

MAT 111

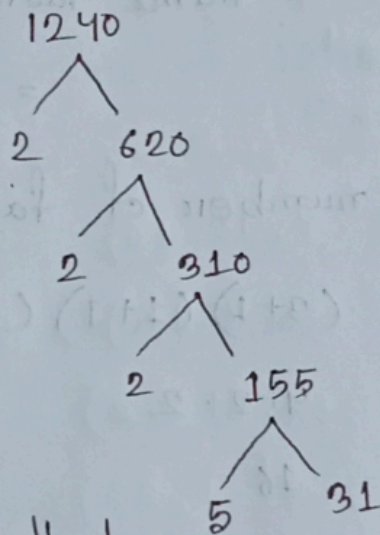
Numbering system.

01.

Division method :-

$$\begin{array}{r} 2 \overline{) 1240} \\ \underline{2 \ 620} \\ 2 \overline{) 310} \\ \underline{2 \ 155} \\ 5 \overline{) 155} \\ \underline{5 \ 31} \\ 31 \end{array}$$

Tree diagram :-

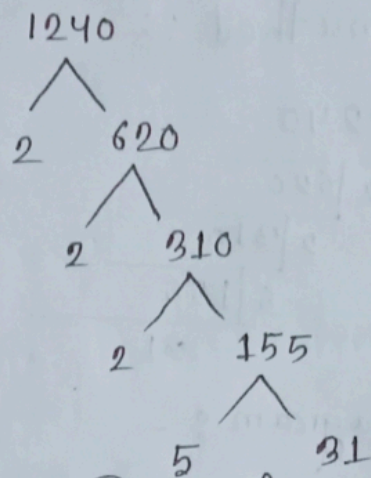


Multiplication method :-

$$\begin{aligned} 1240 &= 2 \times 620 \\ &= 2 \times 2 \times 310 \\ &= 2 \times 2 \times 2 \times 155 \\ &= 2 \times 2 \times 2 \times 5 \times 31 \end{aligned}$$

\therefore Therefore, the factorization of $1240 = 2^3 \cdot 5 \cdot 31$

02.



therefore, the Prime factorization of 1240
 $= 2^3 \cdot 5 \cdot 31$.

the total number of factors are,

$$\begin{aligned} & (3+1)(1+1)(1+1) \\ &= 4 \cdot 2 \cdot 2 \\ &= 16 \end{aligned}$$

Here,

calculation for all factors,

$$\begin{aligned} 1240 &= 1 \times 1240 \\ &= 2 \times 620 \\ &= 4 \times 310 \\ &= 5 \times 248 \\ &= 8 \times 155 \end{aligned}$$

P.

$$= 10 \times 124$$

$$= 20 \times 62$$

$$= 31 \times 40$$

\therefore the factors of 1240 are,

1, 2, 4, 5, 8, 10, 20, 31, 40, 62, 124,
155, 248, 310, 620, 1240.

03. All the Prime factors of 1240 : 2, 5, 31.

04. All the composite factors of 1240 : 4, 8, 10,
20, 40, 62, 124, 155, 248,
310, 620, 1240.

05. $42 = 2 \times 21 = 2 \times 3 \times 7.$

$$63 = 3 \times 21 = 3 \times 3 \times 7.$$

$$140 = 2 \times 70 = 2 \times 2 \times 35 = 2 \times 2 \times 5 \times 7.$$

$$\therefore \text{L.C.M.}(42, 63, 140) = 2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$$

$$\therefore \text{H.C.F.}(42, 63, 140) = 2 \times 7 = 14.$$

06. calculation for numerators,

$$2 = 2^1$$

$$8 = 2^3$$

$$16 = 2^4$$

$$10 = 2^1 \times 5^1$$

calculation for Denominators,

$$3 = 3^1$$

$$9 = 3^2$$

$$81 = 3^4$$

$$27 = 3^3$$

$$\therefore \text{L.C.M} (2, 8, 16, 10) = 2^4 \times 5 = 80$$

$$\therefore \text{H.C.F} (2, 8, 16, 10) = 2$$

$$\therefore \text{L.C.M} (3, 9, 81, 27) = 3^4 = 81$$

$$\therefore \text{H.C.F} (3, 9, 81, 27) = 3$$

Here,

$$\text{H.C.F of } \frac{2}{3}, \frac{8}{9}, \frac{16}{81}, \frac{10}{27}$$

$$= \frac{\text{H.C.F} (2, 8, 16, 10)}{\text{L.C.M} (3, 9, 81, 27)}$$

$$= \frac{2}{81}$$

L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$, $\frac{10}{27}$

$$= \frac{\text{LCM}(2, 8, 16, 10)}{\text{HCF}(3, 9, 81, 27)}$$

$$= \frac{80}{3}$$

Q7.

we have,

$$\begin{aligned} & \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \\ &= \frac{(1 + \sqrt{3}i)(1 + \sqrt{3}i)}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)} \\ &= \frac{1 + 2\sqrt{3}i - 3}{1 - (\sqrt{3}i)^2} \\ &= \frac{-2 + 2\sqrt{3}i}{1 + 3} \\ &= \frac{2(-1 + \sqrt{3}i)}{4} \\ &= -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{aligned}$$

$$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$|z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{\frac{4}{4}}$$

$$= 1$$

\therefore Modulus of $z = 1$

\therefore Argument of z

$$\theta = \pi - \tan^{-1} \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{2\pi}{3}$$

Polar form $\frac{1}{2} + \frac{\sqrt{3}}{2}i$.

Exponential form is $z = re^{i\theta}$
 $= 1 \cdot e^{i2\pi/3}$
 $= e^{2\pi/3 i}$.

08.

We have,

$$\sqrt{-16} \times \sqrt{-4}$$

$$= \sqrt{16i} \times \sqrt{4i}$$

$$= 4i \times 2i$$

$$= 8i^2$$

$$= -8,$$

and,

$$\frac{\sqrt{-16}}{\sqrt{-4}}$$

$$= \frac{4i}{2i}$$

$$= 2.$$

09. we have,

$$z = 2 + i.$$

$$\begin{aligned}\therefore 8z - z^4 &= 8(2+i) - (2+i)^4 \\ &= 16 + 8i - (4 + 4i + i^4) \\ &= 16 + 8i - 4 - 4i - i^4 \\ &= 16 + 8i - 4 - 4i + 1 \\ &= 13 + 4i.\end{aligned}$$

$$\text{Modulus } r = \sqrt{(13)^2 + (4)^2}$$

$$= \sqrt{169 + 16}$$

$$= \sqrt{185}$$

$$\theta = \tan^{-1} \frac{4}{13}$$

$$= 17.102.$$

10.

$$z = 1 + i\sqrt{3}, \quad z = x + iy, \quad |z| = \sqrt{x^2 + y^2}$$

$$\text{modulus of } z = \sqrt{(1)^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$\therefore r = 2.$$

$$\text{Argument of } z = \tan^{-1} \left(\frac{y}{x} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{3}}{1} \right)$$

$$= \tan^{-1} \tan \frac{\pi}{3}$$

$$= \frac{\pi}{3}$$

\therefore therefore, $r(\cos \theta + i \sin \theta)$ form is

$$2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$