



Number Systems

Dr. Sheak Rashed Haider Noori
Associate Professor and Associate Head
Department of CSE
Daffodil International University

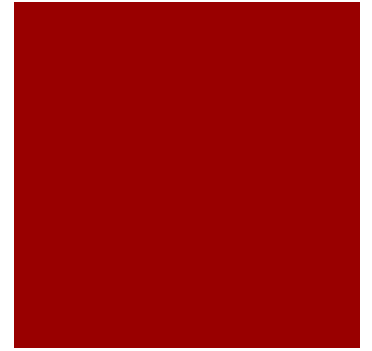
Learning Objectives



In this lecture you will learn about:

- ✓ Non-positional number system
 - ✓ Positional number system
 - ✓ Decimal number system
 - ✓ Binary number system
 - ✓ Octal number system
 - ✓ Hexadecimal number system
- Convert a number's base
 - ✓ Another base to decimal base
 - ✓ Decimal base to another base
 - ✓ Some base to another base
 - Shortcut methods for converting
 - ✓ Binary to octal number
 - ✓ Octal to binary number
 - ✓ Binary to hexadecimal number
 - ✓ Hexadecimal to binary number
 - Fractional numbers in binary number system

Number Systems



Two types of number systems are:

- ① Non-positional number systems
- ② Positional number systems

Non-positional Number Systems



■ Characteristics

- Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc.
- Each symbol represents the same value regardless of its position in the number
- The symbols are simply added to find out the value of a particular number

■ Difficulty

- It is difficult to perform arithmetic with such a number system

Positional Number Systems



■ Characteristics

- Use only a few symbols called digits
- These symbols represent different values depending on the position they occupy in the number

■ The value of each digit is determined by

- ① The digit itself
 - ② The position of the digit in the number
 - ③ The base of the number system (**base** = total number of digits in the number system)
- The maximum value of a single digit is always equal to one less than the value of the base

Decimal Number System



■ Characteristics

- A positional number system
- Has 10 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Hence, its base = 10
- The maximum value of a single digit is 9 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (10)
- We use this number system in our day-to-day life

■ Example

$$\begin{aligned} 2586_{10} &= (2 \times 10^3) + (5 \times 10^2) + (8 \times 10^1) + (6 \times 10^0) \\ &= 2000 + 500 + 80 + 6 \end{aligned}$$

Binary Number System



■ Characteristics

- A positional number system
- Has only 2 symbols or digits (0 and 1). Hence its base = 2
- The maximum value of a single digit is 1 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (2)
- This number system is used in computers

■ Example

$$\begin{aligned}10101_2 &= (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= 16 + 0 + 4 + 0 + 1 \\ &= 21_{10}\end{aligned}$$

Bit



- Bit stands for binary digit
- A bit in computer terminology means either a **0** or a **1**
- A binary number consisting of n bits is called an n -bit number

Representing Numbers in Different Number Systems



- In order to be specific about which number system we are referring to, it is a common practice to indicate the base as a subscript.
- Thus, we write: $10101_2 = 21_{10}$

Octal Number System



■ Characteristics

- A positional number system
- Has total 8 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7).
- Hence, its base = 8
- The maximum value of a single digit is 7 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (8)
- Since there are only 8 digits, 3 bits ($2^3 = 8$) are sufficient to represent any octal number in binary

■ Example

$$\begin{aligned}2057_8 &= (2 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0) \\ &= 1024 + 0 + 40 + 7 \\ &= 1071_{10}\end{aligned}$$

Hexadecimal Number System



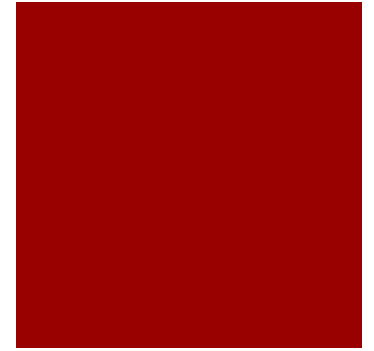
■ Characteristics

- A positional number system
- Has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Hence its base = 16
- The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively
- The maximum value of a single digit is 15 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (16)
- Since there are only 16 digits, 4 bits ($2^4 = 16$) are sufficient to represent any hexadecimal number in binary

