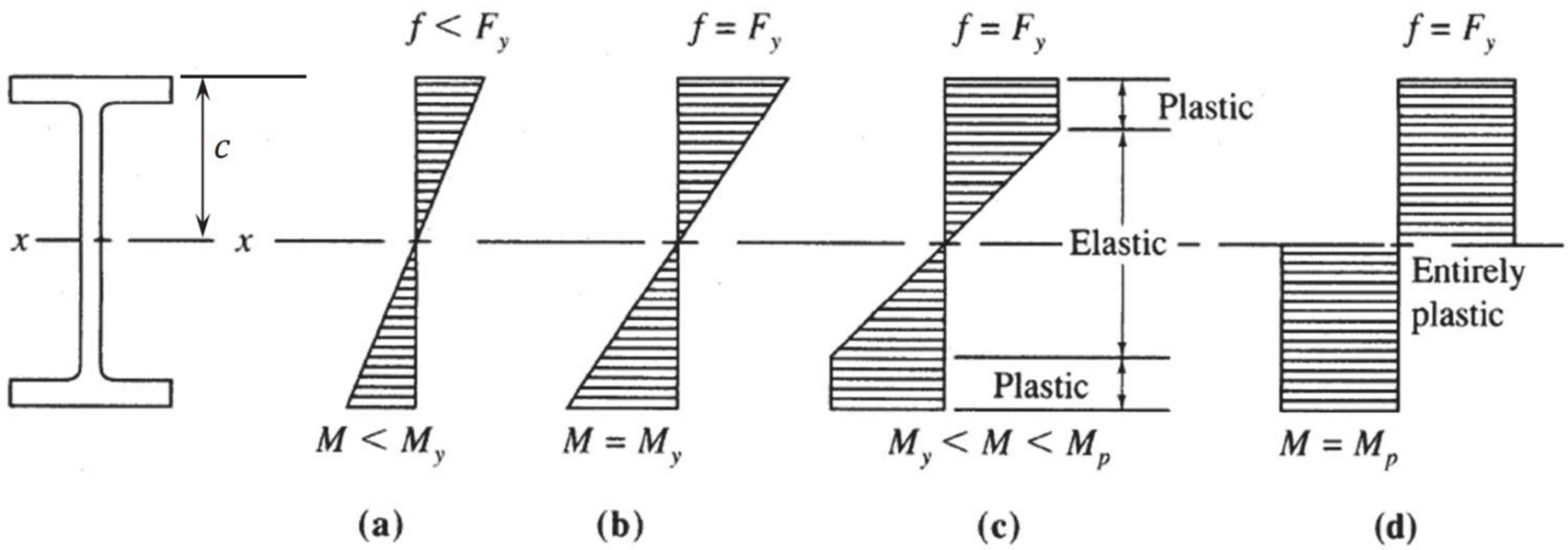
CE 415: Design of Steel Structures

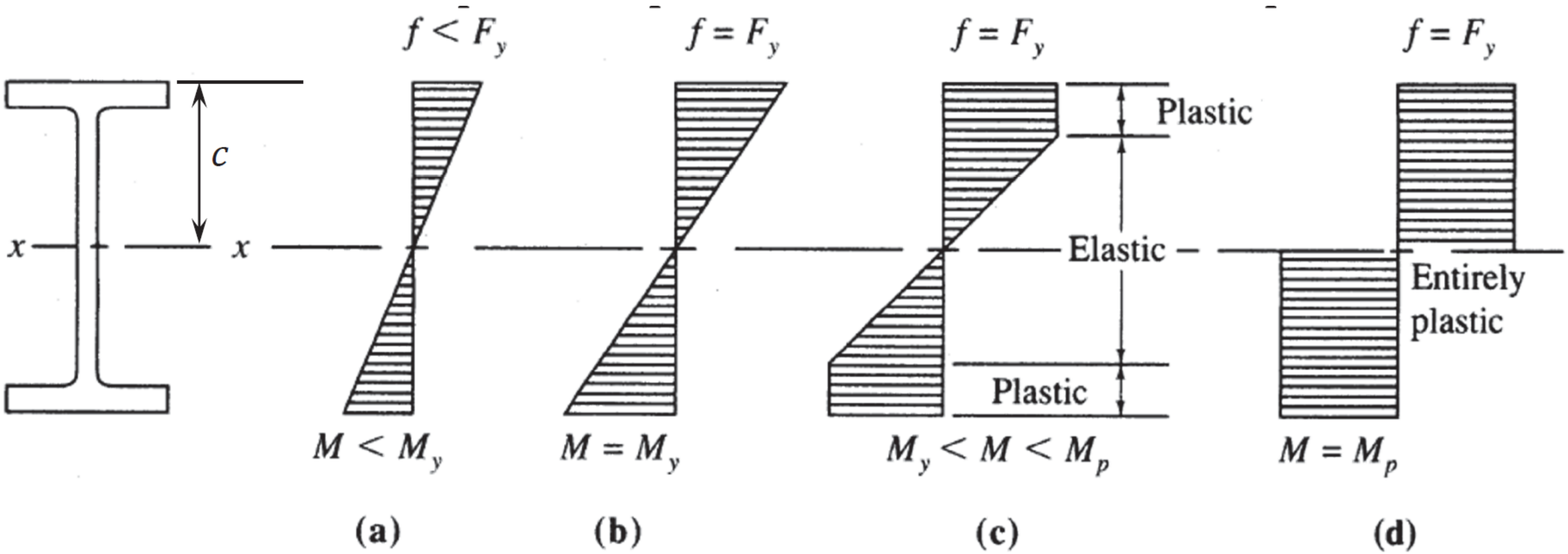
Flexural Member



When the yield stress is reached at the extreme fiber [Fig. (b)], the nominal moment strength M_n is referred to as the yield moment M_y and is computed as

$$M_n = M_y = S_x F_y$$

Where S_x = section modulus = I_x/c



When the condition of Fig. (d) is reached, every fiber has a strain equal to or greater than $\varepsilon_y = F_y/E_s$ i.e., it is in the plastic range. The nominal moment strength M_n is therefore referred to as the plastic moment M_p, and is computed as.

$$M_p = F_y \int_A y\, dA = F_y Z$$

$$Z = \int y\, dA$$ → **Plastic section modulus**

NOMINAL MOMENT CAPACITY OF LATERALLY SUPPORTED BEAMS

Compact Sections

The nominal strength M_n for laterally stable "compact sections" according to AISC may be stated,

$$M_n = M_p = F_y Z_x$$

Where, M_p = Plastic moment capacity

Z_x = Plastic section modulus

F_y = Specified minimum yield stress.

In order to develop full plastic moment, the b/t ratio ($b=b_f/2$) for flange must be smaller than the limit λ_p defined by AISC.

Local buckling in hot-rolled I-shaped sections is, for practical purposes, only possible in the flanges.

Partially Compact Sections

The nominal strength M_n for laterally stable "noncompact sections" whose flange width/thickness ratios λ are less than λ_r but not as low as λ_p must be linearly interpolated between M_p and M_r = 0.7 $F_y S_x$

$$M_n = M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)$$

where $\lambda = b_f/2t_f$ for I-shaped member flanges

b_f = flange width

t_f = flange thickness

λ_{pf} = compact limit for reaching M_p (AISC-Table B4.1)

λ_{rf} = noncompact limit for reaching M_r (AISC-Table B4.1)

