18-10 FLEXURAL STRENGTH OF SHEAR WALLS

Flexure—Nominal Strength, Factored Loads, and Resistance Factors

Cross sections in a wall are designed to satisfy

$$\phi M_n \geq M_u \tag{18-22}$$

$$\phi N_n \geq N_u \tag{18-23}$$

$$\phi V_n \geq V_u \tag{18-24}$$

where M_n is the nominal resistance based on the specified material strengths, M_u is the required resistance computed from the factored loads, and so on. The strength-reduction factor, ϕ, comes from ACI Code Section 9.3.2 for flexure and axial loads and from ACI Code Section 9.3.2.3 for shear. The factored loads are from ACI Code Section 9.2.1.

Strength-Reduction Factors for Flexure and Axial Load—ACI Code Section 9.3.2

The strength-reduction (ϕ) factors for combined flexure and axial loads for a shear wall vary, depending on the maximum steel strains anticipated at ultimate load. As explained in Chapters 5 and 11, the calculation of strength-reduction factors, ϕ, is based on the strain, ε_t, in the layer of steel at the depth, d_t, which is located farthest from the extreme-compression fiber.

Tension-controlled limit for a rectangular wall. The strength-reduction factor, ϕ, can be computed directly from the computed strain in the extreme-tension layer of reinforcement, ε_t. ACI Code Section 10.3.4 defines the tension-controlled limit load as the load causing a strain distribution having a maximum strain of 0.003 *in compression* on the most compressed face of the member, when the steel strain in the extreme layer of tension reinforcement, ε_t, reaches 0.005 in tension. From similar triangles, the neutral axis will be located at $c/d_t = 0.375d_t$ from the compressed face, where d_t is the distance from the extreme-compression fiber to the centroid of the layer of bars farthest from the compression face of the member. Thus, when c is less than or equal to $0.375d_t$, the wall is tension-controlled, and $\phi = 0.9$.

Compression-controlled limit for a rectangular wall. The compression-controlled limit corresponds to a strain distribution with the neutral axis at $0.6d_t$. So, when c is greater than or equal to $0.6d_t$, the wall is compression-controlled, and $\phi = 0.65$.

Flexural Strength of Rectangular Walls with Uniform Curtains of Vertical Distributed Reinforcement

Code guidance on the use of vertical distributed reinforcement in walls loaded cyclically is ambiguous. In seismic regions, the loads resisted by vertical bars that are not tied, are ignored. ACI Code Section 14.3.6 is more lenient. It requires that vertical bars be tied (a) if the total distributed vertical reinforcement, A_s, exceeds $0.01A_g$, or (b) if the vertical reinforcement is included as compression reinforcement in the calculations. The following strength analysis applies to walls with two curtains of reinforcement with ties through the wall to the other curtain of bars. The equations in this subsection also apply if the area of steel is less than 0.01.

ACI Code Section 21.6.4.2 suggests that, in seismic regions, column reinforcement would be adequately tied if the center-to-center spacing of cross-ties or the legs of hoops did

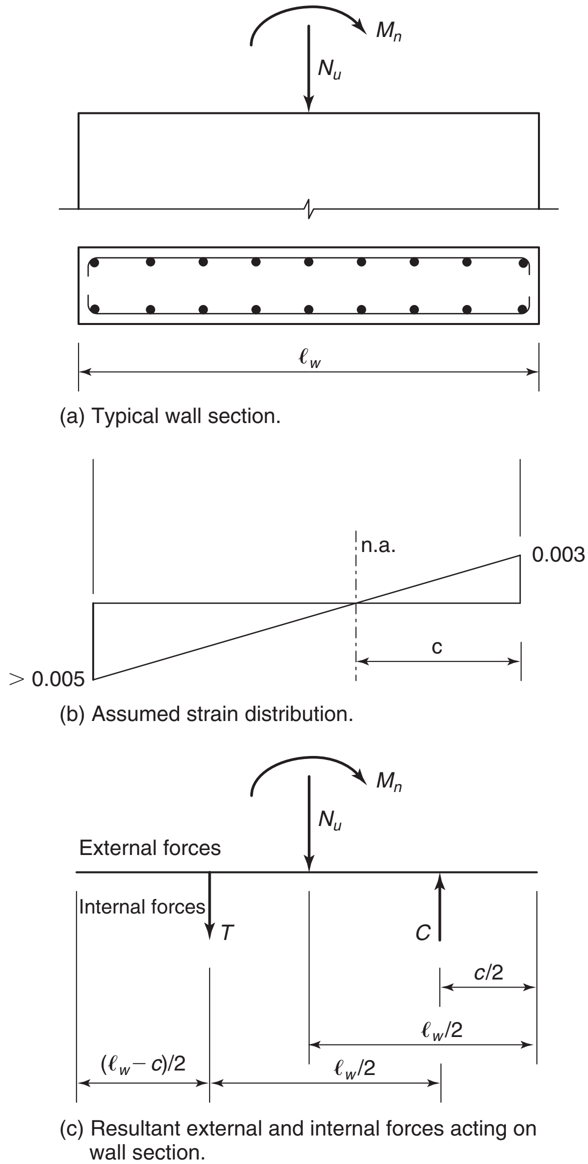


Fig. 18-15
Wall with uniform distribution of vertical reinforcement subjected to axial load and bending.

not exceed 14 in. Given this guidance, the author believes that the vertical reinforcement in a nonseismic wall can be taken as "tied" if at least every second bar in a curtain of reinforcement is tied through the wall to a bar in the other curtain of steel, near the opposite face.

Figure 18-15a shows a rectangular wall section with a uniform distribution of vertical steel. We will assume that the wall is subjected to a factored axial load, N_u, and we want to find the nominal flexural strength, M_n, for this wall using the assumed strain distribution in Fig. 18-15b. We will use a procedure developed by A. E. Cardenas and his colleagues [18-22] and [18-23]. They made the following assumptions at nominal strength conditions for shear wall sections similar to that in Fig. 18-15a.

1. All steel in the tension zone yields in tension.
2. All steel in the compression zone yields in compression.
3. The tension force acts at middepth of the tension zone.

4. The total compression force (sum of steel and concrete contributions) acts at middepth of the compression zone.

From those assumptions and using A_{st} to represent the total area of longitudinal (vertical) reinforcement, we can obtain the following expressions for the vector forces in Fig. 18-15c.

$$T = A_{st} f_y \left(\frac{\ell_w - c}{\ell_w} \right) \tag{18-25a}$$

$$C_s = A_{st} f_y \left(\frac{c}{\ell_w} \right) \tag{18-25b}$$

$$C_c = 0.85 f'_c \, h \beta_1 c \tag{18-25c}$$

and

$$C = C_s + C_c \tag{18-25d}$$

The percentage of total longitudinal reinforcement is

$$\rho_\ell = \frac{A_{st}}{h \ell_w} \tag{18-26a}$$

and the longitudinal reinforcement index is

$$\omega = \rho_\ell \frac{f_y}{f'_c} \tag{18-26b}$$

Finally, Cardenas et al. defined an axial stress parameter as,

$$\alpha = \frac{N_u}{h \ell_w f'_c} \tag{18-27}$$

where N_u represents the factored axial load, positive in compression.

