CE 415
DESIGN OF STEEL STRUCTURES
LECTURE 17
FLEXURAL MEMBER (CONT.)

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OUTLINE

- Deflection of beam
- Allowable stress of beam
- Brace support
- Light weight section selection

DEFLECTION OF BEAM

5-7 DESIGN FOR LIMITED DEFLECTION

Although a beam is unsuitable if it cannot support its loads without excessive deflection, it is not easy to set a dividing line between reasonable and unreasonable deflection. Excessive deflection in floor construction is objectionable not only because of the feeling of softness but also because of undesirable vibration characteristics and the possibility of damage to attached construction such as plaster. Excessive deflection in floor construction supporting machinery may result in misalignments as well as dangerous vibration. Excessive deflection in purlins may cause damage to roofing materials and, on flat roofs, accumulation of water during rainstorms which, under certain conditions, can cause collapse. Retention of water due to the deflection of flat-roof framing is called *ponding*.

The maximum deflection Δ of a simply supported beam uniformly loaded in a principal plane is given by

$$\Delta = \frac{5}{384}\frac{WL^3}{EI} \qquad (a)$$

where W denotes the total load on the span. But since the maximum bending moment $M = WL/8$, we may eliminate W from Eq. (a) to get

$$\Delta = \frac{5}{48}\frac{ML^2}{EI} \qquad (5\text{-}11a)$$

Substituting $M/I = f/c = f/(d/2)$ into Eq. (5-11a) gives

$$\Delta = \frac{5}{24}\frac{fL^2}{Ed} \qquad (5\text{-}11b)$$

ings. The source of this rule seems to be unknown. Presumably it was given originally as a safe limit with respect to cracking of plastered ceilings. This deflection limit is a requirement of the AISC/ASD specification. AASHTO limits deflection due to live load plus impact to not more than $\frac{1}{800}$ of the span, while AREA limits it to $\frac{1}{640}$.

The ratio L/d of beam span to beam depth which corresponds to a specific ratio Δ/L of deflection to span can be determined from Eq. (5-11*b*):

$$\frac{L}{d} = \frac{24}{5} \frac{E}{f} \frac{\Delta}{L} \tag{b}$$

If we wish to limit deflection to, say, $\frac{1}{300}$ of the span for steel beams designed for $f = 0.6F_y$, we find from Eq. (*b*) that

$$\frac{L}{d} = \frac{24}{5} \frac{30{,}000}{0.6F_y} \frac{1}{300} = \frac{800}{F_y} \tag{5-12}$$

The commentary to the AISC/ASD specification suggests this value of L/d as a guide to deflection control of beams in floors, with the proviso that larger values may be used if the allowable bending stress is proportionately reduced. For purlins (except those in flat roofs) the value $1000/F_y$ is suggested. It should be noted that these limits are based on deflection due to the total load, rather than live load only, since the allowable stress $0.6F_y$ was used in deriving them.

Equation (*b*) can be expressed in terms of bending stress by substituting F_b for f. Then for a deflection limit of $\frac{1}{300}$ of the span, and with $E = 30{,}000$ ksi, we get

$$\frac{L}{d} = \frac{480}{F_b} \tag{5-13}$$

This formula is useful in computing the value of L/d for a deflection limit of $L/300$ for live load only by substituting for F_b the live-load bending stress.

The Canadian Standard S16.1-1974 (Ref. 11) suggests maximum values of live-load deflection for a number of types of load on beams.

It should be noted that the deflection limits discussed above are not mandatory. The AISC/ASD Commentary suggests that they be followed "if practicable."

ALLOWABLE STRESS FOR BEAM

Specification Formulas

To facilitate comparison in the discussion to follow, specification formulas are written in forms which are not necessarily the same as in the specifications.

AISC/ASD. The allowable bending stress F_b for channels and I-shaped members of steels with $F_y \le 65$ ksi, supported against lateral buckling and bent about the major axis, are as follows:

Compact section: $F_b = 0.66F_y$ (5-16a)

Noncompact section: $F_b = 0.60F_y$ (5-16b)

If $65/\sqrt{F_y} \le b_f/2t_f \le 95\sqrt{F_y}$:

$$F_b = \begin{cases} F_y\left(0.79 - 0.002\dfrac{b_f}{2t_f}\sqrt{F_y}\right) & \text{(rolled shapes)} \quad 5\text{-}16c) \\ F_y\left(0.79 - 0.002\dfrac{b_f}{2t_f}\sqrt{\dfrac{F_y}{k_c}}\right) & \text{(built-up members)} \quad (5\text{-}16d) \end{cases}$$

where

$$k_c = \begin{cases} 1 & \text{if } \dfrac{h}{t} \le 70 \\ \dfrac{4.05}{(h/t)^{0.46}} & \text{if } \dfrac{h}{t} > 70 \end{cases}$$

Notation in Eqs. (5-16) is as follows:

- b_f = flange width
- t_f = flange thickness
- h = distance between adjacent lines of fasteners, or clear distance between flanges if welds are used
- t = web thickness

The ratio 1.1 of the allowable stresses $0.66F_y$ and $0.60F_y$ is approximately equal to the smallest of the shape factors for standard I-shaped members. It should be noted that Eqs. (5-16b), (5-16c), and (5-16d) are not applicable if $b_f/2t_f$ exceeds 95.

