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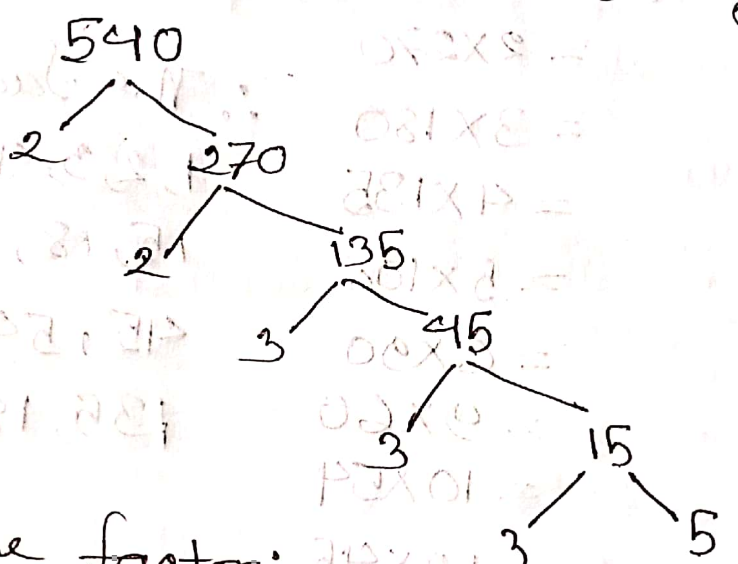
1) Write ~~down~~ down the classification of number System.

<u>Answer:</u>

- Number
  - Real Number
    - Rational
      - Integer
        - Negative
        - Zero
        - positive or Natural Number
          - Prime
          - Neither Prime nor composite
          - Composite
      - Fraction
        - proper
        - Improper
        - Mixed
    - Irrational
  - Complex Number
    - Purely ~~com~~ real
    - Purely Imaginary

2) Find the factorization of 540 using tree.

Answer:



∴ The prime factorization of 540 is,

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 . 3^3 . 5^1$

3) Find out the factors of 540.

Answer: From '2' we find the Prime factorization of 540 is, $540 = 2^2 . 3^3 . 5^1$

$= (2+1).(3+1).(1+1)$

$= 3.4.2$

$= 24$

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Here, $540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

$\therefore$ The factors of 540 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540.

4) What is the GCD and LCM of 240 and 540.

Answer:

2 | 240
2 | 120
2 | 60
2 | 30
3 | 15
5

2 | 540
2 | 270
2 | 135
2 | 45
3 | 15
5

$\therefore 240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 . 3^1 . 5^1$

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^3 . 3^2 . 5^1$

$\therefore$ The GCD of 240 and 540 is $= 2^3 \cdot 3^1 \cdot 5^1$

$= 4 \times 3 \times 5$

$= 60$

and ; The LCM of 240 and 540 is $= 2^4 \cdot 3^3 \cdot 5^1$

$= 16 \cdot 27 \cdot 5$

$= 2160$

5) Find the H.C.F and L.C.M of 42, 63 and 140.

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$42 = 2^1 \cdot 3^1 \cdot 7^1$

$63 = 3^2 \cdot 7^1$

$140 = 2^2 \cdot 5^1 \cdot 7^1$

$\therefore$ The H.C.F of 42, 63 and 140 is $= 7^1 = 7$

$\therefore$ The L.C.M of 42, 63 and 140 is $= 2^2 \cdot 3^2 \cdot 5^1 \cdot 7^1$

$= 4 \times 9 \times 5 \times 7$

$= 1260$

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6) Find the H.C.F and L.C.M of $2/3$, $8/9$, $\frac{16}{81}$ and $\frac{10}{27}$.

Answer:

Factorization of Numerators,

$2 = 2^1$

$8 = 2\times2\times2 = 2^3$

$16 = 2\times2\times2\times2 = 2^4$

$10 = 2\times5 = 2^1.5^1$

H.C.F (2, 8, 16 and 10) = 2

L.C.M (2, 8, 16 and 10) $= 2^4.5^1$

$= 16\times5$

$= 80$

Factorization of Denominators

$3 = 3^1$

$9 = 3\times3 = 3^2$

$81 = 3\times3\times3\times3 = 3^4$

$27 = 3\times3\times3 = 3^3$

HCF (3, 9, 18, 27) = 3

L.C.M (3, 9, 81, 27) $= 3^4$

$= 81$

$\therefore$ The HCF of $2/3$, $8/9$, $16/81$, and $10/27$ is $= \frac{\text{HCF (2, 8, 16 and 10)}}{\text{LCM (3, 9, 81 and 27)}}$

$= \frac{2}{81}$

$\therefore$ The L.C.M of $2/3$, $8/9$, $16/81$ and $10/27$ is $= \frac{\text{L.C.M (2, 8, 16, 10)}}{\text{H.C.F (3, 9, 81, 27)}}$

$= \frac{80}{3}$

7) Find the modulus and argument of $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$ and ~~ald~~ also its polar, exponential form.

Answer: $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} = \frac{(1+\sqrt{3}i)^2}{1+3}$

$= \frac{1+2\sqrt{3}i-3}{1+3} = \frac{-2+2\sqrt{3}i}{4} = \frac{2(-1+\sqrt{3}i)}{4} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

If we compare $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$ with $x+iy$ then

$x = -\frac{1}{2}$ and $y = \sqrt{3}/2$

$\therefore$ modulus, $r = \sqrt{x^2+y^2} = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{\frac{4}{4}}$

$= \sqrt{1} = 1$

$\therefore$ argument, $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}/2}{-1/2} = \tan^{-1}(-\sqrt{3})$

$= \pi - \tan(\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{3\pi-\pi}{3}$

$= \frac{2\pi}{3}$

$\therefore$ The polar form is, $z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

and, Exponential form, $z = re^{i\theta}$

$= \cancel{e}\,1 \cdot e^{i \cdot \frac{2\pi}{3}}$

8) Evaluate $\sqrt{-16} \times \sqrt{-4}$ and $\frac{\sqrt{-16}}{\sqrt{-4}}$

Answer:

$$\sqrt{-16} \times \sqrt{-4}$$
$$= i\sqrt{16} \times i\sqrt{4}$$
$$= i^2 \sqrt{4^2} \times \sqrt{2^2}$$
$$= i^2 \times 4 \times 2$$
$$= -8$$

And,
$$\frac{\sqrt{-16}}{\sqrt{-4}}$$
$$= \frac{i\sqrt{16}}{i\sqrt{4}}$$
$$= \frac{i\sqrt{4^2}}{i\sqrt{2^2}}$$
$$= \frac{4}{2}$$
$$= 2$$

9) Evaluate Modulus and argument of $8z - z^2$ by replacing $z = 2+i$.

Answer:

$\therefore 8(2+i) - (2+i)^2 \quad [\because z = 2+i]$

$= 16 + 8i - (4 + 4i + i^2)$

$= 16 + 8i - 4 - 4i + 1$

$= 13 + 4i$

Here, $x = 13$ and $y = 4$

$\therefore$ Modulus, $r = \sqrt{x^2 + y^2}$

$= \sqrt{13^2 + 4^2}$

$= \sqrt{169 + 16} = \sqrt{185}$

Argument, $\theta = \tan^{-1} \frac{y}{x}$

$= \tan^{-1} \frac{4}{13}$

10) Express $1+i\sqrt{3}$ in the form of $r(\cos\theta + i\sin\theta)$

__Answer:__ $z = 1+i\sqrt{3}$

Here,

$x = 1$, $y = \sqrt{3}$

$\therefore r = \sqrt{x^2+y^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

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