Enforcing section equilibrium leads to

$$C_c + C_s - T = N_u$$

$$0.85 f'_c \, h \beta_1 c + A_{st} f_y \left(\frac{c}{\ell_w} \right) - A_{st} f_y \left(\frac{\ell_w - c}{\ell_w} \right) = N_u$$

Combining some terms and dividing both sides of this force equilibrium expression by $h \ell_w f'_c$ results in

$$0.85 \beta_1 \frac{c}{\ell_w} - \left(1 - \frac{2c}{\ell_w} \right) \frac{A_{st}}{h \ell_w} \frac{f_y}{f'_c} = \frac{N_u}{h \ell_w f'_c}$$

Substituting the definitions from Eqs. (18-26) and (18-27), we can solve for the distance to the neutral axis from the compression edge of the wall,

$$c = \left(\frac{\alpha + \omega}{0.85 \beta_1 + 2\omega} \right) \ell_w \tag{18-28}$$

With this value for c, we can use Eq. (18-25) to find all of the section forces. Then, summing moment about the compression force, C, in Fig. 18-15c, we get the following expression for the nominal moment strength of the wall section.

$$M_n = T\left(\frac{\ell_w}{2}\right) + N_u\left(\frac{\ell_w - c}{2}\right) \tag{18-29}$$

Cardenas and his colleagues were able to show good agreement between measured moment strengths from tests of shear walls and strengths calculated using Eq. (18-29).

The critical load case for evaluating the nominal moment strength of a structural wall normally corresponds to ACI Eq. (9-6) or (9-7) in ACI Code Section 9.2.1.

$$U = 0.9D + 1.0W \quad \text{(ACI Eq. 9-6)}$$

$$U = 0.9D + 1.0E \quad \text{(ACI Eq. 9-7)}$$

If service-level wind forces are specified by the governing building code, use a load factor of 1.6 for W in ACI Eq. (9-6).

Either of these load cases will minimize the factored wall axial load, N_u and thus minimize the wall nominal moment strength. Also, it can be shown for essentially all structural walls that $c < 0.375d$. Thus, the wall section is tension-controlled, and $\phi = 0.9$.

Moment Resistance of Wall Assemblies, Walls with Flanges, and Walls with Boundary Elements

Frequently, shear walls have webs and flanges that act together to form H-, C-, T-, and L-shaped wall cross sections referred to as wall *assemblies*. The effective flange widths can be taken from ACI Code Sections 8.12.2 and 8.12.3. In regions subject to earthquakes, ACI Code Section 21.9.5.2 limits the flange widths to the smaller of

(a) half the distance to an adjacent web or

(b) 25 percent of the total height of the wall.

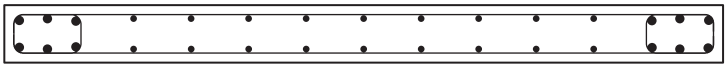
We shall use the limiting flange thicknesses from ACI Code Section 21.9.5.2 for both seismic and nonseismic walls.

Frequently, the thickness is increased at the ends of a wall to give room for concentrated vertical reinforcement that is tied like a tied column (see ACI Code Section 7.10.5). The increased thickness also helps to prevent buckling of the flanges. Regions containing concentrated and tied reinforcement are known as *boundary elements*, regardless of whether or not they are thicker than the rest of the wall.

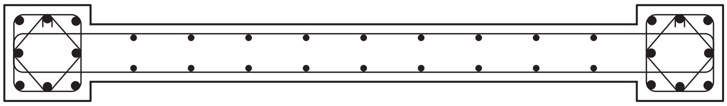
Nominal Moment Strength of Walls with Boundary Elements or Flanges

In this subsection, we will discuss structural walls that have longitudinal reinforcement concentrated at the edges to increase their nominal moment strength. A few typical examples of such walls are shown in Fig. 18-16. The longitudinal reinforcement in boundary elements similar to those in Fig. 18-16a and b will need to be tied with transverse reinforcement that satisfies ACI Code Section 7.10.5 if the walls are in regions of low or no seismic risk. The confinement requirements are more stringent for boundary elements of structural walls in regions of high seismic risk, as will be discussed in Chapter 19.

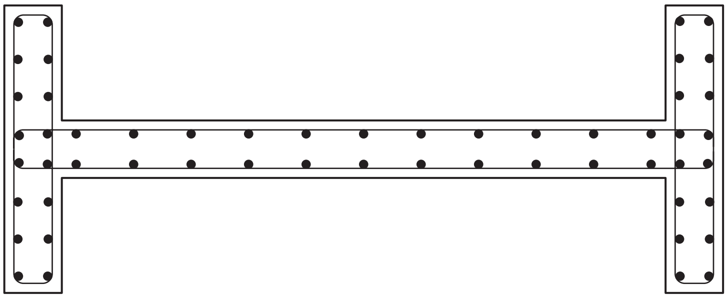
When calculating the nominal moment strength, M_n, for walls similar to those in Fig. 18-16, the contribution from the vertical reinforcement in the web is usually ignored because its contribution to M_n is quite small compared to the contribution from the vertical reinforcement concentrated at the edges of the wall. For some flanged sections or wall assemblies, as shown in Fig. 18-16c, ignoring the vertical reinforcement in the web may be



(a) Boundary element within dimensions of wall.



(b) Wall with enlarged boundary element.



(c) Wall with reinforcement concentrated in flanges.

Fig. 18-16
Structural walls with concentrated reinforcement at their edges.

too conservative. An alternative procedure for analyzing such wall assemblies in flexure is given in the next subsection.

The model used to analyze the nominal moment strength of a structural wall with boundary elements is shown in Fig. 18-17. For the boundary element in tension, the tension force is

$$T = A_s f_y \tag{18-30}$$

where A_s is the total area of longitudinal steel in the boundary element. The longitudinal steel in the compression boundary element is ignored. Using the compression stress-block model from Chapter 4, the compression force is

$$C = 0.85 f'_c \, ba \tag{18-31}$$

where b is the width of the boundary element. Enforcing section equilibrium for the vertical forces in Fig. 18-17 results in

$$a = \frac{T + N_u}{0.85 f'_c \, b} \tag{18-32}$$

Normally, the compression stress block is contained within the boundary element, as shown in Fig. 18-17. If the compression force required for section equilibrium is large enough to cause the value of a to exceed the length of the boundary element shown as b' in Fig. 18-17, then a section analysis similar to that discussed in Section 4-8 for flanged sections will be required.

