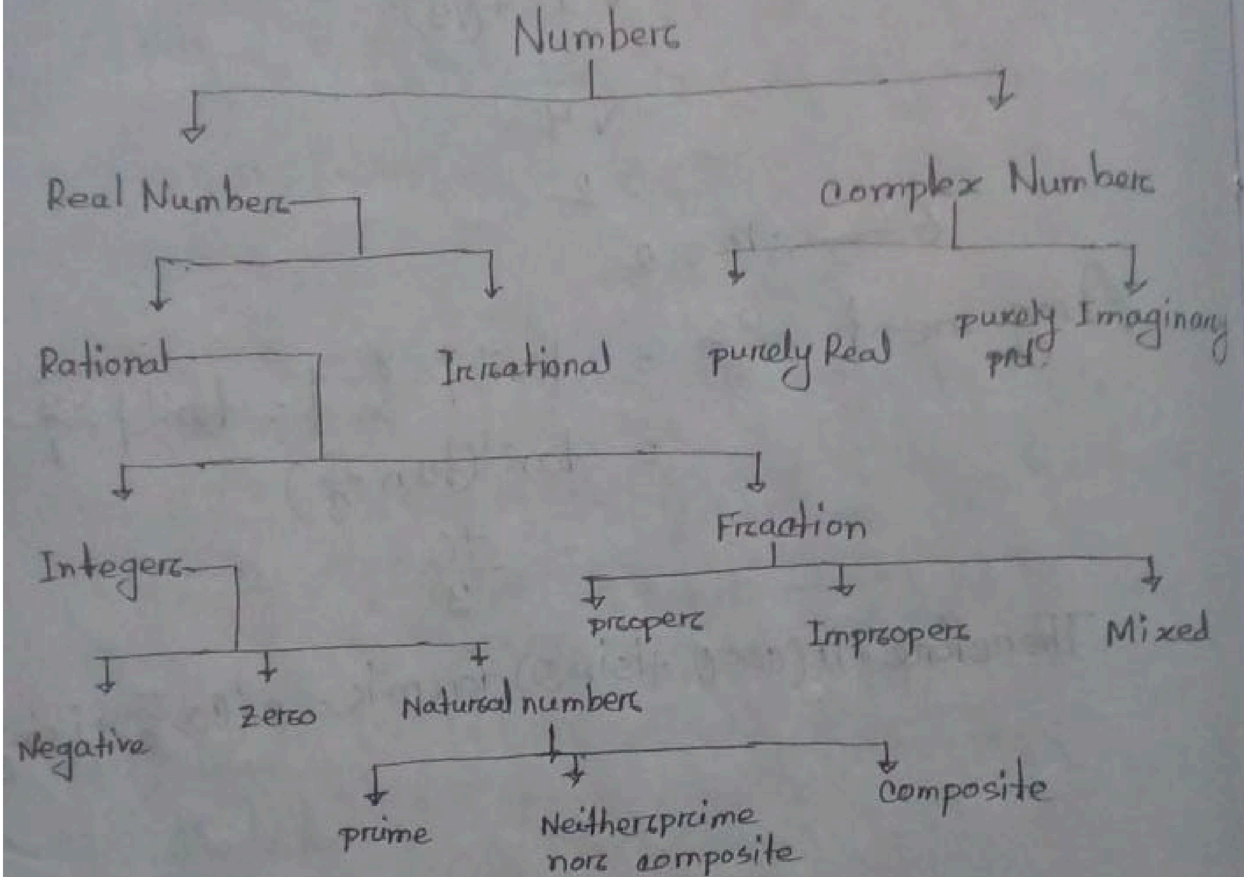
Homework

1. Write down the classification of numbers system.

→ Here it is :-



2. Find the prime factorization of 540 using tree.

→ 540

2 270

2 135

5 27

3 9

3 3

3 1

∴ The prime factorization of 540 are : $2^2 . 3^3 . 5^1$

3. Find out the all factors of 540

$$
\begin{array}{r|l}
2 & 540 \\
2 & 270 \\
5 & 135 \\
3 & 27 \\
3 & 9 \\
3 & 3 \\
 & 1
\end{array}
$$