Elements as slender as this buckle at stresses less than the yield stress and are treated as described in Chap. 10.

The value of k_c in Eq. (5-16*d*) is derived from results of tests on built-up members with relatively slender webs and flanges, in which the compression flange was braced against lateral buckling.[12] However, premature buckling in the form of a rotation of the flange and web about their intersection occurred, and Eq. (5-16*d*) gives the reduced value of F_b to conform to the test results. There is an unexplained discontinuity in the value of k_c at $h/t = 70$. Thus, for $F_y = 36$ ksi and $b/2t_f = 15$, $k_c = 1$ and $F_b = 21.9$ ksi if $h/t = 70$, but for $h/t = 70.1$, $k_c = 0.57$ and $F_b = 19.9$ ksi.

Lateral support may be continuous, as for a beam which is the direct support of a floor, or by bracing members. Lateral-support spacing for beams designed for $F_b = 0.66F_y$ must not exceed the smaller of the values of L_c given by the following:

smaller of

$$L_c = \frac{76b_f}{\sqrt{F_y}} \qquad (5\text{-}17a)$$

$$L_c = \frac{20{,}000}{F_y d/A_f} \qquad (5\text{-}17b)$$

Beams with $L \geq L_c$. Allowable bending stresses for members with laterally unsupported lengths greater than those given by Eqs. (5-17) are based on the bending strengths given by ACE of Fig. 5-12a and $ABDE$ of Fig. 5-13.

In Fig. 5-13b CDE is a plot of Eq. (g) with $K = 1$ and r_y replaced by r_T, where r_T is the radius of gyration of a section consisting of the compression flange and one-third the compression web area. This definition enables the formula to be used for beams with unequal flanges, as explained in Art. 5-10. These are elastic-buckling values. Inelastic buckling is represented by ABD, where BD is a segment of the parabola GBD with vertex at G where $F_{cr} = \frac{10}{9}F_y$. The abscissa $\sqrt{510{,}000C_b/F_y}$ is found by equating the Euler stress to $\frac{5}{9}F_y$. The abscissa $\sqrt{102{,}000C_b/F_y}$ is found similarly by equating F_{cr} in the inelastic-buckling formula shown in the figure to F_y. The allowable stress $A'B'D'E'$ is obtained by multiplying the ordinates to $ABDE$ by 0.6 to give a factor of safety of 1.67. The results are

$$F_b = \begin{cases} 0.6F_y & 0 \leq \dfrac{L}{r_T} \leq \sqrt{\dfrac{102{,}000C_b}{F_y}} \quad (5\text{-}19a) \\[2ex] \left[\dfrac{2}{3} - \dfrac{F_y(L/r_T)^2}{1530 \times 10^3 C_b}\right]F_y & \sqrt{\dfrac{102{,}000C_b}{F_y}} \leq \dfrac{L}{r_T} \leq \sqrt{\dfrac{510{,}000C_b}{F_y}} \quad (5\text{-}19b) \\[2ex] \dfrac{170{,}000C_b}{(L/r_T)^2} & \sqrt{\dfrac{510{,}000C_b}{F_y}} \leq \dfrac{L}{r_T} \quad (5\text{-}19c) \end{cases}$$

These equations are not to be used for channels, because the formula for C_w, which is involved in their derivation, is quite different for channels than for I-shaped members (Fig. 4-46).