As discussed in the prior subsection, the critical load case for evaluating the nominal moment strength of a structural wall normally corresponds to ACI Eq. (9-6) or (9-7) in ACI Code Section 9.2.1. Either of these load cases will minimize the factored axial load, N_u, and thus minimize the wall nominal moment strength. Summing the moment about the compression force in Fig. 18-17 leads to the following expression for M_n.

$$M_n = T\left(d - \frac{a}{2}\right) + N_u\left(\frac{\ell_w - a}{2}\right) \tag{18-33}$$

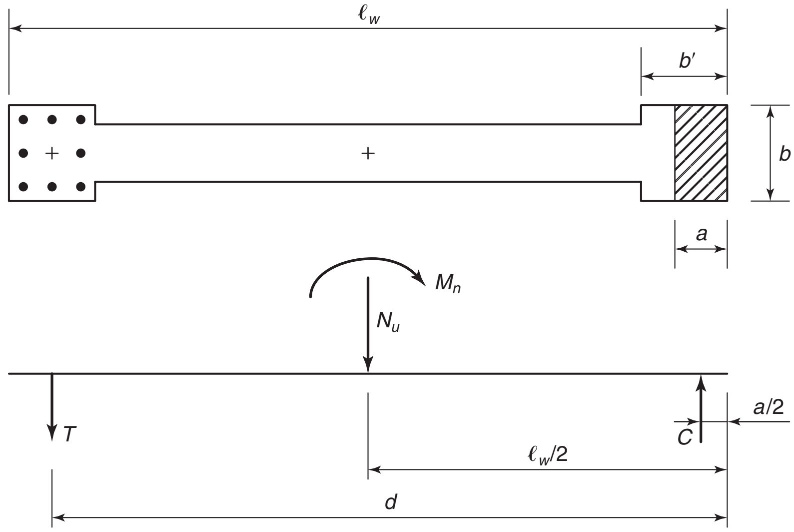


Fig. 18-17
Flexural strength model for wall with boundary elements.

For essentially all structural walls, it can be shown that the neutral-axis depth, c, is well less than $0.375d$, so the section is tension-controlled, and $\phi = 0.9$.

Nominal Moment Strength of Wall Assemblies

Paulay and Priestley [18-17] present the following method of computing the required reinforcement in a wall assembly. Figure 18-18 shows a plan of a wall consisting of a web and two flanges. The wall is loaded with a factored axial load, N_u, and a moment, M_{ua}, that causes compression in flange 1; and a shear, V_u, parallel to the web. The axial load and the moment act through the centroid of the area of the wall. The moment can be replaced by an eccentric axial load located at

$$e_a = \frac{M_{ua}}{N_u} \tag{18-34}$$

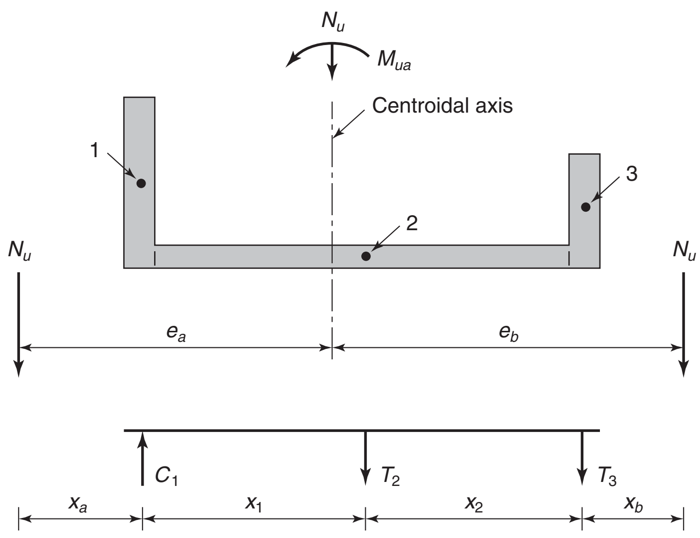


Fig. 18-18
Reinforcement forces acting on a wall segment. (From *Seismic Design of Reinforced Concrete and Masonry Buildings*, Paulay and Priestley, Copyright John Wylie & Sons, 1992. Reprinted by permission of John Wylie & Sons, Inc.)

from the centroid, which is equivalent to it acting at $x_a = e_a - x_1$ from the centroid of the flange that is in compression.

Because of the shear, V_u, the vertical reinforcement in the web will be assumed to be the minimum steel required by ACI Code Section 11.9.9.4. The calculations will be simplified by assuming that all of the web steel yields in tension. We can assume this because under cyclic loads, the neutral axis will alternately be close to the left and the right ends of the web. The steel in the web resists an axial force of

$$T_2 = A_{s2} f_y \tag{18-35}$$

where A_{s2} is the area of distributed steel in wall element 2, the web. Summing moments about the centroid of the compression flange gives the following equation for the tension, T_3, in the right-hand flange:

$$T_3 \approx \frac{N_u x_a - T_2 x_1}{x_1 + x_2} \tag{18-36}$$

In a similar manner, the force in the tension reinforcement, required in the left-hand flange for the axial load and the moment M_{ub}, causing compression in the right-hand flange can be computed as

$$T_1 \approx \frac{N_u x_b - T_2 x_2}{x_1 + x_2} \tag{18-37}$$

The required area of concentrated vertical reinforcement in the flanges can be computed by dividing the tension forces from Eqs. (18-36) and (18-37) by the product, $\phi \times f_y$.

Biaxially loaded walls. A wall is said to be biaxially loaded if it resists axial load plus moments about two axes. One method of computing the strength of such walls is the equivalent eccentricity method presented in Section 11-7. In this method, a fraction between 0.4 and 0.8 times the weak-axis moment is added to the strong-axis moment. The wall is then designed for the axial load and the combined biaxial moment treated as a case of uniaxial bending and compression.

Strictly speaking, the elastic moment resistance of an unsymmetrical wall should be computed allowing for moments about both principal axes of the cross section. This is not widely done in practice. It is generally assumed that cracking of the walls and the proportioning of the vertical wall reinforcement can be done considering moments about one orthogonal axis at a time.

Shear Transfer between Wall Segments in Wall Assemblies

For the flanges to work with the rest of the cross section of a wall assembly, so-called "vertical shear stresses" must exist on the interface between the flange and web, even when the wall and the wall segments are constructed monolithically. The stresses to be transferred are calculated in the same manner as for a composite beam, by using Eqs. (16-13) and (16-15). The reinforcement should satisfy ACI Code Section 11.6, *Shear Friction*.

