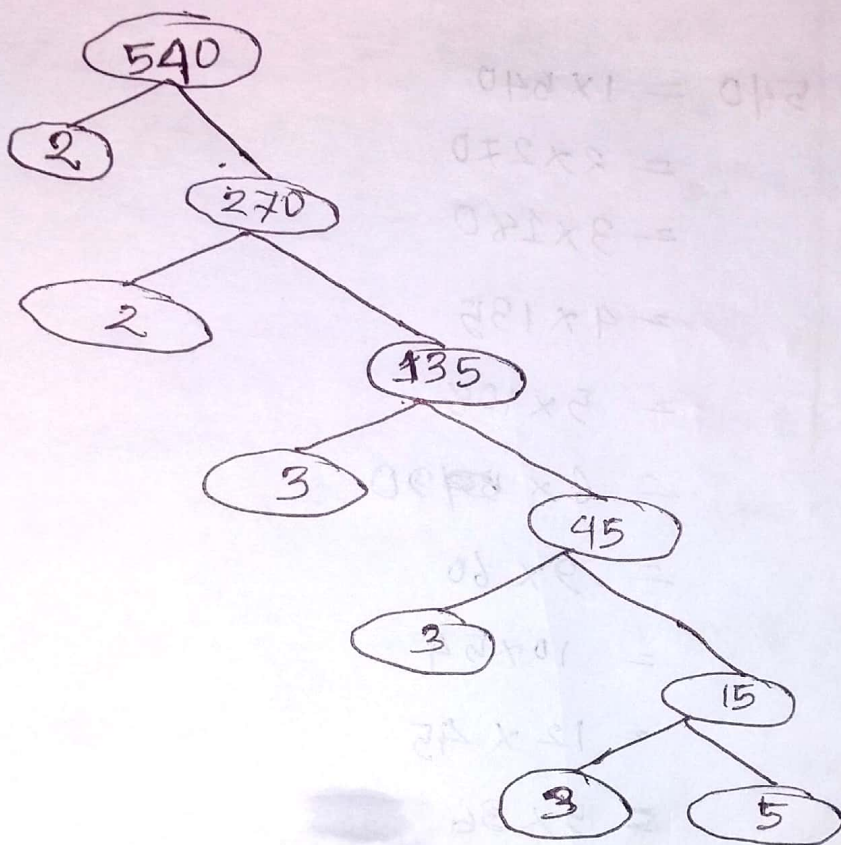


① Write down the classification of number system



② Find the prime factorization of 540 using tree:



therefore the prime factorization of $540 = 2^2 \cdot 3^3 \cdot 5$

③ Find the all factors of 540:-

The prime factorization of 540 is $= 2^2 \cdot 3^3 \cdot 5$

So, the total numbers of 540 is

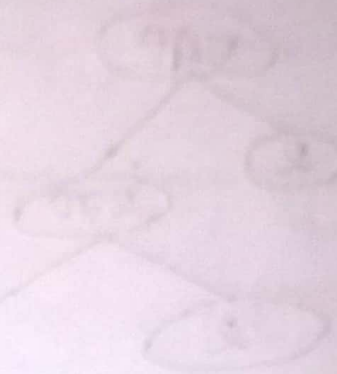
$$= (2+1)(3+1)(1+1)$$

$$= 3 \times 4 \times 2$$

$$= 24$$

calculation for all factors:

$$\begin{aligned}540 &= 1 \times 540 \\ &= 2 \times 270 \\ &= 3 \times 180 \\ &= 4 \times 135 \\ &= 5 \times 108 \\ &= 6 \times 90 \\ &= 9 \times 60 \\ &= 10 \times 54 \\ &= 12 \times 45 \\ &= 15 \times 36 \\ &= 18 \times 30 \\ &= 20 \times 27\end{aligned}$$



The factors of 540 are:-

$$\{1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540\}$$

Q) what is the GCD and LCM of 240 and 540:-

$$\begin{aligned}\therefore 240 &= 2 \times 120 = 2 \times 2 \times 60 = 2 \times 2 \times 2 \times 30 = 2 \times 2 \times 2 \times 2 \times 15 \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \\ &= 2^4 \cdot 3 \cdot 5\end{aligned}$$

$$\therefore 540 = 2^2 \cdot 3^3 \cdot 5$$

[following previous work]

$$\begin{aligned}\therefore 240 &= 2^4 \cdot 3 \cdot 5 \\ \therefore \text{LCM} &= 2^4 \cdot 3^3 \cdot 5 = 2160 \\ \therefore \text{GCD} &= 2^2 \cdot 3 \cdot 5 = 60\end{aligned}$$