■ Example

$$\begin{aligned}1AF_{16} &= (1 \times 16^2) + (A \times 16^1) + (F \times 16^0) \\ &= 1 \times 256 + 10 \times 16 + 15 \times 1 \\ &= 256 + 160 + 15 \\ &= 431_{10}\end{aligned}$$

Converting a Number of Another Base to a Decimal Number



■ Method

- Step 1: Determine the column (positional) value of each digit
- Step 2: Multiply the obtained column values by the digits in the corresponding columns
- Step 3: Calculate the sum of these products

Converting a Number of Another Base to a Decimal Number



■ Example

$$4706_8 = ?_{10}$$

$$\begin{aligned} 4706_8 &= 4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0 \\ &= 4 \times 512 + 7 \times 64 + 0 + 6 \times 1 \\ &= 2048 + 448 + 0 + 6 \leftarrow \text{Sum of these products} \\ &= 2502_{10} \end{aligned}$$

Common values multiplied by the corresponding digits

Converting a Decimal Number to a Number of Another Base



■ Division-Remainder Method

- **Step 1:** Divide the decimal number to be converted by the value of the new base
 - **Step 2:** Record the remainder from Step 1 as the rightmost digit (least significant digit) of the new base number
 - **Step 3:** Divide the quotient of the previous divide by the new base
 - **Step 4:** Record the remainder from Step 3 as the next digit (to the left) of the new base number
 - Repeat Steps 3 and 4, recording remainders from right to left, until the quotient becomes zero in Step 3
- Note that the last remainder thus obtained will be the most significant digit (MSD) of the new base number

Converting a Decimal Number to a Number of Another Base

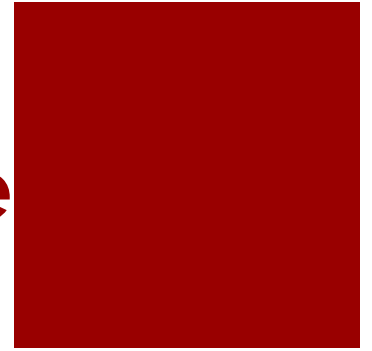
■ Example: $952_{10} = ?_8$

Solution:

8	952	Remainder
	<hr/>	
	119	S 0
	<hr/>	
	14	7
	<hr/>	
	1	6
	<hr/>	
	0	1

Hence, $952_{10} = 1670_8$

Converting a Number of Some Base to a Number of Another Base



■ Method

- **Step 1:** Convert the original number to a decimal number (base 10)
- **Step 2:** Convert the decimal number so obtained to the new base number

Converting a Number of Some Base to a Number of Another Base



■ Example:

$$545_6 = ?_4$$

Solution:

Step 1: Convert from base 6 to base 10

$$\begin{aligned} 545_6 &= 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 \\ &= 5 \times 36 + 4 \times 6 + 5 \times 1 \\ &= 180 + 24 + 5 \\ &= 209_{10} \end{aligned}$$

Converting a Number of Some Base to a Number of Another Base



Step 2: Convert 209_{10} to base 4

4	209	Remainders
	52	1
	13	0
	3	1
	0	3

Hence, $209_{10} = 3101_4$

So, $545_6 = 209_{10} = 3101_4$

Thus, $545_6 = 3101_4$

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number



■ Method

- Step 1: Divide the digits into groups of three starting from the right
- Step 2: Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion

Shortcut Method for Converting a Binary Number to its Equivalent Octal Number

■ Example:

$$1101010_2 = ?_8$$

Step 1: Divide the binary digits into groups of 3 starting from right

$$\underline{001} \quad \underline{101} \quad \underline{010}$$

Step 2: Convert each group into one octal digit

$$001_2 = 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1$$

$$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5$$

$$010_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$$

$$\text{Hence, } 1101010_2 = 152_8$$

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number



■ Method

- Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)
- Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number

■ Example:

$$562_8 = ?_2$$

Step 1: Convert each octal digit to 3 binary digits

$$5_8 = 101_2, \quad 6_8 = 110_2, \quad 2_8 = 010_2$$

Step 2: Combine the binary groups

$$562_8 = \begin{array}{ccc} \underline{101} & \underline{110} & \underline{010} \\ 5 & 6 & 2 \end{array}$$

$$\text{Hence, } 562_8 = 101110010_2$$

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number



■ Method

- Step 1: Divide the binary digits into groups of four starting from the right
- Step 2: Combine each group of four binary digits to one hexadecimal digit

