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Theory of Computing (CSE 221)

Lecture - 2: Finite Automata

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Topic Contents

- ❖Finite Automata
- ❖Deterministic Finite Automata (DFA)
- ❖ Nondeterministic Finite Automata (NFA)

Finite Automaton

- Finite automata are computing devices that accept/recognize regular languages and are used to model operations of many systems
- A finite automaton has a set of states, and moves from one state to another in response to external inputs

Finite Automaton cont.

- Two types of automata:

1. **Deterministic**: the automaton cannot be in more than one state at any one time
2. **Non-deterministic**: the automaton may be in several states at once

If the internal state, input and contents of the storage are known, it is possible to predict the future behaviour of the automaton. This is said to be deterministic automata otherwise it is nondeterminist automata.

Deterministic Finite Automata (DFA)

A DFA is a quintuple

$$A = (Q, \Sigma, \delta, q_0, F)$$

- Q is a finite set of *states*
- Σ is a *finite alphabet* (=input symbols)
- δ is a *transition function*
- $q_0 \in Q$ is the *start state*
- $F \subseteq Q$ is a set of *final states*

DFA: Example 1

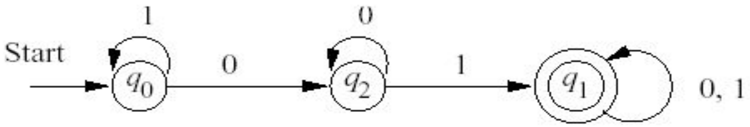
Example: An automaton A that accepts

$L = \{x01y : x, y \in \{0,1\}^*\}$

The automaton $A = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$
as a *transition table*:

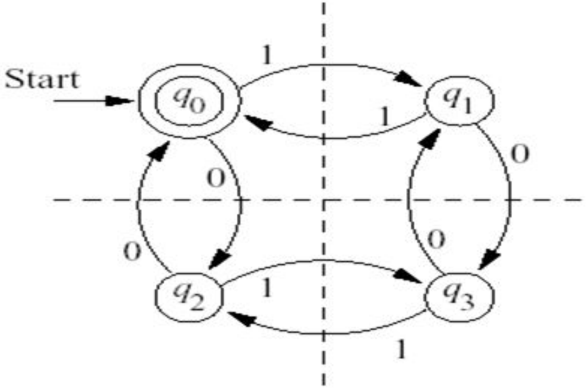
	0	1
$\rightarrow q_0$	q_2	q_0
$\star q_1$	q_1	q_1
q_2	q_2	q_1

The automaton as a *transition diagram*:



DFA: Example 2

Example: DFA accepting all and only strings with an even number of 0's and an even number of 1's



Tabular representation of the Automaton

	0	1
$\star \rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

DFA: Example 3

Example 1.1.2: Determine the DFA schematic for $M = (Q, \Sigma, \delta, q, F)$ where $Q = \{q_1, q_2, q_3\}$, $\Sigma = \{0,1\}$, q_1 is the start state, $F = \{q_2\}$ and δ is given by the table below.

Initial state q	Symbol σ	Final state $\delta(q,\sigma)$
q_1	0	q_1
q_1	1	q_2
q_2	0	q_3
q_2	1	q_2
q_3	0	q_2
q_3	1	q_2

Also determine a Language L recognized by the DFA.

Solution

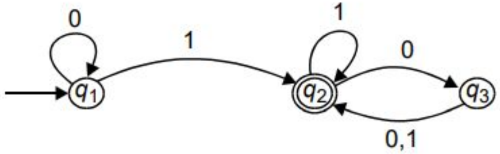


Fig. Finite Automaton having three states.

From the given table for δ, the DFA is drawn, where q_2 is the only final state.

(It is to be noted that a DFA can "accept" a string and it can "recognize" a language. Catch here is that "accept" is used for strings and "recognize" for that of a language).

It could be seen that the DFA accepts strings that has at least one 1 and an even number of 0s following the last 1.

Hence the language L is given by

$$L = \{w \mid w \text{ contains at least one 1 and an even number of 0s follow the last 1}\}$$

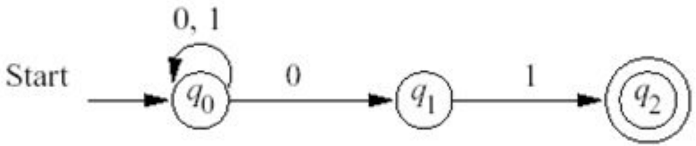
where $L = L(M)$ and M recognized the RHS of the equation above.