18-11 SHEAR STRENGTH OF SHEAR WALLS

The design of structural walls for shear in nonseismic regions is covered in ACI Code Section 11.9, *Provisions for Walls*. The basis for this code section was a series of tests of one-third-size, planar, shear walls, reported in [18-22], [18-23], [18-24]. The test specimens were divided between flexural shear walls with ratios of $M_u/(V_u\ell_w)$ of 1.0, 2.0, and higher and short shear walls with $M_u/(V_u\ell_w) = 0.50$. The basic shear-design equations are similar to those for the shear design of prestressed concrete beams:

$$\phi V_n \geq V_u \tag{18-38}$$
(ACI Eq. 11-1)

$$V_n = V_c + V_s \tag{18-39}$$
(ACI Eq. 11-2)

$$V_s \geq \left(\frac{V_u}{\phi} - V_c\right) \tag{18-40}$$

ACI Code Section 11.9.3 limits V_n to a maximum value of $10\sqrt{f'_c}\,hd$, where d shall be taken as $0.8\ell_w$ unless a strain–compatibility analysis is used to define the centroid of the tension force in bending. For walls with concentrated vertical reinforcement in boundary elements at the edges of the walls, d may be measured from the extreme compression edge to the centroid of the concentrated vertical reinforcement near the tension edge.

V_c for Shear Walls

As with beam design, the concrete contribution to shear strength is set approximately equal to the value of shear that causes shear (inclined) cracking in a structural wall. For walls subjected to axial compression, a designer is permitted to use Eq. (18-41), unless a more detailed analysis is made, as will be discussed in the next paragraph,

$$V_c = 2\lambda\sqrt{f'_c}\,hd \tag{18-41}$$

where λ is the factor for lightweight aggregate concrete. It is taken as 1.0 for normal-weight concrete and shall be used as defined in ACI Code Section 8.6.1 for lightweight concrete. For walls subjected to axial tension, a designer must use ACI Code Eq. (11-8) with h substituted for b_w,

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\lambda\sqrt{f'_c}\,hd \tag{18-42}$$

where N_u is negative for tension and N_u/A_g shall be expressed in psi units.

For structural walls subjected to axial compression, ACI Code Section 11.9.6 permits V_c to be taken as the smaller of

$$V_c = 3.3\lambda\sqrt{f'_c}\,hd + \frac{N_u d}{4\ell_w} \tag{18-43}$$
(ACI Eq. 11-27)

or,

$$V_c = \left[0.6\lambda\sqrt{f'_c} + \frac{\ell_w\left(1.25\lambda\sqrt{f'_c} + 0.2\dfrac{N_u}{\ell_w h}\right)}{\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2}}\right]hd \tag{18-44}$$
(ACI Eq. 11-28)

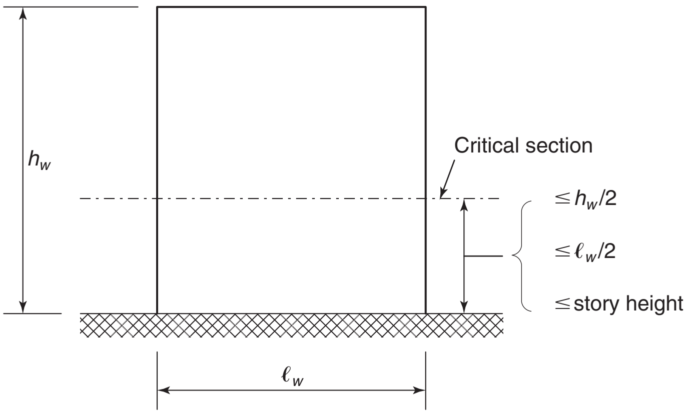


Fig. 18-19
Location of critical section for checking flexural-shear strength.

Equation (18-43) corresponds to the shear force at the initiation of *web-shear* cracking and normally will govern for short walls. Equation (18-44) normally will govern for slender walls and corresponds to the shear force at the initiation of *flexural-shear* cracking at a section approximately $\ell_w/2$ above the base of the wall. If the quantity $(M_u/V_u - \ell_w/2)$ in Eq. (18-44) is negative, then Eq. (18-44) does not apply to the wall being analyzed. The value for M_u/V_u is to be evaluated at a section above the base of the wall, and the distance to that section is to be taken as the smallest of $\ell_w/2$, $h_w/2$, and one story height (Fig. 18-19). The value of V_c computed at that section may be used throughout the height of the shear wall.

Shear Reinforcement for Structural Walls

Shear reinforcement for structural walls always consists of evenly distributed vertical and horizontal reinforcement. In many cases, shear cracks in walls are relatively shallow (i.e., their inclination with respect to a horizontal line is less than 45°), so vertical reinforcement will be just as effective—if not more effective—as horizontal reinforcement in controlling the width and growth of such cracks. However, the shear-strength contribution from wall reinforcement is based on the size and spacing of the horizontal reinforcement.

In many cases, only minimum amounts of shear reinforcement are required in structural walls, and those minimum amounts are a function of the amount of shear being resisted by the structural wall. If the factored shear force, V_u, is less than $\phi V_c/2$, then the distributed vertical and horizontal wall reinforcement must satisfy the requirements of ACI Code Section 14.3, as summarized in Table 18-1. If a designer uses Grade-60 reinforcement and bar sizes not larger than No. 5, then the minimum percentage of vertical steel is 0.0012, and the minimum percentage of horizontal steel is 0.0020. Referring to Fig. 18-20 for notation definitions, the percentage of vertical (longitudinal) steel is

$$\rho_\ell = \frac{A_{v,\text{vert}}}{hs_1} \tag{18-45a}$$

And the percentage of horizontal (transverse) steel is

$$\rho_t = \frac{A_{v,\text{horiz}}}{hs_2} \tag{18-45b}$$

For both the horizontal and vertical steel, ACI Code Section 14.3.5 limits the bar spacing to the smaller of $3h$ and 18 in.

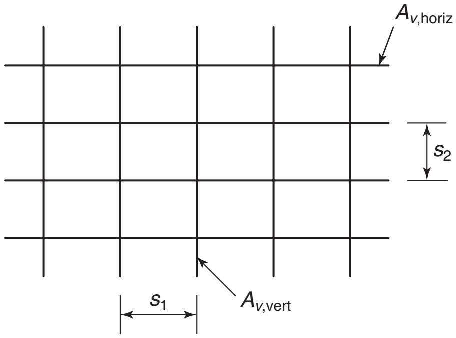


Fig. 18-20
Distribution of vertical and horizontal shear reinforcement in the web of a shear wall.

For walls resisting a higher factored shear force (i.e., $\phi V_c/2 < V_u \leq \phi V_c$), the vertical and horizontal steel in the wall must satisfy the minimum percentages and the maximum spacing requirements of ACI Code Section 11.9.9, as summarized in Table 18-1. The minimum percentage of horizontal (transverse) reinforcement, ρ_t, is 0.0025 and is to be placed at a spacing that does not exceed the smallest of $\ell_w/5$, $3h$, and 18 in. Vertical reinforcement is to be placed at a spacing that does not exceed the smallest of $\ell_w/3$, $3h$, and 18 in. The percentage of vertical (longitudinal) steel, ρ_ℓ, shall not be less than the *larger* of 0.0025 and that calculated using ACI Code Eq. (11-30):

$$\rho_\ell = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_t - 0.0025) \tag{18-46}$$

(ACI Eq. 11-30)

For walls with $h_w/\ell_w \geq 2.5$, this equation will not govern. In shorter walls where the horizontal reinforcement percentage, ρ_t, exceeds 0.0025, the value of ρ_ℓ calculated in Eq. (18-46) does not need to exceed the amount (percentage) of horizontal reinforcement required for shear strength, as given next.

