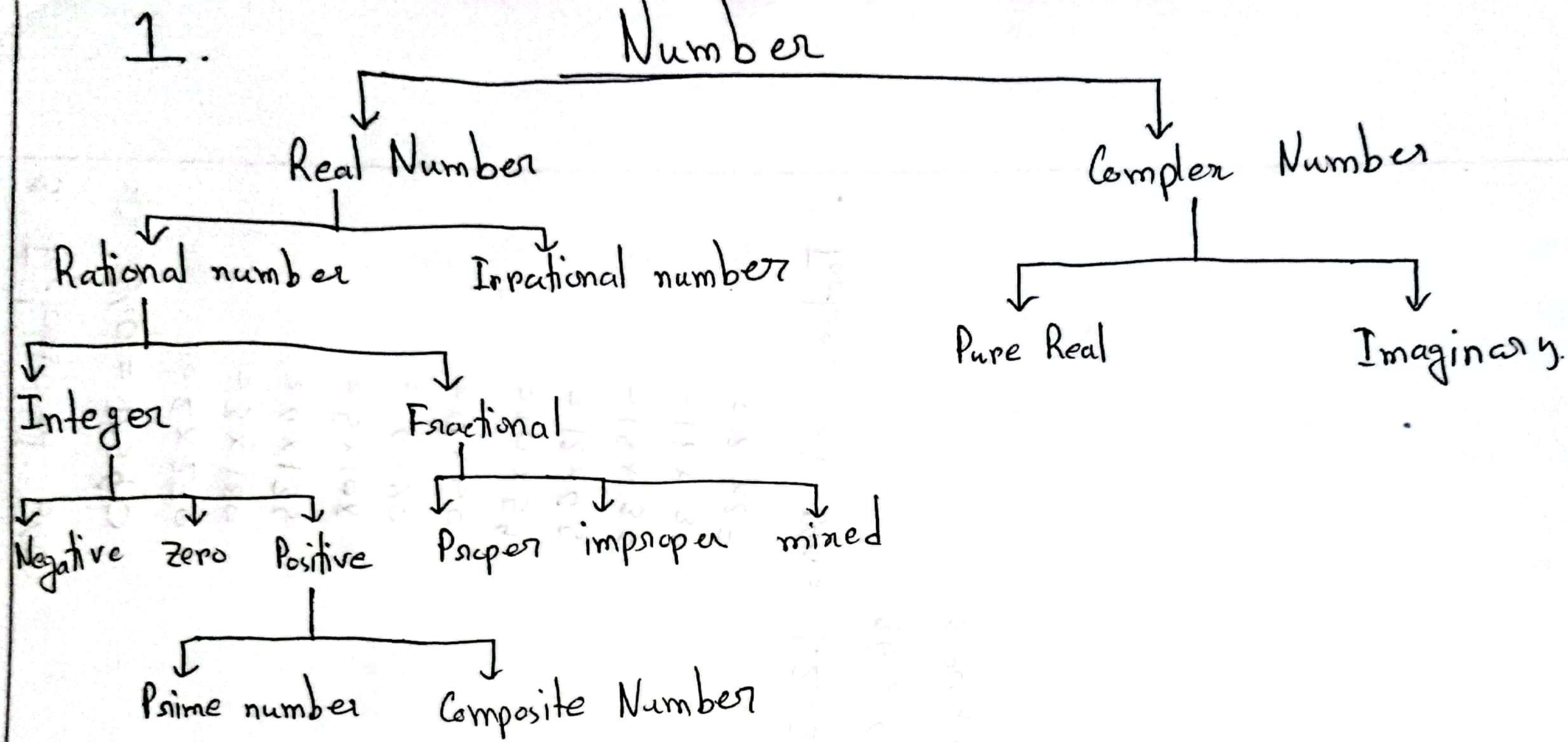
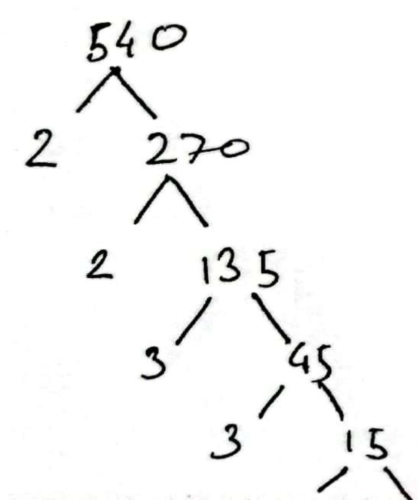
1.