Slender Flange Sections

When the width/thickness ratio λ [$=b_f/(2t_f)$] exceeds the limit λ_r of AISC-B4, the section is referred to as "slender" and must be treated in accordance with AISC-F3.2(b). The nominal strength of such a section is

$$M_n = \frac{0.9 E k_c S_x}{\lambda^2}$$ [Eq. F3-2, page 49, AISC 360-05]

$$k_c = \frac{4}{\sqrt{h/t_w}}, \text{ where } 0.35 \le k_c \le 0.763$$

LATERALLY SUPPORTED BEAMS: LRFD Design

The strength requirement for beams in load and resistance factor design according to AISC-F1 may be stated

$$\phi_b M_n \geq M_u$$

where ϕ_b = resistance (i.e., strength reduction) factor for flexure = 0.90

M_n = nominal moment strength

M_u = factored service load moment

LATERALLY SUPPORTED BEAMS: ASD Design

The strength requirement for beams in allowable strength design according to AISC-FI may be stated

$$\frac{M_n}{\Omega_b} \geq M_a$$

where M_a = required strength, which equals the service load moment

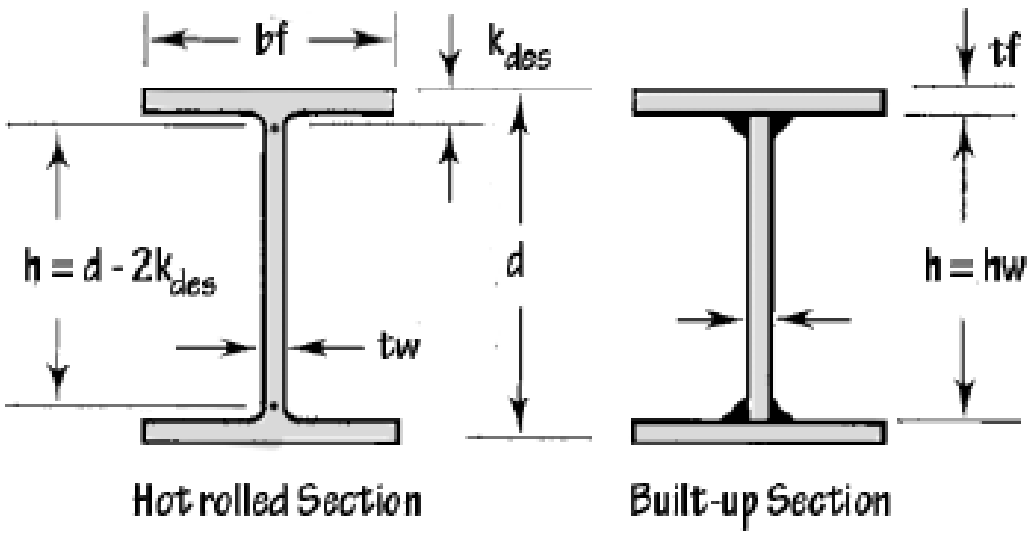
M_n/Ω_b = allowable flexural strength

M_n = nominal flexural strength,

Ω_b = safety factor equal to 1.67 according to AISC-F1

AISC classifies cross-sectional shapes in following three categories based on width-to-thickness ratio (λ).

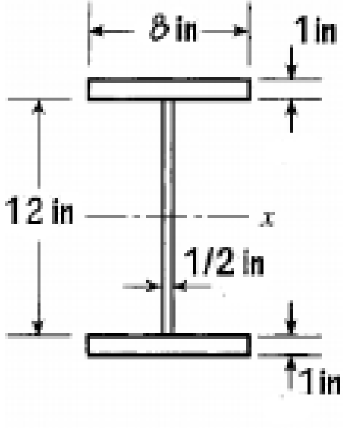
- *Compact*, if $\lambda < \lambda_p$. The section can fully utilize its material strength (plastic moment) and there is no local buckling
- *Noncompact*, if $\lambda_p < \lambda < \lambda_r$. The section cannot fully utilize its material strength. Buckling may occur inelastically or elastically before reaching to plastic moment
- *Slender*, if $\lambda_r < \lambda$. The section definately reaches elastic buckling prior to plastic moment.