If walls are required to resist a factored shear force that exceeds ϕV_c, then horizontal reinforcement must be provided to satisfy the strength requirement expressed in Eq. (18-40). The shear strength, V_s, provided by the horizontal reinforcement is given by ACI Code Eq. (11-29):

$$V_s = \frac{A_v f_y d}{s} \tag{18-47}$$

(ACI Code Eq. 11-29)

In this equation, A_v is the same as $A_{v,\text{horiz}}$ shown in Fig. 18-20, and s is the same as s_2 in that figure. As was done for shear-strength requirements in Chapter 11, the designer has the option to select a bar size and spacing to satisfy the strength requirement in Eq. (18-40). In addition to satisfying the shear-strength requirement, a designer must check that both the horizontal and vertical wall reinforcement satisfy the minimum reinforcement percentages and maximum spacing limits given in ACI Code Section 11.9.9, which were discussed in the prior paragraph.

Shear Strength of Structural Walls Resisting Seismic Loads

Chapter 19 discusses *Design for Earthquake Effects*, but the author thought it was essential to include here the modified shear-strength requirements given in ACI Code Chapter 21 for structural walls. Although the equation that defines the nominal shear strength in ACI Code

Chapter 21 appears to be significantly different from that given by the equations in ACI Code Chapter 11, the final values for V_n are not substantially different.

The nominal shear strength, V_n, for a wall designed to resist shear forces due to earthquake ground motions is given by ACI Code Eq. (21-7):

$$V_n = A_{cv}\,(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y) \tag{18-48}$$

(ACI Eq. 21-7)

In this equation, A_{cv} is taken as the width of the web of the wall, h, multiplied by the total length of the wall, ℓ_w. This area is larger than the effective shear area, hd, used for the equations in ACI Code Chapter 11.

The first term inside the parenthesis of Eq. (18-48) represents the concrete contribution to shear strength, V_c. The coefficient, α_c, represents the difference between the expected occurrence of *flexure-shear* cracking in slender walls and *web-shear* cracking in short walls. The value of α_c is taken as 2.0 for walls with $h_w/\ell_w \geq 2.0$ and as 3.0 for walls with $h_w/\ell_w \leq 1.5$. A linear variation for the value of α_c is to be used for walls with h_w/ℓ_w ratios between 1.5 and 2.0.

The second term inside the parenthesis of Eq. (18-48) represents the shear-strength contribution from the horizontal wall reinforcement, V_s. Multiplying the definition of ρ_t given in Eq. (18-45b) by the definition of $A_{cv}(h\ell_w)$ and the yield strength, f_y, results in the following equivalent value for V_s.

$$V_{s,\text{equiv}} = \frac{A_{v,\text{horiz}}}{hs_2} f_y h \ell_w$$

$$V_{s,\text{equiv}} = \frac{A_{v,\text{horiz}} f_y \ell_w}{s_2} \tag{18-49}$$

This equation is similar to Eq. (18-47), which gives the value for V_s used in ACI Code Chapter 11. However, because $\ell_w > d$, the value of V_s used in ACI Code Chapter 21 exceeds the value of V_s used in ACI Code Chapter 11.

The maximum allowable value for the nominal shear strength, V_n, from Eq. (18-48) is limited to $8A_{cv}\sqrt{f'_c}$ in ACI Code Section 21.9.4.4. Again, this value is similar, but not the same as the upper limit of $10\sqrt{f'_c}\,hd$ given in ACI Code Section 11.9.3.

The requirements for minimum vertical and horizontal reinforcement percentages and for maximum permissible spacing are the same as those given in ACI Code Section 11.9.9, which were discussed previously. The only modification is given in ACI Code Section 21.9.4.3, which states that for walls with $h_w/\ell_w \leq 2.0$, the value of ρ_ℓ (vertical steel) shall not be less that the value of ρ_t. This requirement reflects the fact that in short walls, the vertical reinforcement is equal to or more efficient than the horizontal reinforcement in controlling the width and growth of inclined shear cracks.

The discrepancies noted here between the shear-strength equations given in ACI Code Chapters 11 and 21 can create a dilemma for structural designers. The shear strength of walls designed to resist lateral wind forces should be determined from the equations in ACI Code Chapter 11. However, if the same wall is designed to also resist equivalent lateral forces due to earthquake ground motions, the shear strength must be checked using the equations in ACI Code Chapter 21. A unification of these different shear-strength equations is a major goal of the Code Committee as it works toward future editions of the ACI Code.

Shear Transfer across Construction Joints

The shear force transferred across construction joints can be designed using shear friction. The clamping force is the sum of the tensile reinforcement force components of all the tensile bar forces and permanent compressive forces acting on the joint. This includes all the reinforcement perpendicular to the joint, regardless of the primary use of this steel.

EXAMPLE 18-2 Structural Wall Subjected to Lateral Wind Loads

The moment and shear strengths of the structural wall shown in Fig. 18-21 are to be evaluated for the combined gravity and lateral loads applied to the wall. The wall is 18 ft long and is 10 in. thick. A uniform distribution of vertical and horizontal reinforcement is used in two layers, one near the front face of the wall and the other near the back face. The vertical reinforcement consists of No. 5 bars at 18 in. on centers in each face, and the horizontal reinforcement consists of No. 4 bars at 16 in. on centers in each face. Normal-weight concrete with a compressive strength of 4000 psi is used in the wall. All of the wall reinforcement is Grade-60 steel ($f_y = 60$ ksi).

The gravity loads applied at each floor level, as shown in Fig. 18-21, are due to dead load. The live loads are not shown but are assumed to be equal to approximately one-half of the dead loads. The lateral wind loads are based on service-level wind forces and did include the directionality factor.