Acceptance

An FA *accepts* a string $w = a_1 a_2 \cdots a_n$ if there is a path in the transition diagram that

1. Begins at a start state
2. Ends at an accepting state
3. Has sequence of labels $a_1 a_2 \cdots a_n$

Example: The FA



accepts e.g. the string 01101

Nondeterministic Finite Automata (NFA)

Formally, a NFA is a quintuple

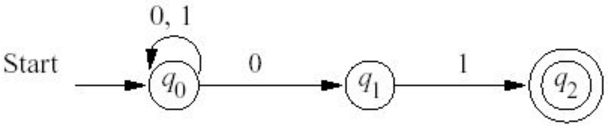
$$A = (Q, \Sigma, \delta, q_0, F)$$

- Q is a finite set of states
- Σ is a finite alphabet
- δ is a transition function from $Q \times \Sigma$ to the *powerset* of Q
- $q_0 \in Q$ is the *start state*
- $F \subseteq Q$ is a set of *final states*

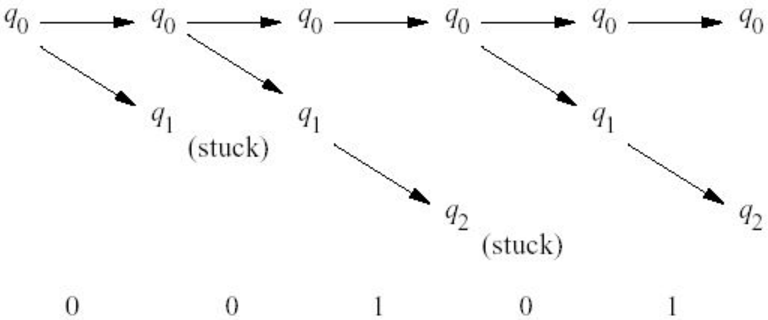
NFA cont.

A NFA can be in several states at once, or, viewded another way, it can "guess" which state to go to next

Example: An automaton that accepts all and only strings ending in 01.



Here is what happens when the NFA processes the input 00101



NFA: Example 1

Example: The NFA from the previous slide is

$$(\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_2\})$$

where δ is the transition function

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\emptyset$	$\{q_2\}$
$\star q_2$	$\emptyset$	$\emptyset$

NFA: Example 2

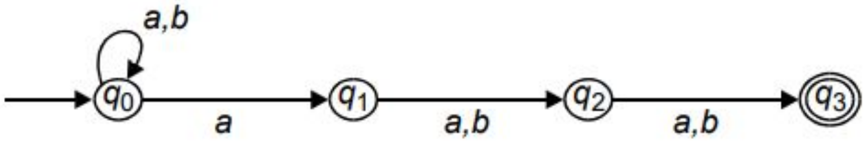
Example 1.2.4: Obtain an NFA which should accept a language L_A, given by $L_A = \{x \in \{a, b\}^* : |x| \geq 3$ and third symbol of x from the right is $\{‘a’\}$.

Solution

The conditions are

(a) the last two symbols can be ‘a’ or ‘b’.
(b) third symbol from the right is ‘a’
(c) symbol in any position but for the last three position can be ‘a’ or ‘b’.

The NFA is shown in fig. below.



NFA: Example 3

Example 1.2.5: Sketch the NFA state diagram for

$$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\})$$

with the state table as given below.

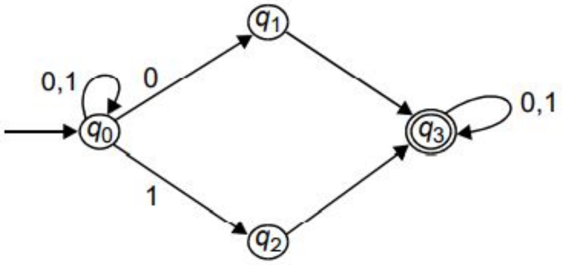
δ	0	1
q_0	q_0, q_1	q_0, q_2
q_1	q_3	$\varnothing$
q_2	$\varnothing$	q_3
q_3	q_3	q_3

Solution

The NFA states are q_0, q_1, q_2 and q_3.

$$\delta(q_0,0) = \{q_0, q_1\} \qquad \delta(q_0,1) = \{q_0, q_2\}$$
$$\delta(q_1,0) = \{q_3\} \qquad \delta(q_2,1) = \{q_3\}$$
$$\delta(q_3,0) = \{q_3\} \qquad \delta(q_3,1) = \{q_3\}\ .$$

The NFA is as shown below.



Thank You