Element	λ	λ_p	λ_r
Flange	$\frac{b_f}{2t_f}$	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$
Web	$\frac{h}{t_w}$	$3.76\sqrt{\frac{E}{F_y}}$	$5.7\sqrt{\frac{E}{F_y}}$

b_f	Flange width
t_f	Flange thickness
h_w	Web height
t_w	Web thickness
d	Total depth
h	Unstiffened web height
k_{des}	Distance from outer surface of flange to depth of fillet radius

Ques. Investigate the local stability of the following section.



Flange Buckling Check

$$\lambda = \frac{b_f}{2t_f} = \frac{8}{2\times 1} = 4$$

$$\lambda_p = 0.38\sqrt{E/F_y} = 0.38\sqrt{29000/50} = 9.15$$

Since $\lambda(4) < \lambda_p(9.15)$, flange is compact.

Web Buckling Check

$$\lambda = \frac{h}{t_w} = \frac{12}{0.5} = 24$$

$$\lambda_p = 3.76\sqrt{E/F_y} = 3.76\sqrt{29000/50} = 90.6$$

Since $\lambda(24) < \lambda_p(90.6)$, web is also compact.

Ans. Section is compact.

Ques. Investigate the local stability of section W14 × 90

Solution.
From Table 1-1 of AISC Manual, we find,

Section	b_f	t_f	d	k_{des}	t_w
W14 × 90	14.5	0.71	14	1.31	0.44

Flange Buckling Check

$$\lambda = \frac{b_f}{2t_f} = \frac{14.5}{2\times 0.71} = 10.2$$

$$\lambda_p = 0.38\sqrt{E/F_y} = 0.38\sqrt{29000/50} = 9.15$$

$$\lambda_r = 1.00\sqrt{E/F_y} = 0.38\sqrt{29000/50} = 24.1$$

Since $\lambda_p(9.15) < \lambda(10.2) < \lambda_r(24.1)$, flange is noncompact.

Web Buckling Check

$$\lambda = \frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{14 - 2\times 1.31}{0.44} = 25.86$$

$$\lambda_p = 3.76\sqrt{E/F_y} = 3.76\sqrt{29000/50} = 90.6$$

Since $\lambda(25.8) < \lambda_p(90.6)$, web is compact.

Ans. Section is noncompact (flange governs).

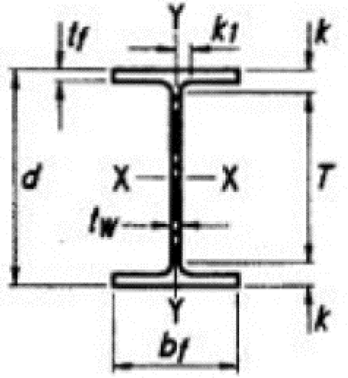
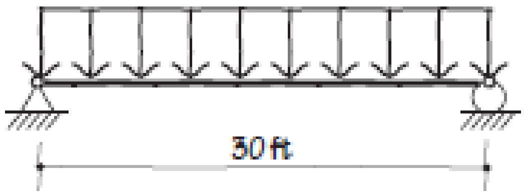