1. Make an Initial Check of Wall Reinforcement. The percentage and spacing of the vertical and horizontal reinforcement will be checked before we do the strength calculations. The percentage of horizontal reinforcement is found using Eq. (18-45b):

$$\rho_t = \frac{A_{v,\text{horiz}}}{hs_2} = \frac{2 \times 0.20 \text{ in.}^2}{10 \text{ in.} \times 16 \text{ in.}} = 0.0025$$

This satisfies the minimum requirement in ACI Code Section 11.9.9.2, so it should be acceptable unless a larger amount is required to satisfy shear strength requirements. The maximum center-to-center spacing for the horizontal reinforcement is the smallest of $\ell_w/5$

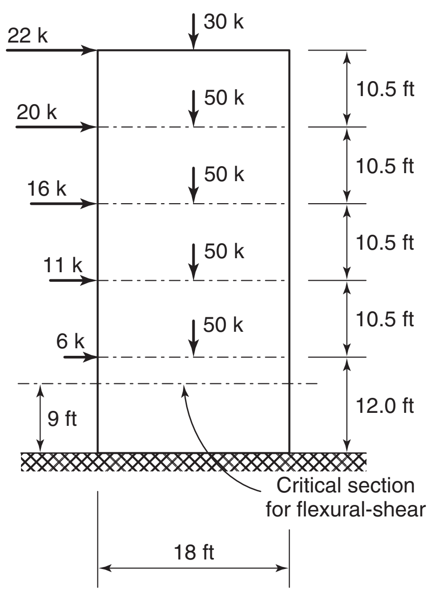


Fig. 18-21
Structural wall for
Example 18-2.

(3.6 ft = 43.2 in.), $3h$ (30 in.), and 18 in. Thus, the provided spacing for the horizontal reinforcement is o.k.

Although it is good practice to use larger vertical bars at the edges of the wall, say No. 6 or No. 7 bars, we will calculate the percentage of vertical reinforcement assuming only No. 5 bars are used. From Eq. (18-45a),

$$\rho_\ell = \frac{A_{v,\text{vert}}}{hs_1} = \frac{2 \times 0.31 \text{ in.}^2}{10 \text{ in.} \times 18 \text{ in.}} = 0.00344$$

Because the wall has an aspect ratio, $h_w/\ell_w = 3.0 > 2.5$, Eq. (18-46) will not govern for the minimum percentage of vertical reinforcement. Thus, the minimum percentage of vertical reinforcement is 0.0025 (ACI Code Section 11.9.9.4), which is less than what is provided. The spacing limit for the vertical reinforcement is the smallest of $\ell_w/3$ (6 ft = 72 in.), $3h$ (30 in.), and 18 in. Thus, the provided spacing of vertical reinforcement is o.k.

2. Check Moment Strength. The moment at the base of the wall is equal to the sum of the products of the lateral forces times their respective distances to the base of the wall:

$$\begin{aligned} M(\text{base}) &= 22 \times 54 + 20 \times 43.5 + 16 \times 33 + 11 \times 22.5 + 6 \times 12 \\ &= 2910 \text{ kip-ft} \end{aligned}$$

The appropriate load factor for service-level wind forces is 1.6. Thus, the factored moment at the base of the wall is

$$M_u(\text{base}) = 1.6 \times 2910 = 4660 \text{ kip-ft}$$

The analysis given in [18-23] will be used to evaluate the moment strength of the wall. Using ACI Code Eq. (9-6), the factored axial load is

$$N_u = 0.9N_D = 0.9 \times 230 \text{ kips} = 207 \text{ kips}$$

For 4000-psi concrete, $\beta_1 = 0.85$. Other required parameters are

$$\omega = \rho_\ell \frac{f_y}{f'_c} = 0.00344 \frac{60 \text{ ksi}}{4 \text{ ksi}} = 0.0516 \tag{18-26b}$$

and

$$\alpha = \frac{N_u}{h\ell_w f'_c} = \frac{207 \text{ kip}}{10 \text{ in.} \times 216 \text{ in.} \times 4 \text{ ksi}} = 0.0240 \tag{18-27}$$

With these parameters, we can use Eq. (18-28) to find the depth to the neutral axis:

$$c = \left(\frac{\alpha + \omega}{0.85\beta_1 + 2\omega}\right)\ell_w$$

$$c = \left(\frac{0.0240 + 0.0516}{0.85 \times 0.85 + 2 \times 0.0516}\right)216 \text{ in.} = 19.8 \text{ in.}$$

If we assume that the effective flexural depth, d, is approximately equal to $0.8\ell_w = 173$ in., it is clear that c is significantly less than $0.375d$. Thus, this is a tension-controlled section, and we will use a strength reduction factor, ϕ, equal to 0.9.

To calculate the nominal moment strength, we first must determine the tension force at nominal strength conditions. The value of A_{st} for the vertical steel can be calculated as

$$A_{st} = 2A_b \frac{\ell_w}{s_1} = 2 \times 0.31 \text{ in.}^2 \times \frac{216 \text{ in.}}{18 \text{ in.}} = 7.44 \text{ in.}^2$$

Then, from Eq. (18-25a),

$$T = A_{st} f_y \left(\frac{\ell_w - c}{\ell_w} \right)$$

$$T = 7.44 \text{ in.}^2 \times 60 \text{ ksi} \left(\frac{216 \text{ in.} - 19.8 \text{ in.}}{216 \text{ in.}} \right) = 405 \text{ kips}$$

Now, referring to the vector forces in Fig. 18-15c, we can use Eq. (18-29) to calculate the nominal moment strength of the structural wall.

$$\begin{aligned} M_n &= T\left(\frac{\ell_w}{2} \right) + N_u \left(\frac{\ell_w - c}{2} \right) \\ &= 405 \text{ k} \left(\frac{216 \text{ in.}}{2} \right) + 207 \text{ k} \left(\frac{216 \text{ in.} - 19.8 \text{ in.}}{2} \right) \\ &= 64{,}000 \text{ kip-in.} = 5340 \text{ kip-ft} \end{aligned}$$

Using the strength reduction factor, ϕ, we get

$$\phi M_n = 0.9 \times 5340 = 4800 \text{ kip-ft}$$

Because ϕM_n is larger than M_u (4660 kip-ft), the wall has adequate flexural strength.

3. Check Shear Strength. The factored shear at the base of the wall is

$$V_u = 1.6(6 + 11 + 16 + 20 + 22) = 120 \text{ kips}$$

Because this wall is slender, $h_w/\ell_w = 3.0$, we can assume that Eq. (18-44) will govern for the shear-strength contribution from the concrete, V_c. However, we will check both Eqs. (18-43) and (18-44) to demonstrate how each is used. For normal-weight concrete, $\lambda = 1.0$. Using $d = 0.8\ell_w$ (173 in.) and putting all of the quantities into units of pounds and inches, the value of V_c from Eq. (18-43) is

$$\begin{aligned} V_c &= 3.3\lambda\sqrt{f'_c}\, hd + \frac{N_u d}{4\ell_w} \\ &= 3.3 \times 1\sqrt{4000} \times 10 \times 173 + \frac{207{,}000 \times 173}{4 \times 216} \\ &= (361{,}000 + 41{,}400) \text{ lbs} = 402 \text{ kips} \end{aligned}$$

