Chapter 4: Measures of Dispersions

Part-1

For ungrouped data

Learning Outcomes

After Completing the chapter ,you will able to :

- Necessity of measures of Dispersion.
- What is measures of Dispersion?
  and it’s purpose.
- Different types of measures of Dispersion and their application.
- Their uses and Limitations.

Contents

From this lecture, you are going to learn…

- What is dispersion?
- Types of measures of dispersion.
- Discussion on Range, Mean deviation, Population variance and standard deviation.
- Examples, Uses and limitations

What is measures of Dispersion?

Dispersion measures the spread or variability of a set of observations among themselves or about some central values.

Small Dispersion/ variation	→	High uniformity

Example: **Group-1**
Marks of 4 students out of 100.
50, 49, 51, 50.
Mean = 50

Large Dispersion/ variation	→	Low uniformity

Example: **Group-2**
Marks of 4 students out of 100.
100, 100, 0, 0.
Mean = 50

Types of measures of dispersion

Measures of Dispersion

Absolute measures

1. Range
2. Mean Deviation
3. Variance
4. Standard deviation($\sqrt{Variance}$)
5. Quartile deviation

Relative measures

1. Coefficient of Range
2. Coefficient of Mean Deviation
3. Coefficient of Variation(c.v.)
4. Coefficient of Quartile deviation

Purpose of Studying Dispersion

Purposes of measures of dispersions:

- To measure the spread of the data set.
- To determine the reliability of an average.
- To compare two or more data sets according to their variability.

Range

1. Range: Simplest measure of dispersion is the range.

Range = Largest value – Smallest value

Example: suppose the marks of 8 students in a class are: 65, 20,55,80,42,35,77,68.
Calculate Range.
Solution:

$$R = X_{\max} - X_{\min}$$

$$R = 80 - 20 = 60$$

Limitation:
Range cannot tell us anything about the character of the distribution within two extreme observations

Example of Range

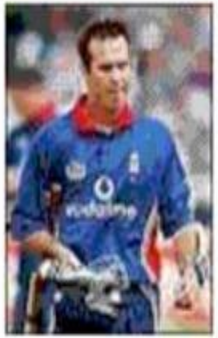
Average run of Batsman A =36.73

The variation of the run of Batsman A = 86-10=76

20	35	22	55	60
10	17	32	64	86
14	32	50	24	30

Average run of Batsman B = 43.4

The variation of the run of Batsman B = 370-0=370



0	0	0	2	0
15	5	370	250	
0	3	5	0	1

Mean deviation

Mean Deviation: The arithmetic mean of the absolute values of the deviations from the arithmetic mean.

Mean deviation is obtained by calculating the absolute deviations of each observation from mean and then averaging these deviations by taking arithmetic mean.

Formula:

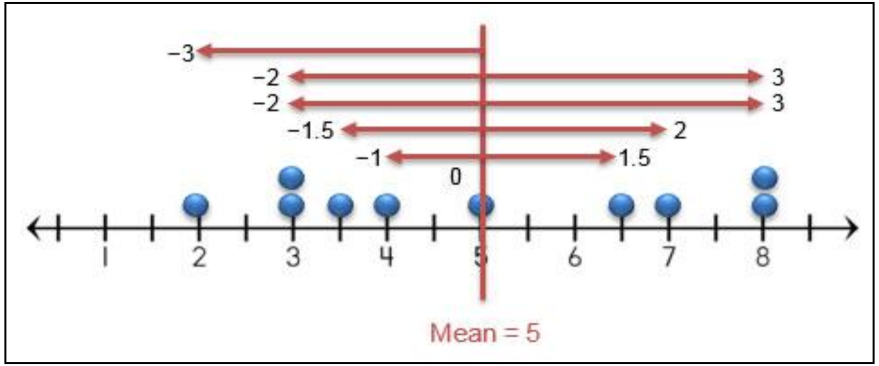
$$M.D. = \frac{\sum_{i=1}^{n}|X_i - \bar{X}|}{n}$$

$$= \frac{|X_1 - \bar{X}| + |X_2 - \bar{X}| + \cdots + |X_n - \bar{X}|}{n}$$

Example of Mean deviation

Example:

Find the Mean Deviation of data values are 2, 3, 3, 3.5, 4, 5, 6.5, 7, 8, 8.