Table 1–1 (continued)
W Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	Distance k k_{det}	Distance k_1	Distance T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W14×132	38.8	14.7	14 5/8	0.645	5/8	5/16	14.7	14 3/4	1.03	1	1.63	2 5/16	1 9/16	10	5 1/2
×120	35.3	14.5	14 1/2	0.590	9/16	5/16	14.7	14 5/8	0.940	15/16	1.54	2 1/4	1 1/2	↓	↓
×109	32.0	14.3	14 3/8	0.525	1/2	1/4	14.6	14 5/8	0.860	7/8	1.46	2 3/16	1 1/2	↓	↓
×99[f]	29.1	14.2	14 1/8	0.485	1/2	1/4	14.6	14 5/8	0.780	3/4	1.38	2 1/16	1 7/16	↓	↓
×90[f]	26.5	14.0	14	0.440	7/16	1/4	14.5	14 1/2	0.710	11/16	1.31	2	1 7/16	↓	↓
W14×82	24.0	14.3	14 1/4	0.510	1/2	1/4	10.1	10 1/8	0.855	7/8	1.45	1 11/16	1 1/16	10 7/8	5 1/2
×74	21.8	14.2	14 1/8	0.450	7/16	1/4	10.1	10 1/8	0.785	13/16	1.38	1 5/8	1 1/16	↓	↓
×68	20.0	14.0	14	0.415	7/16	1/4	10.0	10	0.720	3/4	1.31	1 9/16	1 1/16	↓	↓
×61	17.9	13.9	13 7/8	0.375	3/8	3/16	10.0	10	0.645	5/8	1.24	1 1/2	1	↓	↓
W14×53	15.6	13.9	13 7/8	0.370	3/8	3/16	8.06	8	0.660	11/16	1.25	1 1/2	1	10 7/8	5 1/2
×48	14.1	13.8	13 3/4	0.340	5/16	3/16	8.03	8	0.595	5/8	1.19	1 7/16	1	↓	↓
×43[c]	12.6	13.7	13 5/8	0.305	5/16	3/16	8.00	8	0.530	1/2	1.12	1 3/8	1	↓	↓

Problem. The following beam is W 16×31 of A992 steel. It supports a reinforced concrete floor slab that provides continuous lateral support of compression flange. The service dead load is 450 lb/ft and service live load is 550 lb/ft. Does the beam has adequate moment strength?



Flange Check

From Table 1-1, $b_f = 5.53$ in, $t_f = 0.44$ in.

$$\lambda = \frac{b_f}{2t_f} = \frac{5.53}{2\times0.44} = 6.28$$

$$\lambda_p = 0.38\sqrt{E/F_y} = 0.38\sqrt{29000/50} = 9.15 > 6.28$$

Since, $\lambda < \lambda_p$ for flange, there is no local buckling in flange.

Web Check

From Table 1-1, $d = 15.9$ in, $k_{des} = 0.842$ in and $t_w = 0.275$ in

$$\lambda = \frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{15.9 - 2\times0.842}{0.275} = 51.7$$

$$\lambda_p = 3.76\sqrt{E/F_y} = 3.76\sqrt{29000/50} = 90.5 > 51.7$$

Since, $\lambda < \lambda_p$ for web, there is no local buckling in web either.

Determine Capacity

Since, both flange and web have no local buckling, the section can reach up to plastic moment before failure.
From Table 1-1, $Z_x = 54$ in^3.

$$M_p = F_y Z_x = 50\times54 = 2700 \text{ k-in}$$
$$\phi_b M_n = \phi_b M_p = 0.9\times2700 \text{ k-in} = 2430 \text{ k-in} 202.5 \text{ k-ft}$$

Determine Demand

The dead load should be increased by self weight (31 lb/ft) of the beam since given dead load (450 lb/ft) is excluded of self weight.

$$w_D = 450 + 31 = 481 \text{ lb/ft}$$
$$w_L = 550 \text{ lb/ft}$$
$$w_u = 1.2w_D + 1.6w_L$$
$$= 1.2\times481 + 1.6\times550 = 1457 \text{ lb/ft} = 1.46 \text{ k/ft}$$

$$M_u = \frac{w_u L^2}{8} = \frac{1.46\times30^2}{8} = 164.3 \text{ k-ft} < \phi_b M_n$$

Since, $\phi_b M_n(202.5$ k-ft$) > M_u(164.3$ k-ft$)$, the section W 16×31 has adequate moment capacity.

Ans. Yes. The beam has adequate moment strength.