For Eq. (18-44), we need to evaluate the ratio of M_u/V_u at the critical section above the base of the wall. The distance to that section is the smallest of $\ell_w/2$ (9 ft), $h_w/2$ (27 ft), and one story height (12 ft). In this case, $\ell_w/2$ governs, as shown in Fig. 18-21. The factored moment at that section can be found from the following as

$$\begin{aligned} M_u(\text{crit. sect.}) &= M_u\,(\text{base}) - V_u\,(\text{base})\frac{\ell_w}{2} \\ &= 4660 \text{ kip-ft} - 120 \text{ kip} \times 9 \text{ ft} = 3580 \text{ kip-ft} \end{aligned}$$

Thus, the ratio of $M_u/V_u = 3580/120 = 29.8$ ft. Using this value in Eq. (18-44) and expressing all of the quantities in pounds and inches, we have

$$V_c = \left[0.6\lambda\sqrt{f'_c} + \frac{\ell_w\left(1.25\lambda\sqrt{f'_c} + 0.2\dfrac{N_u}{\ell_w h}\right)}{\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2}}\right]hd$$

$$= \left[37.9 \text{ psi} + \frac{216 \text{ in.}(79.1 \text{ psi} + 19.2 \text{ psi})}{358 \text{ in.} - 108 \text{ in.}}\right] \times 10 \text{ in.} \times 173 \text{ in.}$$

$$= 212{,}000 \text{ lbs} = 212 \text{ kips}$$

This value governs for V_c, as expected. Using $\phi = 0.75$, as defined in ACI Code Section 9.3.2.3 for shear and torsion, $\phi V_c = 0.75 \times 212 = 159$ kips. This exceeds V_u, so no calculation of the value of V_s for the provided horizontal (transverse) reinforcement is required. It is clear that the value of $0.5\phi V_c$ is less than V_u, so the requirements for horizontal and vertical reinforcement in ACI Code Section 11.9.9 will govern for this wall. Those requirements were checked in step 1 and were found to be at or above the ACI Code required values.

Thus, the 10-in. thick structural wall with the indicated vertical and horizontal reinforcement has adequate moment and shear strength and the reinforcement satisfies all of the ACI Code requirements. ■

EXAMPLE 18-3 Structural Wall Subjected to Equivalent Lateral Earthquake Loads

The structural wall shown in Fig. 18-22 is to be analyzed for a combined gravity load and equivalent lateral load due to earthquake ground motions. The dimensions of the wall and the section reinforcement (Fig. 18-22b) are taken from a paper by Wallace and Thomsen [18-25], in which the authors analyzed the need for special confinement reinforcement for the boundary elements at the edges of the wall. The details for special confinement reinforcement will be discussed in Chapter 19.

This wall is one of several walls resisting lateral loads for the structure that was analyzed by Wallace and Thomsen [18-25]. The lateral earthquake force, E, assigned to this wall is 205 kips. The dead and reduced live load at the base of the wall are $N_D = 1000$ kips and $N_L = 450$ kips. Use $f'_c = 4000$ psi (normal-weight concrete) and $f_y = 60$ ksi for all reinforcement.

1. Check Moment Strength at Base of Wall. ACI Code Eq. (9-7) will be used to find the factored loads for this analysis.

$$U = 0.9D + 1.0E \qquad \text{(ACI Eq. 9-7)}$$

Seismic lateral loads, as defined in [18-18], typically have an inverted triangular distribution over the height of the building. Therefore, we will assume that the variable x shown in Fig. 18-22a, which is taken as the assumed distance from the base of the wall to the centroid of the lateral force, has a value of $\frac{2}{3}\,h_w$. Thus, the factored moment at the base of the wall is

$$M_u = 1.0 \times 205 \text{ k} \times \tfrac{2}{3} \times 120 \text{ ft}$$

$$= 16{,}400 \text{ kip-ft}$$

The nominal moment strength of the wall will be analyzed by the procedure shown in Fig. 18-17 for a wall with a vertical reinforcement concentrated in a boundary element. The tension strength for the concentrated reinforcement is

$$T = A_s f_y = 10 \times 1.27 \text{ in.}^2 \times 60 \text{ ksi} = 762 \text{ kips}$$

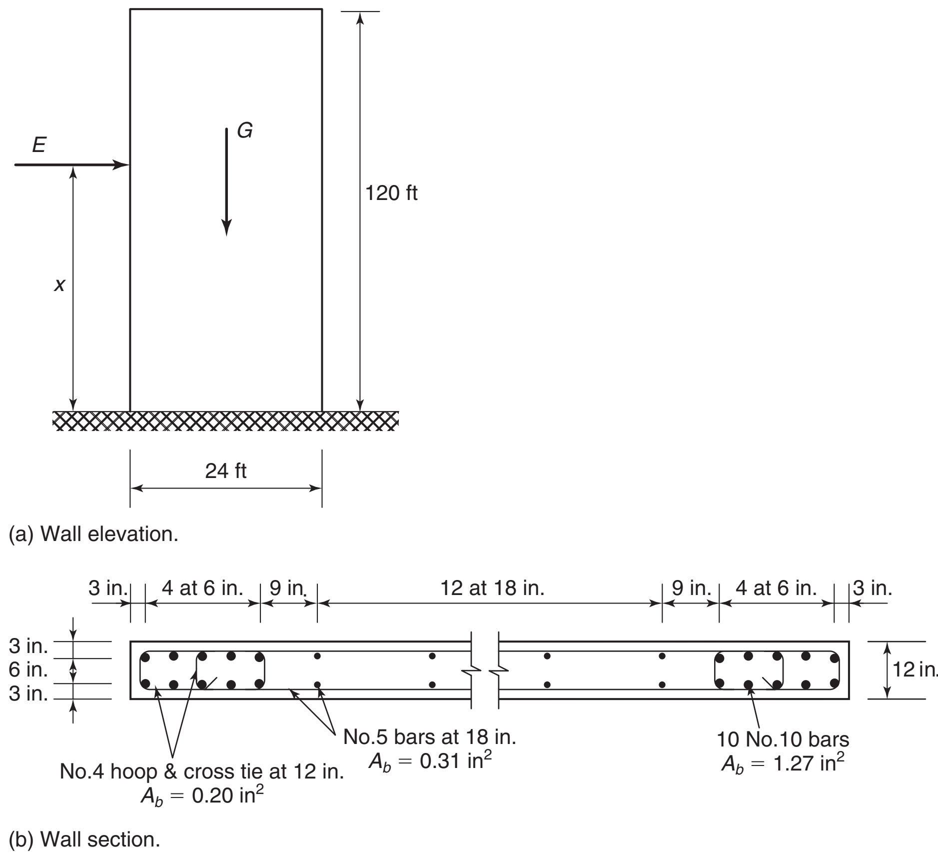


Fig. 18-22
Structural wall for
Example 18-3.