Now, mean, $\bar{X} = \frac{2+3+3+3.5+4+5+6.5+7+8+8}{10}$

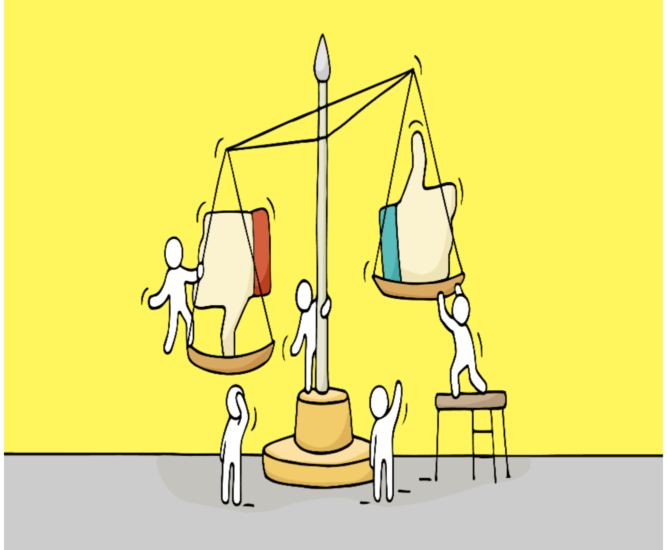
=5

X_i	$X_i - \bar{X}$	$\lvert X_i - \bar{X}\rvert$
2	-3	3
3	-2	2
3	-2	2
3.5	-1.5	1.5
4	-1	1
5	0	0
6.5	1.5	1.5
7	2	2
8	3	3
8	3	3
Total		$\sum_{i=1}^{n}\lvert X_i - \bar{X}\rvert = 19$

$$\therefore \text{M.D.} = \frac{\sum_{i=1}^{n}\lvert X_i - \bar{X}\rvert}{n}$$

$$= \frac{19}{10} = 1.9.$$

Merits and limitations of Mean Deviation



Merits:

Less affected by the values of extreme observation.

limitations:

The greatest limitation of this method is that algebraic sings are ignored while taking the deviations of the items.

Population Variance and Standard Deviation

Population variance,

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

$$= \frac{(X_1 - \mu)^2 + (X_2 - \mu)^2 + \cdots \ldots + (X_N - \mu)^2}{N}$$

Where $X_1, X_2, \ldots\ldots X_N$ are Population observation

N= Population size.

μ = Population mean

∴ Population Standard deviation,

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}}$$

$$= \sqrt{\frac{(X_1 - \mu)^2 + (X_2 - \mu)^2 + \cdots \ldots + (X_N - \mu)^2}{N}}$$

Example of Population Variance and Standard Deviation

Example: Calculate the population variance and standard deviation for the data set 1, 2, 2, 3, 4, 5.

Solution:

Population variance,

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

Where, Population Mean, $\mu = \frac{\sum_{i=1}^{N} X_i}{N}$

$$= \frac{1+2+2+3+4+5}{6}$$

$= 2.83$

$$\therefore \sigma^2 = \frac{(1-2.83)^2+(2-2.83)^2+\cdots\ldots+(5-2.83)^2}{6}$$

$$= \frac{10.84}{6} = 1.81$$

$\therefore$ Population standard deviation, $\sigma = \sqrt{1.81} = 1.35$

Sample Variance and Standard Deviation

Sample variance, $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$

$$= \frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \cdots \ldots + (X_n - \bar{X})^2}{n-1}$$

$\therefore$ **Sample Standard deviation,**

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

$$= \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \cdots \ldots + (X_n - \bar{X})^2}{n-1}}$$

Where $X_1, X_2, \ldots.. X_N$ are Population observation
N= Population size.
$\bar{X}$= Sample Mean.

Why Sample standard deviation?
When dealing with large population and need to draw conclusion based on sample we use sample standard deviation .

Why use n-1?
To make the estimator unbiased.

Example of Sample Variance and Standard Deviation

Example: Calculate the Sample variance and standard deviation for the data set 1, 2, 2, 3, 4, 5.

Solution:

Sample variance, $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$

Where, Sample Mean, $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

$= \frac{1+2+2+3+4+5}{6}$

$= 2.83$

$\therefore S^2 = \frac{(1-2.83)^2 + (2-2.83)^2 + \cdots \ldots + (5-2.83)^2}{6-1}$

$= \frac{10.84}{6-1} = 2.87$

$\therefore$ Sample standard deviation, $\sigma = \sqrt{2.87} = 1.69$

Exercise

1: Set A: 18, 25, 10, 12.   Set B: 32, 30, 20, 10.
Find population standard deviation and compare the variability between these two data sets.

**Hints: Find Standard deviation for both the data. Then the data set with lower standard deviation will have higher uniformity.

2. **Find the (a) Range, (b) Mean deviation (c) Variance and (d) Standard deviation (e) Sample variance and Sample Standard deviation.**

12	36	40	16	10
19	27	15	21	33
26	37	6	5	20

3. **Find the (a) Range, (b) Mean deviation (c) Variance and (d) Standard deviation (e) Sample variance and Sample Standard deviation.**

10	19	20	28	30
45	7	19	20	26
30	37	17	11	20

4. **Find the (a) Range, (b) Mean deviation (c) Variance and (d) Standard deviation (e) Sample variance and Sample Standard deviation.**

30	42	30	54	40
30	27	42	36	28
36	22	30	31	19

5. **Find the (a) Range, (b) Mean deviation (c) Variance and (d) Standard deviation (e) Sample variance and Sample Standard deviation.**

48	15	17	51	42
26	37	54	44	31
48	16	42	32	21


Thank you