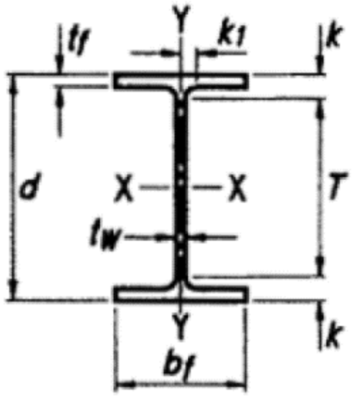
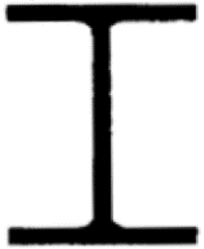


Table 1–1 (continued)
W Shapes
Dimensions

Shape	Area, A	Depth, d		Web: Thickness, t_w		Web: $\frac{t_w}{2}$	Flange: Width, b_f		Flange: Thickness, t_f		Distance: k: k_{des}	Distance: k: k_{det}	Distance: k_1	Distance: T	Distance: Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W16×100	29.5	17.0	17	0.585	9/16	5/16	10.4	10 3/8	0.985	1	1.39	1 7/8	1 1/8	13 1/4	5 1/2
×89	26.2	16.8	16 3/4	0.525	1/2	1/4	10.4	10 3/8	0.875	7/8	1.28	1 3/4	1 1/16	↓	↓
×77	22.6	16.5	16 1/2	0.455	7/16	1/4	10.3	10 1/4	0.760	3/4	1.16	1 5/8	1 1/16		
×67[c]	19.7	16.3	16 3/8	0.395	3/8	3/16	10.2	10 1/4	0.665	11/16	1.07	1 9/16	1		
W16×57	16.8	16.4	16 3/8	0.430	7/16	1/4	7.12	7 1/8	0.715	11/16	1.12	1 3/8	7/8	13 5/8	3 1/2[g]
×50[c]	14.7	16.3	16 1/4	0.380	3/8	3/16	7.07	7 1/8	0.630	5/8	1.03	1 5/16	13/16	↓	↓
×45[c]	13.3	16.1	16 1/8	0.345	3/8	3/16	7.04	7	0.565	9/16	0.967	1 1/4	13/16		
×40[c]	11.8	16.0	16	0.305	5/16	3/16	7.00	7	0.505	1/2	0.907	1 3/16	13/16		
×36[c]	10.6	15.9	15 7/8	0.295	5/16	3/16	6.99	7	0.430	7/16	0.832	1 1/8	3/4		
W16×31[c]	9.13	15.9	15 7/8	0.275	1/4	1/8	5.53	5 1/2	0.440	7/16	0.842	1 1/8	3/4	13 5/8	3 1/2
×26[c,v]	7.68	15.7	15 3/4	0.250	1/4	1/8	5.50	5 1/2	0.345	3/8	0.747	1 1/16	3/4	13 5/8	3 1/2
W14×730[h]	215	22.4	22 3/8	3.07	3 1/16	1 9/16	17.9	17 7/8	4.91	4 15/16	5.51	6 3/16	2 3/4	10	3-7 1/2-3[g]
×665[h]	196	21.6	21 5/8	2.83	2 13/16	1 7/16	17.7	17 5/8	4.52	4 1/2	5.12	5 13/16	2 5/8	|	3-7 1/2-3[g]

Table 1–1 (continued)
W Shapes
Properties



W16 – W14

Nom-inal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r_{ts}	h_o	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z	I	S	r	Z			J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
100	5.29	24.3	1490	175	7.10	198	186	35.7	2.51	54.9	2.92	16.0	7.73	11900
89	5.92	27.0	1300	155	7.05	175	163	31.4	2.49	48.1	2.88	15.9	5.45	10200
77	6.77	31.2	1110	134	7.00	150	138	26.9	2.47	41.1	2.85	15.8	3.57	8590
67	7.70	35.9	954	117	6.96	130	119	23.2	2.46	35.5	2.82	15.7	2.39	7300
57	4.98	33.0	758	92.2	6.72	105	43.1	12.1	1.60	18.9	1.92	15.7	2.22	2660
50	5.61	37.4	659	81.0	6.68	92.0	37.2	10.5	1.59	16.3	1.89	15.6	1.52	2270
45	6.23	41.1	586	72.7	6.65	82.3	32.8	9.34	1.57	14.5	1.88	15.6	1.11	1990
40	6.93	46.5	518	64.7	6.63	73.0	28.9	8.25	1.57	12.7	1.86	15.5	0.794	1730
36	8.12	48.1	448	56.5	6.51	64.0	24.5	7.00	1.52	10.8	1.83	15.4	0.545	1460
31	6.28	51.6	375	47.2	6.41	54.0	12.4	4.49	1.17	7.03	1.42	15.4	0.461	739
26	7.97	56.8	301	38.4	6.26	44.2	9.59	3.49	1.12	5.48	1.38	15.3	0.262	565
730	1.82	3.71	14300	1280	8.17	1660	4720	527	4.69	816	5.68	17.5	1450	362000
665	1.95	4.03	12400	1150	7.98	1480	4170	472	4.62	730	5.57	17.1	1120	305000