The distance from the compression edge to the centroid of the tension reinforcement is

$$d = \ell_w - (3 \text{ in.} + 2 \times 6 \text{ in.}) = 288 - 15 = 273 \text{ in.}$$

The factored axial load for this load case is

$$N_u = 0.9N_D = 0.9 \times 1000 \text{ k} = 900 \text{ kips}$$

Now, we will use Eq. (18-32) to determine the depth of the compression stress block. In this case, $b = h = 12$ in., giving

$$a = \frac{T + N_u}{0.85f'_c\, b} = \frac{762 \text{ k} + 900 \text{ k}}{0.85 \times 4 \text{ ksi} \times 12 \text{ in.}} = 40.7 \text{ in.}$$

The distance to the neutral axis is, $c = a/\beta_1 = 40.7/0.85 = 47.9$ in., which is well less than $0.375d = 0.375 \times 273 = 102$ in. Therefore, this is a tension-controlled section, and $\phi = 0.9$. Eq. (18-33) will be multiplied by ϕ to find ϕM_n:

$$\phi M_n = \phi\left[T\left(d - \frac{a}{2}\right) + N_u\left(\frac{\ell_w - a}{2}\right)\right]$$

$$= 0.9\left[762\text{ k}\left(273 - \frac{40.7}{2}\right)\text{in.} + 900\text{ k}\left(\frac{288 - 40.7}{2}\right)\text{ in.}\right]$$
$$= 0.9[193{,}000\text{ k-in.} + 111{,}000\text{ k-in.}]$$
$$= 273{,}000\text{ kip-in.} = 22{,}800\text{ kip-ft}$$

Because ϕM_n exceeds M_u, the wall has adequate moment strength.

2. Capacity-Based Design Shear. In seismic design, the design shear force, V_u, is not often based on the factored loads but rather on the *probable flexural strength* of the member in question. This is referred to as the *capacity-based design* procedure and is intended to ensure a ductile flexural response in the member—as opposed to a brittle shear failure—if the member is loaded beyond its elastic range of behavior during an earthquake.

For the wall in this example, the probable flexural strength should be based on a *probable* axial load that the wall will be carrying at the time of an earthquake. A value for such an axial load is not defined in either the ACI Code or ASCE/SEI 7-10 [18-18]. The author will assume that a reasonable value for the probable axial load is the sum of the unfactored dead load plus the unfactored reduced live load.

$$N_{pr} = N_D + N_L = 1000\text{ k} + 450\text{ k} = 1450\text{ kips}$$

With this axial load, the wall moment strength will be reevaluated and referred to as the probable moment strength, M_{pr}. First, the depth of the compression stress block is

$$a = \frac{T + N_u}{0.85 f'_c\, b} = 54.2\text{ in.}$$

Although this is larger than calculated previously, it still is clear that the tension steel in the boundary element will be *yielding*. Note, we do not need to show that this is a tension-controlled section. We only need to show that this is an *underreinforced section* and that the tension steel will be yielding when the concrete in the compression zone reaches the maximum useable strain of 0.003. With this calculated value of a, we are now ready to calculate the moment strength.

$$M_{pr} = T\left(d - \frac{a}{2}\right) + N_{pr}\left(\frac{\ell_w - a}{2}\right)$$
$$= 187{,}000\text{ k-in.} + 169{,}000\text{ k-in.}$$
$$= 357{,}000\text{ kip-in.} = 29{,}700\text{ kip-ft}$$

The final step is to find the lateral shear force required to develop this moment at the base of the wall. As originally discussed by Bertero et al. [18-26], the lateral load distribution over the height of a shear wall that is part of a complete structural system will tend to be closer to a uniform distribution at peak response of the structure when it is subjected to large earthquake motions. Therefore, to more conservatively predict the capacity-based shear force acting on the wall when the base moment reaches M_{pr}, the author recommends using a value of $x = 0.5h_w$ in Fig. 18-22a. Using that value, the capacity-based design shear is

$$V_u(\text{cap-based}) = \frac{M_{pr}}{0.5h_w} = \frac{29{,}700\text{ k-ft}}{0.5 \times 120\text{ ft}} = 495\text{ kips}$$

For this particular structural wall, the capacity-based design shear is approximately 2.5 times the design shear used to check the flexural strength.

3. Check Shear Strength. Eq. (18-48), which is the same as ACI Code Eq. (21-7), will be used to check the shear strength of the wall. For this wall, the value of A_{cv} in that equation is equal to the gross wall area, so

$$A_{cv} = h\ell_w = 12 \text{ in.} \times 288 \text{ in.} = 3460 \text{ in.}^2$$

Because this is a slender wall, $\alpha_c = 2.0$. Also, because the wall is constructed with normal-weight concrete, $\lambda = 1.0$. Eq. (18-45b) will be used to determine ρ_t for the distributed horizontal reinforcement in Fig. 18-22b as

$$\rho_t = \frac{A_{v,\text{horiz}}}{hs_2} = \frac{2 \times 0.31 \text{ in.}^2}{12 \text{ in.} \times 18 \text{ in.}} = 0.00287$$

This is greater than the minimum required percentage of 0.0025 from ACI Code Section 11.9.9.2, and the 18-in. spacing satisfies the limits in ACI Code Section 11.9.9.3. Using the values calculated here and units of pounds and inches, the nominal shear strength of the wall from Eq. (18-48) is

$$\begin{aligned} V_n &= A_{cv}(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y) \\ &= 3460 \text{ in.}^2 \,(2 \times 1\sqrt{4000} + 0.00287 \times 60{,}000) \\ &= 3460 \text{ in.}^2 \,(126 \text{ psi} + 172 \text{ psi}) \\ &= 1.03 \times 10^6 \text{ lbs} = 1030 \text{ kips} \end{aligned}$$

ACI Code Section 21.9.4.4 limits the value of V_n to $8A_{cv}\sqrt{f'_c} = 1750$ kips. Thus, we can use the value calculated here. ACI Code Section 9.3.2.3 states that for shear and torsion, $\phi = 0.75$. So, the reduced, nominal shear strength of the wall is

$$\phi V_n = 0.75 \times 1030 \text{ k} = 773 \text{ kips}$$

Because $\phi V_n > V_u$(cap-based), the wall shear strength is o.k.

Thus, the 12-in. thick structural wall with the indicated vertical and horizontal reinforcement has adequate moment and shear strength, and the reinforcement satisfies all of the ACI Code requirements. ■

18-12 CRITICAL LOADS FOR AXIALLY LOADED WALLS

Buckling of Compressed Walls

The critical stress for buckling of a hinged column or a one-way wall, with a rectangular cross sectional area $(b \times h)$ is

$$\sigma_{cr} = \frac{P_{cr}}{bh} = \frac{\pi^2 EI}{(k\ell)^2}\left(\frac{1}{bh}\right) \tag{18-50}$$

where b is the width of the wall, and $k\ell$ is the *effective length* of the wall. The flexural stiffness of a wall section of width b and thickness h is.

$$EI = \frac{Ebh^3}{12}$$