The prime factorization of 540 are : $2^2 . 3^3 . 5^1$

∴ The total factor of 540 is $= (2+1) . (3+1) . (1+1)$

$= 3 . 4 . 2$

$= 24$

Calculation for all factors:-

$540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

∴ The factors of 540 are:-

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

4. What is the GCD & LCM of 240 & 540.

$\Rightarrow$ Here,

$240 = 2 \times 120 = 2 \times 2 \times 60 = 2^2 \times 2 \times 30 = 2^3 \times 2 \times 3 \times 5$
$= 2^4 \times 3 \times 5$

$540 = 2 \times 270 = 2 \times 2 \times 135 = 2^2 \times 5 \times 27 = 2^2 \times 3^3 \times 5$

$\therefore$ LCM $(240, 540) = 2^4 \times 3^3 \times 5 = 2160$

$\therefore$ GCD $(240, 540) = 2 \times 3 \times 5 = 30$

5. Find the HCF & LCM of 42, 63 & ~~540~~ 140.

$\Rightarrow$ Here,

$42 = 2 \times 21 = 2 \times 3 \times 7$

$63 = 3 \times 3 \times 7 = 3^2 \times 7$

$140 = 2 \times 70 = 2 \times 2 \times 35 = 2^2 \times 7 \times 5$

$\therefore$ LCM $(42, 63, 140) = 3^2 \times 2^2 \times 7 \times 5 = 1260$

$\therefore$ HCF $(42, 63, 140) = 2 \times 7 = 14$

6. Calculation for Numerators.

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

$10 = 2^1 \times 5^1$

$\therefore$ LCM (2, 8, 16 and 10) $= 2^4 \times 5 = 80$

$\therefore$ HCF (2, 8, 16 and 10) $= 2$

Calculation for Denominators,

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

$\therefore LCM(3, 9, 81, 27) = 3^4 = 81$

$\therefore HCF(3, 9, 81, 27) = 3$

$\therefore$ HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{HCF(2, 8, 16, 10)}{LCM(3, 9, 81, 27)} = \frac{2}{81}$

$\therefore$ LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27} = \frac{80}{3}$

7. We have,

$\frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{1+\sqrt{3}i+\sqrt{3}i+3i^2}{1-(\sqrt{3}i)^2}$

$= \frac{1+2\sqrt{3}i-3}{1+3}$

$= \frac{-2+2\sqrt{3}i}{4}$

$= \frac{2(-1+\sqrt{3}i)}{4}$

$= \frac{-1+\sqrt{3}i}{2}$

$= \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

$\therefore$ polar form $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Let $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$|z| = \sqrt{(-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

$= \sqrt{\frac{1}{4} + \frac{3}{4}}$

$= \sqrt{\frac{4}{4}}$

$= 1$

$\therefore$ Modulus of z is 1

And Argument of z will :-

$$\theta = \pi - \tan^{-1}\left|\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right|$$

$$= \pi - \tan^{-1}(\sqrt{3})$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Exponential form is $z = re^{i\theta}$

$$= 1 \cdot e^{i \cdot \frac{2\pi}{3}}$$

$$= e^{\frac{2\pi}{3}i}$$

8. We have,

$$\sqrt{-16} \times \sqrt{-4}$$

$$= \sqrt{16}\, i \times \sqrt{4}\, i$$

$$= 4i \times 2i$$

$$= -8$$

and,

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{\sqrt{16}\, i}{\sqrt{4}\, i}$$

$$= \frac{4i}{2i}$$

$$= 2$$

Ans.

9. We have,

$z = 2+i$

$\therefore 8z - z^2 = 8(2+i) - (2+i)^2$

$= 16 + 8i - 4 - 4i - i^2$

$= 12 + 4i + 1.$

$= 13 + 4i$

Modulus $r = \sqrt{(13)^2 + (4)^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

$\theta = \tan^{-1}\left(\frac{4}{13}\right)$

$= 17.102$

10. Let $z = 1 + i\sqrt{3}$

Modulous of $z = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\therefore r = 2$

Argument of $z = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| = \tan^{-1}(\sqrt{3})$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

$\therefore$ Therefore, $r(\cos\theta + i\sin\theta)$ form is =

$2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Ans.