Problem. The beam shown in following figure supports a reinforced concrete floor slab that provides continuous lateral support of compression flange. Does the beam has adequate moment strength?

Flange Check
From Table 1-1, $b_f = 14.5$ in, $t_f = 0.71$ in.

$$\lambda = \frac{b_f}{2t_f} = \frac{14.5}{2\times 0.71} = 10.21$$

$$\lambda_p = 0.38\sqrt{E/F_y} = 0.38\sqrt{29000/50} = 9.15 < 10.21$$

$$\lambda_r = 1.0\sqrt{E/F_y} = 1.0\sqrt{29000/50} = 24.08 > 10.21$$

Since, $\lambda_p < \lambda < \lambda_r$, flange is non-compact.

Web Check
From Table 1-1, $d = 14$ in, $k_{des} = 1.31$ in and $t_w = 0.44$ in

$$\lambda = \frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{14 - 2\times 1.31}{0.44} = 25.8$$

$$\lambda_p = 3.76\sqrt{E/F_y} = 3.76\sqrt{29000/50} = 90.5 > 25.8$$

Since, $\lambda < \lambda_p$, web is compact.

The shape is therefore non-compact.

Determine Capacity
Since, flange has local buckling, the section cannot reach up to plastic moment before failure.

From Table 1-1, $S_x = 143$ in^3, $Z_x = 157$ in^3.

$$M_p = F_y Z_x = 50\times 157 = 7850 \text{ k-in}$$

$$\phi_b M_n = \phi_b \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right]$$

$$= 0.9\times \left[7850 - (7850 - 0.7\times 50\times 143)\left(\frac{10.2 - 9.15}{24.08 - 9.15}\right)\right]$$

$$= 7650 \text{ k-in} = 637.5 \text{ k-ft}$$

Determine Demand
The dead load should be increased by self weight (90 lb/ft) of the beam since given dead load (600 lb/ft) is excluded of self weight.

$$w_D = 600 + 90 = 690 \text{ lb/ft}$$

$$w_L = 1200 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L$$

$$= 1.2\times 690 + 1.6\times 1200 = 2748 \text{ lb/ft} = 2.75 \text{ k/ft}$$

$$M_u = \frac{w_u L^2}{8} = \frac{2.75\times 45^2}{8} = 696.1 \text{ k-ft} > \phi_b M_n$$

Since, $\phi_b M_n (637.5 \text{ k-ft}) < M_u (696.1 \text{ k-ft})$, the section W14×90 has not adequate moment capacity.

Answer. No. The beam has not adequate moment strength.

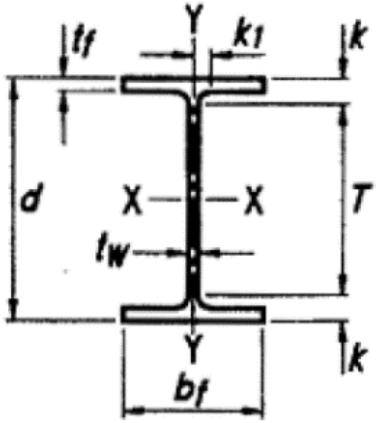
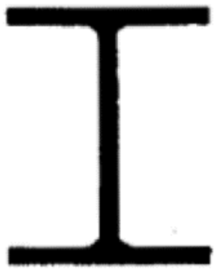


