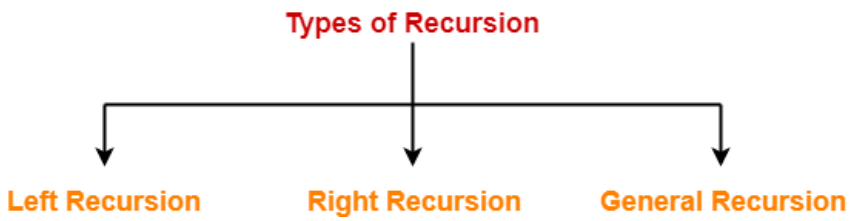
Left Recursion | Left Recursion Elimination

Theory of Automata & Computation

Recursion-

Recursion can be classified into following three types-



1. Left Recursion
2. Right Recursion
3. General Recursion

1. Left Recursion-

- A production of grammar is said to have **left recursion** if the leftmost variable of its RHS is same as variable of its LHS.
- A grammar containing a production having left recursion is called as Left Recursive Grammar.

Example-

$$S \rightarrow Sa / \in$$

(**Left Recursive Grammar**)

- Left recursion is considered to be a problematic situation for Top down parsers.
- Therefore, left recursion has to be eliminated from the grammar.

Elimination of Left Recursion

Left recursion is eliminated by converting the grammar into a right recursive grammar.

If we have the left-recursive pair of productions-

$$A \rightarrow A\alpha / \beta$$

(Left Recursive Grammar)

where β does not begin with an A.

Then, we can eliminate left recursion by replacing the pair of productions with-

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' / \in$$

(Right Recursive Grammar)

This right recursive grammar functions same as left recursive grammar.

2. Right Recursion-

- A production of grammar is said to have **right recursion** if the rightmost variable of its RHS is same as variable of its LHS.
- A grammar containing a production having right recursion is called as Right Recursive Grammar.

Example-

$$S \rightarrow aS / \in$$

(**Right Recursive Grammar**)

- Right recursion does not create any problem for the Top down parsers.
- Therefore, there is no need of eliminating right recursion from the grammar.

Also Read- **Types of Recursive Grammar**

3. General Recursion-

- The recursion which is neither left recursion nor right recursion is called as general recursion.

Example-

$$S \rightarrow aSb / \in$$

PRACTICE PROBLEMS BASED ON LEFT RECURSION ELIMINATION-

Problem-01:

Consider the following grammar and eliminate left recursion-

$$A \rightarrow ABd / Aa / a$$

$$B \rightarrow Be / b$$

Solution-

The grammar after eliminating left recursion is-

A → aA’

A’ → BdA’ / aA’ / ∈

B → bB’

B’ → eB’ / ∈

Problem-02:

Consider the following grammar and eliminate left recursion-

$$E \rightarrow E + E / E \times E / a$$

Solution-

The grammar after eliminating left recursion is-

E → aA

A → +EA / xEA / ∈

Problem-03:

Consider the following grammar and eliminate left recursion-

E → E + T / T

T → T x F / F

F → id

Solution-

The grammar after eliminating left recursion is-

E → TE’

E’ → +TE’ / ∈

T → FT’

T’ → xFT’ / ∈

F → id

Problem-04:

Consider the following grammar and eliminate left recursion-

S → (L) / a

L → L , S / S

Solution-

The grammar after eliminating left recursion is-

S → (L) / a

L → SL’

L’ → ,SL’ / ∈

Problem-05:

Consider the following grammar and eliminate left recursion-

S → S0S1S / 01

Solution-

The grammar after eliminating left recursion is-

S → 01A

A → 0S1SA / ∈

Problem-06:

Consider the following grammar and eliminate left recursion-

$$S \rightarrow A$$

$$A \rightarrow Ad / Ae / aB / ac$$

$$B \rightarrow bBc / f$$

Solution-

The grammar after eliminating left recursion is-

$S \rightarrow A$

$A \rightarrow aBA' / acA'$

$A' \rightarrow dA' / eA' / \in$

$B \rightarrow bBc / f$

Problem-07:

Consider the following grammar and eliminate left recursion-

$$A \rightarrow AA\alpha / \beta$$

Solution-

The grammar after eliminating left recursion is-

$A \rightarrow \beta A'$

$A' \rightarrow \alpha A' / \in$

Problem-08:

Consider the following grammar and eliminate left recursion-

$$A \rightarrow Ba / Aa / c$$

$$B \rightarrow Bb / Ab / d$$

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $A \rightarrow Ba / Aa / c$

Eliminating left recursion from here, we get-

A → BaA' / cA'

A' → aA' / ∈

Now, given grammar becomes-

A → BaA' / cA'

A' → aA' / ∈

B → Bb / Ab / d

Step-02:

Substituting the productions of A in B → Ab, we get the following grammar-

A → BaA' / cA'

A' → aA' / ∈

B → Bb / BaA'b / cA'b / d

Step-03:

Now, eliminating left recursion from the productions of B, we get the following grammar-

A → BaA' / cA'

A' → aA' / ∈

B → cA'bB' / dB'

B' → bB' / aA'bB' / ∈

This is the final grammar after eliminating left recursion.

Problem-09:

Consider the following grammar and eliminate left recursion-

X → XSb / Sa / b

S → Sb / Xa / a

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from X → XSb / Sa / b

Eliminating left recursion from here, we get-

X → SaX' / bX'

X' → SbX' / ∈

Now, given grammar becomes-

X → SaX' / bX'

X' → SbX' / ∈

S → Sb / Xa / a

Step-02:

Substituting the productions of X in S → Xa, we get the following grammar-

X → SaX' / bX'

X' → SbX' / ∈

S → Sb / SaX'a / bX'a / a

Step-03:

Now, eliminating left recursion from the productions of S, we get the following grammar-

X → SaX' / bX'

X' → SbX' / ∈

S → bX'aS' / aS'

S' → bS' / aX'aS' / ∈

This is the final grammar after eliminating left recursion.

Problem-10:

Consider the following grammar and eliminate left recursion-

S → Aa / b

A → Ac / Sd / ∈

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from S → Aa / b

This is already free from left recursion.

Step-02:

Substituting the productions of S in A → Sd, we get the following grammar-

S → Aa / b

A → Ac / Aad / bd / ∈

Step-03:

Now, eliminating left recursion from the productions of A, we get the following grammar-

S → Aa / b

A → bdA' / A'

A' → cA' / adA' / ∈

This is the final grammar after eliminating left recursion.

Also Read- Left Factoring

To gain better understanding about Left Recursion Elimination,

Watch this Video Lecture

Next Article- Types of Grammars

Get more notes and other study material of **Theory of Automata and Computation**.

Watch video lectures by visiting our YouTube channel **LearnVidFun**.

Summary