Allowable stresses corresponding to ACE of Fig. 5-12a are found by multiplying the ordinates by 0.6 to give a factor of safety of 1.67. This means that

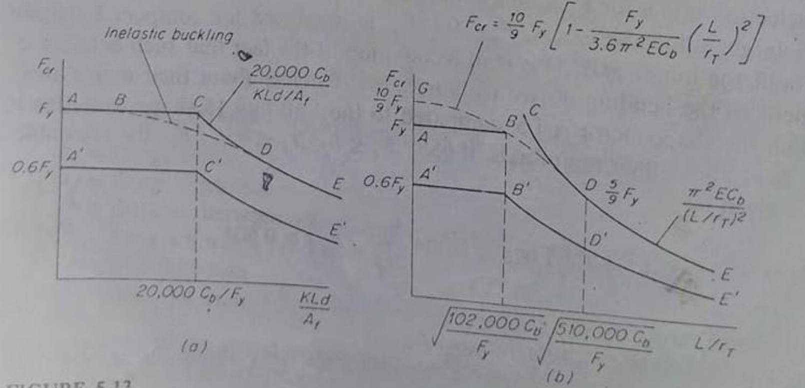

FIGURE 5-13

inelastic buckling is not considered. The results are

$$F_b = \begin{cases} 0.6F_y & 0 \le \dfrac{Ld}{A_f} \le \dfrac{20{,}000C_b}{F_y} & (5\text{-}20a) \\ \dfrac{12{,}000C_b}{Ld/A_f} & \dfrac{Ld}{A_f} \ge \dfrac{20{,}000C_b}{F_y} & (5\text{-}20b) \end{cases}$$

Equation (5-20*b*) is to be used only if the compression flange is solid and approximately rectangular in cross section and is not smaller in area than the tension flange. It is applicable for channels in bending about the major axis as well as for I-shaped members.

The larger of the values of F_b given by Eq. (5-20*b*) and the applicable Eq. (5-19) should be used, as was explained earlier. C_b in these equations is given by Eq. (5-5*a*). However, the specification requires that C_b be taken as unity if the bending moment at any point within the unbraced length is larger than that at both ends of the unbraced length.

The largest lateral-brace spacing L for which the allowable stress $0.6F_y$ may be used is given by Eqs. (5-19*a*) and (5-20*b*):

$$L = r_T\sqrt{\frac{102{,}000C_b}{F_y}} \qquad (5\text{-}21a)$$

$$L = \frac{20{,}000C_b}{F_y d/A_f} \qquad (5\text{-}21b)$$

Since Eqs. (5-19) and (5-20) underestimate the stress that can be allowed, the larger value of L from Eqs. (5-21) should be used. This value is tabulated, for $C_b = 1$, as L_u in the AISC/ASD Manual's beam-selection tables for $F_y = 36$ and 50 ksi.

Since each of Eqs. (5-20) and the applicable Eq. (5-19) is a conservative value of the allowable stress, the specification allows a higher value if it can be "justified on the basis of a more precise analysis," and in the commentary suggests using an equivalent radius of gyration [for example, Eq. (5-7*a*)] to obtain the higher value. With this procedure the allowable stress may be obtained from Eqs. (5-19), using L/r_{eq} for L/r_T, and Eqs. (5-20) should be ignored.

Select the lightest W section to carry a uniformly distributed live load 1.5 k/ft and dead load (not including weight of beam) of 0.5k/ft on a 30 ft simply supported beam. The beam does not have continuous lateral support and so must be braced. Assuming that it will be braced to satisfy compact section requirements, design the beam using f_y = 36 ksi. Use AISC/ASD method.

Section	Unit weight (lb/ft)	S_x (in^3)	b_f (in)	d (in)	A_f (in^2)
W21 x 62	62	127	8.24	21	5.072
W14 x 53	53	77.8	8.06	13.92	5.32
W24 x 68	68	154	8.965	23.73	5.24

Solution:

Assume, beam weight = 60 lb/ft

Design load = 1.5+0.5+0.06

= 2.06 k/ft

$M = 1/8 \times w \times L^2 = 1/8 \times 2.06 \times 30^2 = 231.8$ k-ft

Allowable stress, $f_b = 0.66 \times fy$

$= 0.66 \times 36$

$= 24$ ksi

$S_x = M/f_b$

$= 231.8/24$

$= 115.9\ in^3$

So, Select W21 x 62 which has $S_x = 127\ in^3 > S_x$ (required)

Since, assuming beam weight is very close to selected beam weight, only 2 lb/ft difference. So no need to revise beam weight.

Deflection check:

$L/d < 480/f_b$

$L/d = 30 \times 12/\ 21$

$= 17.1$

$480/f_b = 480/24$

$= 20$

Since, $L/d < 480/f_b$, deflection is less than limiting value.

Lateral support:

$L_c = 76\ b_f \sqrt{F_y}$

$= 76 \times 8.26 / \sqrt{36}$

$= 104$ in $= 8.7$ ft

$L_c = 20000/(F_y \times d/A_f)$

$= 20000/(36 \times 21 / 5.072)$

$= 134$ in $= 11.2$ ft

Provide spacing of bracing = 7.5ft.

No. of bracing = span/spacing -1 = 30/7.5 -1 = 3

So, Use W21 x 62 with three braces spaced at 7.5 ft.