Table 1–1 (continued)
W Shapes
Dimensions

Shape	Area, A	Depth, d		Web: Thickness, t_w		Web: $\frac{t_w}{2}$	Flange: Width, b_f		Flange: Thickness, t_f		Distance: k: k_{des}	Distance: k: k_{det}	Distance: k_1	Distance: T	Distance: Workable Gage
	in.²	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W14×132	38.8	14.7	14 5/8	0.645	5/8	5/16	14.7	14 3/4	1.03	1	1.63	2 5/16	1 9/16	10	5 1/2
×120	35.3	14.5	14 1/2	0.590	9/16	5/16	14.7	14 5/8	0.940	15/16	1.54	2 1/4	1 1/2	↓	↓
×109	32.0	14.3	14 3/8	0.525	1/2	1/4	14.6	14 5/8	0.860	7/8	1.46	2 3/16	1 1/2		
×99[f]	29.1	14.2	14 1/8	0.485	1/2	1/4	14.6	14 5/8	0.780	3/4	1.38	2 1/16	1 7/16		
×90[f]	26.5	14.0	14	0.440	7/16	1/4	14.5	14 1/2	0.710	11/16	1.31	2	1 7/16		
W14×82	24.0	14.3	14 1/4	0.510	1/2	1/4	10.1	10 1/8	0.855	7/8	1.45	1 11/16	1 1/16	10 7/8	5 1/2
×74	21.8	14.2	14 1/8	0.450	7/16	1/4	10.1	10 1/8	0.785	13/16	1.38	1 5/8	1 1/16	↓	↓
×68	20.0	14.0	14	0.415	7/16	1/4	10.0	10	0.720	3/4	1.31	1 9/16	1 1/16		
×61	17.9	13.9	13 7/8	0.375	3/8	3/16	10.0	10	0.645	5/8	1.24	1 1/2	1		
W14×53	15.6	13.9	13 7/8	0.370	3/8	3/16	8.06	8	0.660	11/16	1.25	1 1/2	1	10 7/8	5 1/2
×48	14.1	13.8	13 3/4	0.340	5/16	3/16	8.03	8	0.595	5/8	1.19	1 7/16	1	↓	↓
×43[c]	12.6	13.7	13 5/8	0.305	5/16	3/16	8.00	8	0.530	1/2	1.12	1 3/8	1		

Table 1–1 (continued)
W Shapes
Properties



W14 – W12

Nominal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r_{ts}	h_o	Torsional Properties	
	$\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	I	S	r	Z	I	S	r	Z			J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
132	7.15	17.7	1530	209	6.28	234	548	74.5	3.76	113	4.23	13.6	12.3	25500
120	7.80	19.3	1380	190	6.24	212	495	67.5	3.74	102	4.20	13.5	9.37	22700
109	8.49	21.7	1240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.5	7.12	20200
99	9.34	23.5	1110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18000
90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.11	13.3	4.06	16000
82	5.92	22.4	881	123	6.05	139	148	29.3	2.48	44.8	2.85	13.5	5.07	6710
74	6.41	25.4	795	112	6.04	126	134	26.6	2.48	40.5	2.82	13.4	3.87	5990
68	6.97	27.5	722	103	6.01	115	121	24.2	2.46	36.9	2.80	13.3	3.01	5380
61	7.75	30.4	640	92.1	5.98	102	107	21.5	2.45	32.8	2.78	13.2	2.19	4710
53	6.11	30.9	541	77.8	5.89	87.1	57.7	14.3	1.92	22.0	2.22	13.3	1.94	2540
48	6.75	33.6	484	70.2	5.85	78.4	51.4	12.8	1.91	19.6	2.20	13.2	1.45	2240
43	7.54	37.4	428	62.6	5.82	69.6	45.2	11.3	1.89	17.3	2.18	13.1	1.05	1950