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number



■ Example:

$$111101_2 = ?_{16}$$

Step 1: Divide the binary digits into groups of four starting from the right

$$\underline{0011} \quad \underline{1101}$$

Step 2: Convert each group into a hexadecimal digit

$$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$$

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10} = D_{16}$$

$$\text{Hence, } 111101_2 = 3D_{16}$$

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number



■ Method

- Step 1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion)
- Step 2: Combine all the resulting binary groups (of 3 digits each) into a single binary number

Shortcut Method for Converting an Octal Number to Its Equivalent Binary Number



■ Example:

$$562_8 = ?_2$$

Step 1: Convert each octal digit to 3 binary digits

$$5_8 = 101_2, \quad 6_8 = 110_2, \quad 2_8 = 010_2$$

Step 2: Combine the binary groups

$$562_8 = \begin{array}{ccc} \underline{101} & \underline{110} & \underline{010} \\ 5 & 6 & 2 \end{array}$$

$$\text{Hence, } 562_8 = 101110010_2$$

Shortcut Method for Converting a Binary Number to its Equivalent Hexadecimal Number



■ Example:

$$111101_2 = ?_{16}$$

Step 1: Divide the binary digits into groups of four starting from the right

$$\underline{0011} \quad \underline{1101}$$

Step 2: Convert each group into a hexadecimal digit

$$0011_2 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3_{10} = 3_{16}$$

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10} = D_{16}$$

$$\text{Hence, } 111101_2 = 3D_{16}$$

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number



■ Method

- Step 1: Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number
- Step 2: Combine all the resulting binary groups (of 4 digits each) in a single binary number

Shortcut Method for Converting a Hexadecimal Number to its Equivalent Binary Number



■ **Example:** $2AB_{16} = ?_2$

Step 1: Convert each hexadecimal digit to a 4 digit binary number

$$2_{16} = 210 = 0010_2$$

$$A_{16} = 1010 = 1010_2$$

$$B_{16} = 1110 = 1011_2$$

Step 2: Combine the binary groups

$$2AB_{16} = \begin{array}{ccc} \underline{0010} & \underline{1010} & \underline{1011} \\ & 2 & A & B \end{array}$$

Hence, $2AB_{16} = 001010101011_2$

Fractional Numbers



- **Fractional numbers** are formed same way as decimal number system

In general, a number in a number system with base b would be written as:

$$a_n a_{n-1} \dots a_0 . a_{-1} a_{-2} \dots a_{-m}$$

And would be interpreted to mean:

$$a_n \times b^n + a_{n-1} \times b^{n-1} + \dots + a_0 \times b^0 + a_{-1} \times b^{-1} + a_{-2} \times b^{-2} + \dots + a_{-m} \times b^{-m}$$

The symbols $a_n, a_{n-1}, \dots, a_{-m}$ in above representation should be one of the b symbols allowed in the number system

Formation of Fractional Numbers in Binary Number System



						Binary Point ↓				
Position	4	3	2	1	0	.	-1	-2	-3	-4
Position Value	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
Quantity Represented	16	8	4	2	1		$1/2$	$1/4$	$1/8$	$1/16$

■ Example:

$$\begin{aligned} 110.101_2 &= 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} \\ &= 4 + 2 + 0 + 0.5 + 0 + 0.125 \\ &= 6.625_{10} \end{aligned}$$

Formation of Fractional Numbers in Octal Number System



	Octal Point							
Position	3	2	1	0	•	-1	-2	-3
Position Value	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}
Quantity Represented	512	64	8	1		$1/8$	$1/64$	$1/512$

■ Example:

$$\begin{aligned}127.54_8 &= 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 5 \times 8^{-1} + 4 \times 8^{-2} \\ &= 64 + 16 + 7 + \frac{5}{8} + \frac{4}{64} \\ &= 87 + 0.625 + 0.0625 \\ &= 87.6875_{10}\end{aligned}$$

Key Words/Phrases



- Base
- Binary number system
- Binary point
- Bit
- Decimal number system
- Division-Remainder technique
- Fractional numbers
- Hexadecimal number system
- Least Significant Digit (LSD)
- Memory dump
- Most Significant Digit (MSD)
- Non-positional number system
- Number system
- Octal number system
- Positional number system