5) Find the HCF and LCM of 42, 63 and 140.

$$\begin{aligned} \therefore 42 &= 2 \times 21 \\ &= 2 \times 3 \times 7 \end{aligned}$$

$$\begin{aligned} \therefore 63 &= 3 \times 21 \\ &= 3 \times 3 \times 7 \\ &= 3^2 \times 7 \end{aligned}$$

$$\begin{aligned} \therefore 140 &= 2 \times 70 \\ &= 2 \times 2 \times 35 \\ &= 2^2 \times 5 \times 7 \end{aligned}$$

$$\therefore 42 = 2 \cdot 3 \cdot 7$$

$$\therefore 63 = 3^2 \cdot 7$$

$$\therefore 140 = 2^2 \cdot 5 \cdot 7$$

$$\therefore \text{LCM} = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260 \text{ (Ans)}$$

$$\therefore \text{HCF} = 7 \text{ (Ans)}$$

6) Find the H.C.F and L.C.M of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$

Calculation for Numerators:

$$\text{LCM } 2 = 2^1$$

$$8 = 2^3$$

$$16 = 2^4$$

$$10 = 2^1 \times 5^1$$

$$\text{LCM} = 2^4 \times 5 = 80$$

$$\text{HCF} = 2^1 = 2$$

$$\therefore \text{LCM of } \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} \right) = \frac{80}{3} \text{ (Ans)}$$

$$\therefore \text{HCF of } \left(\frac{2}{3}, \frac{8}{9}, \frac{16}{81} \text{ and } \frac{10}{27} \right) = \frac{2}{81} \text{ (Ans)}$$

Calculation of denominators:

$$3 = 3^1$$

$$9 = 3^2$$

$$81 = 3^4$$

$$27 = 3^3$$

$$\text{LCM} = 3^4 = 81$$

$$\text{HCF} = 3 = 3$$

Q7) Find the modulus and Argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ also its polar, exponential form:-

$$\begin{aligned} z &= \frac{1+\sqrt{3}i}{1-\sqrt{3}i} \\ &= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)} \\ &= \frac{1+2\sqrt{3}i-3}{1+3} \\ &= \frac{-2+2\sqrt{3}i}{4} \\ &= \frac{2(-1+\sqrt{3}i)}{4} \\ &= \frac{-1+\sqrt{3}i}{2} \\ &= -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{aligned}$$

$$\begin{aligned} z &= -\frac{1}{2} + \frac{\sqrt{3}}{2}i \\ |z| &= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ \therefore r &= \sqrt{\frac{1}{4} + \frac{3}{4}} \\ &= 1 \end{aligned}$$

\therefore modulus of z is $= 1$
And argument of z will-

$$\begin{aligned} \theta &= \pi - \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) \\ &= \pi - \tan^{-1}(\sqrt{3}) \\ &= \pi - \frac{\pi}{3} = \frac{2\pi}{3} \end{aligned}$$

\therefore Polar form, $z = r(\cos\theta + i\sin\theta)$
 $= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

\therefore Exponential form, $z = r e^{i\theta}$
 $= 1 e^{i\frac{2\pi}{3}}$
 $= e^{\frac{2\pi}{3}i}$

(8) Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\sqrt{\frac{-16}{-4}}$

Answer:

$$\sqrt{-16} \times \sqrt{-4}$$

$$= 4i \times 2i$$

$$= 8i^2$$

$$= -8$$

(Ans)

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{4i}{2i}$$

$$= 2 \text{ (Ans)}$$

(9) Evaluate modulus and Argument of $8z - z^2$

replacing, $z = 2+i$

$$8z - z^2 = 8(2+i) - (2+i)^2$$

$$= 16 + 8i - 4 - 4i + 1$$

$$= 13 + 4i$$

$$\text{modulus of } 8z - z^2 = \sqrt{(13)^2 + (4)^2} = \sqrt{169 + 16}$$

$$\text{Argument of } 8z - z^2 = \tan^{-1}\left(\frac{4}{13}\right)$$

$$= 17.1^\circ \text{ (Ans)}$$

(10) Express $1 + \sqrt{3}i$ in the form of $r(\cos\theta + i\sin\theta)$

$$z = 1 + \sqrt{3}i$$

Hence, modulus of $|z| = \sqrt{(1)^2 + (\sqrt{3})^2}$

$$= \sqrt{4} = 2$$

$$\therefore r = 2$$

\therefore Argument, $\theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \left(\frac{\sqrt{3}}{1} \right)$

$$= \frac{\pi}{3}$$

therefore,

$r(\cos\theta + i\sin\theta)$ form $z = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$