Number Systems

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Learning Objectives

In this lecture you will learn about:

- ✓ Non-positional number system
- ✓ Positional number system
- ✓ Decimal number system
- ✓ Binary number system
- ✓ Octal number system
- ✓ Hexadecimal number system

- Convert a number's base
  - ✓ Another base to decimal base
  - ✓ Decimal base to another base
  - ✓ Some base to another base
- Shortcut methods for converting
  - ✓ Binary to octal number
  - ✓ Octal to binary number
  - ✓ Binary to hexadecimal number
  - ✓ Hexadecimal to binary number
- Fractional numbers in binary number system

Number Systems

Two types of number systems are:

① Non-positional number systems

② Positional number systems

Non-positional Number Systems

- **Characteristics**
  - Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc.
  - Each symbol represents the same value regardless of its position in the number
  - The symbols are simply added to find out the value of a particular number
- **Difficulty**
  - It is difficult to perform arithmetic with such a number system

Positional Number Systems

- **Characteristics**
  - Use only a few symbols called digits
  - These symbols represent different values depending on the position they occupy in the number
- **The value of each digit is determined by**
  1. The digit itself
  2. The position of the digit in the number
  3. The base of the number system (**base** = total number of digits in the number system)
- The maximum value of a single digit is always equal to one less than the value of the base

Decimal Number System

- **Characteristics**
  - A positional number system
  - Has 10 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Hence, its base = 10
  - The maximum value of a single digit is 9 (one less than the value of the base)
  - Each position of a digit represents a specific power of the base (10)
  - We use this number system in our day-to-day life
- **Example**

$$2586_{10} = (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (6 \times 10^0)$$
$$= 2000 + 500 + 80 + 6$$

Binary Number System

- **Characteristics**
  - A positional number system
  - Has only 2 symbols or digits (0 and 1). Hence its base = 2
  - The maximum value of a single digit is 1 (one less than the value of the base)
  - Each position of a digit represents a specific power of the base (2)
  - This number system is used in computers
- **Example**

$$10101_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) \times (1 \times 2^0)$$

$$= 16 + 0 + 4 + 0 + 1$$

$$= 21_{10}$$

Bit

- Bit stands for binary digit
- A bit in computer terminology means either a **0** or a **1**
- A binary number consisting of *n* bits is called an n-bit number

Representing Numbers in Different Number Systems

- In order to be specific about which number system we are referring to, it is a common practice to indicate the base as a subscript.
- Thus, we write: $10101_2 = 21_{10}$

Octal Number System

- **Characteristics**
  - A positional number system
  - Has total 8 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7).
  - Hence, its base = 8
  - The maximum value of a single digit is 7 (one less than the value of the base)
  - Each position of a digit represents a specific power of the base (8)
  - Since there are only 8 digits, 3 bits ($2^3 = 8$) are sufficient to represent any octal number in binary
- **Example**

$$
\begin{aligned}
2057_8 &= (2 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0) \\
&= 1024 + 0 + 40 + 7 \\
&= 1071_{10}
\end{aligned}
$$

Hexadecimal Number System

- **Characteristics**
  - A positional number system
  - Has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Hence its base = 16
  - The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively
  - The maximum value of a single digit is 15 (one less than the value of the base)
  - Each position of a digit represents a specific power of the base (16)
  - Since there are only 16 digits, 4 bits ($2^4 = 16$) are sufficient to represent any hexadecimal number in binary

- **Example**

$$
\begin{aligned}
1AF_{16} &= (1 \times 16^2) + (A \times 16^1) + (F \times 16^0) \\
&= 1 \times 256 + 10 \times 16 + 15 \times 1 \\
&= 256 + 160 + 15 \\
&= 431_{10}
\end{aligned}
$$

Converting a Number of Another Base to a Decimal Number

- **Method**
  - Step 1: Determine the column (positional) value of each digit
  - Step 2: Multiply the obtained column values by the digits in the corresponding columns
  - Step 3: Calculate the sum of these products

Converting a Number of Another Base to a Decimal Number

- Example

$$4706_8 = ?_{10}$$

Common values multiplied by the corresponding digits

$$4706_8 = 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0$$

$$= 4 \times 512 + 7 \times 64 + 0 + 6 \times 1$$

$$= 2048 + 448 + 0 + 6$$

Sum of these products

$$= 2502_{10}$$

Converting a Decimal Number to a Number of Another Base

- **Division-Remainder Method**
  - **Step 1:** Divide the decimal number to be converted by the value of the new base
  - **Step 2:** Record the remainder from Step 1 as the rightmost digit (least significant digit) of the new base number
  - **Step 3:** Divide the quotient of the previous divide by the new base
  - **Step 4:** Record the remainder from Step 3 as the next digit (to the left) of the new base number
  - Repeat Steps 3 and 4, recording remainders from right to left, until the quotient becomes zero in Step 3
- Note that the last remainder thus obtained will be the most significant digit (MSD) of the new base number

Converting a Decimal Number to a Number of Another Base

- **Example:** $952_{10} = ?_8$

Solution:

8	952	Remainder s
	119	0
	14	7
	1	6
	0	1

Hence, $952_{10} = 1670_8$

Converting a Number of Some Base to a Number of Another Base

- **Method**
  - **Step 1:** Convert the original number to a decimal number (base 10)
  - **Step 2:** Convert the decimal number so obtained to the new base number

Converting a Number of Some Base to a Number of Another Base

- **Example:**

$$545_6 = ?_4$$

Solution:

Step 1: Convert from base 6 to base 10

$$\begin{aligned} 545_6 &= 5 \text{ x } 6^2 + 4 \text{ x } 6^1 + 5 \text{ x } 6^0 \\ &= 5 \text{ x } 36 + 4 \text{ x } 6 + 5 \text{ x } 1 \\ &= 180 + 24 + 5 \\ &= 209_{10} \end{aligned}$$

Converting a Number of Some Base to a Number of Another Base

Step 2: Convert 209_{10} to base 4

4	209	Remainders
	52	1
	13	0
	3	1
	0	3

Hence, $209_{10} = 3101_4$

So, $545_6 = 209_{10} = 3101_4$

Thus, $545_6 = 3101_4$

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

- **Method**

  - Step 1: Divide the digits into groups of three starting from the right
  - Step 2: Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

- **Example:**

$1101010_2 = ?_8$

Step 1: Divide the binary digits into groups of 3 starting from right

<u>001</u> <u>101</u> <u>010</u>

Step 2: Convert each group into one octal digit

$$001_2 = 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1$$
$$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5$$
$$010_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$$

Hence, $1101010_2 = 152_8$

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

- **Method**
  - Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)
  - Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

- **Example:**

$562_8 = ?_2$

Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2, \quad 6_8 = 110_2, \quad 2_8 = 010_2$

Step 2: Combine the binary groups

$$562_8 = \underset{5}{\underline{101}} \quad \underset{6}{\underline{110}} \quad \underset{2}{\underline{010}}$$

Hence, $562_8 = 101110010_2$

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

- **Method**
  - Step 1: Divide the binary digits into groups of four starting from the right
  - Step 2: Combine each group of four binary digits to one hexadecimal digit

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

- **Example:**

$111101_2 = ?_{16}$

Step 1: Divide the binary digits into groups of four starting from the right

<u>0011</u> <u>1101</u>

Step 2: Convert each group into a hexadecimal digit

$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 3_{10} = D_{16}$

Hence, $111101_2 = 3D_{16}$

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

- **Method**
  - Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)
  - Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

- **Example:**

$562_8 = ?_2$

Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2, \quad 6_8 = 110_2, \quad 2_8 = 010_2$

Step 2: Combine the binary groups

$562_8 = \underbrace{101}_{5} \quad \underbrace{110}_{6} \quad \underbrace{010}_{2}$

Hence, $562_8 = 101110010_2$

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number

- **Example:**

$111101_2 = ?_{16}$

Step 1: Divide the binary digits into groups of four starting from the right

<u>0011</u> <u>1101</u>

Step 2: Convert each group into a hexadecimal digit

$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 3_{10} = D_{16}$

Hence, $111101_2 = 3D_{16}$

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

- **Method**
  - Step 1: Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number
  - Step 2: Combine all the resulting binary groups (of 4 digits each) in a single binary number

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number

- **Example:** $2AB_{16} = ?_2$

Step 1: Convert each hexadecimal digit to a 4 digit binary number

$$2_{16} = 210 = 0010_2$$

$$A_{16} = 1010 = 1010_2$$

$$B_{16} = 1110 = 1011_2$$

Step 2: Combine the binary groups

$$2AB_{16} = \underset{2}{\underline{0010}} \quad \underset{A}{\underline{1010}} \quad \underset{B}{\underline{1011}}$$

Hence, $2AB_{16} = 001010101011_2$

Fractional Numbers

- **Fractional numbers** are formed same way as decimal number system

In general, a number in a number system with base *b* would be written as:

$a_n\, a_{n-1} \ldots a_0 \,.\, a_{-1}\, a_{-2} \ldots a_{-m}$

And would be interpreted to mean:

$a_n \times b^n + a_{n-1} \times b^{n-1} + \ldots + a_0 \times b^0 + a_{-1} \times b^{-1} + a_{-2} \times b^{-2} + \ldots + a_{-m} \times b^{-m}$

The symbols a_n, a_{n-1}, …, a_{-m} in above representation should be one of the *b* symbols allowed in the number system

Formation of Fractional Numbers in Binary Number System

Binary Point

Position	4	3	2	1	0	.	-1	-2	-3	-4
Position Value	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
Quantity Represented	16	8	4	2	1		$^1/_2$	$^1/_4$	$^1/_8$	$^1/_{16}$

- **Example:**

$$
\begin{aligned}
110.101_2 &= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} \\
&= 4 + 2 + 0 + 0.5 + 0 + 0.125 \\
&= 6.625_{10}
\end{aligned}
$$

Formation of Fractional Numbers in Octal Number System

					Octal Point			
Position	3	2	1	0	.	-1	-2	-3
Position Value	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}
Quantity Represented	512	64	8	1		$^1/_8$	$^1/_{64}$	$^1/_{512}$

- **Example:**

$$
\begin{aligned}
127.54_8 &= 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 5 \times 8^{-1} + 4 \times 8^{-2} \\
&= 64 + 16 + 7 + {}^5/_8 + {}^4/_{64} \\
&= 87 + 0.625 + 0.0625 \\
&= 87.6875_{10}
\end{aligned}
$$

Key Words/Phrases

- Base
- Binary number system
- Binary point
- Bit
- Decimal number system
- Division-Remainder technique
- Fractional numbers
- Hexadecimal number system
- Least Significant Digit (LSD)
- Memory dump
- Most Significant Digit (MSD)
- Non-positional number system
- Number system
- Octal number system
- Positional number system