2. Factorization of 540 with tree structure.

=>



∴ Prime Factorization of 540 = $2^2 \times 3^3 \times 5$

3. Factors of 540

$\Rightarrow 540 = 1 \times 540$

$= 2 \times 270$

$= 3 \times 180$

$= 4 \times 135$

$= 5 \times 108$

$= 6 \times 90$

$= 9 \times 60$

$= 10 \times 54$

$= 12 \times 45$

$= 15 \times 36$

$= 18 \times 30$

$= 20 \times 27$

$\therefore$ Factors of 540 = 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 27, 30, 36, 45, 54, 60, 90, 108, 135, 180, 270, 540

4.

$$\begin{array}{r|l} 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$\therefore$ Prime factorization of $240 = 2^4 \times 3^1 \times 5^1$

$$\begin{array}{r|l} 2 & 540 \\ \hline 2 & 270 \\ \hline 3 & 135 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$\therefore$ Prime factorization of $540 = 2^2 \times 3^3 \times 5$

$\therefore$ GCD of 240 and 540 $= 2^2 \times 3 \times 5$

$= 60$

LCM " 540 " 240 $= 2^4 \times 3^3 \times 5$

$= 2160$

5. HCF and LCF of 42, 63, 140

$42 = 2 \times 3 \times 7$

$63 = 7 \times 3 \times 3$

$140 = 2 \times 2 \times 5 \times 7$

$\therefore$ HCF $= 7$

LCM $= 2^2 \times 3^3 \times 5 \times 7$

$= 1260$

6. LCM and HCF of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

Numerator side,

$2 = 2 \times 1$

$8 = 2 \times 2 \times 2$

$16 = 2 \times 2 \times 2 \times 2$

$10 = 2 \times 5$

$\therefore LCM = 2^4 \times 5 = 80$

$\therefore HCF = 2$

Denominator side,

$3 = 3 \times 1$

$9 = 3 \times 3$

$81 = 3 \times 3 \times 3 \times 3$

$27 = 3 \times 3 \times 3$

$\therefore LCM = 3^4 = 81$

$\therefore HCF = 3$

$\therefore$ LCM of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{80}{3}$

HCF " $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27} = \frac{81}{3}$

7. $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$= \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}$

$= \frac{(1+\sqrt{3}i)^2}{1-(\sqrt{3}i)^2}$

$= \frac{1+2\sqrt{3}i+3i^2}{1-3i^2}$

$= \frac{2\sqrt{3}i+1-3}{4}$

$= \frac{2\sqrt{3}i-2}{4}$

$\Rightarrow \frac{\cancel{2}(\sqrt{3}i-1)}{\cancel{4}2}$

$= \frac{\sqrt{3}i-1}{2}$

$= -\frac{1}{2} + \frac{\sqrt{3}i}{2}$

$\therefore |z| = \sqrt{x^2+y^2}$

$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{1}{4}+\frac{3}{4}}$

$= \sqrt{\frac{1+3}{4}}$

$= \sqrt{\frac{4}{4}} = 1$

$\theta = \pi - \tan^{-1}\left(\frac{y}{x}\right)$

$= \pi - \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right)$

$= \pi - \tan^{-1}(-\sqrt{3})$.

$\cancel{= \pi - \tan^{-1}}$ $\Rightarrow \pi - \frac{\pi}{3}$

$\Rightarrow \frac{3\pi - \pi}{3} = \frac{2\pi}{3}$

$\therefore$ Polar form,

$z = r(\cos\theta + i\sin\theta)$

$= 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$.

8. $\sqrt{-16} \times \sqrt{-4}$

$\Rightarrow 4i \times 2i$

$= 8i^2$

$= -8$

$\frac{\sqrt{-16}}{\sqrt{-4}}$

$= \frac{4i}{2i}$

$= 2$.

9. $8z - z^2$ [where $z = 2+i$].

$\Rightarrow 8(2+i) - (2+i)^2$

$\Rightarrow 16 + 8i - (4 + 4i + i^2)$

$\Rightarrow 16 + 8i - 4 - 4i - i^2$

$= 12 + 4i + i$

$= 13 + 4i$

$\therefore z = \sqrt{13^2 + 4^2}$

$= \sqrt{169 + 16}$

$= \sqrt{185}$

$\theta = \tan^{-1} \frac{y}{x}$

$= \tan^{-1} \frac{4}{13}$.

10. Given,

$1 + i\sqrt{3}$.

$z = \sqrt{1^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= 2$.

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \cancel{\tan^{-1}} \times \cancel{\tan} \frac{\pi}{3}$

$= \frac{\pi}{3}$

$\therefore r(\cos\theta + i\sin\theta)$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 2\cos\frac{\pi}{3} \cancel{+ 2i\sin} + 2i\sin\frac{\pi}{3}$