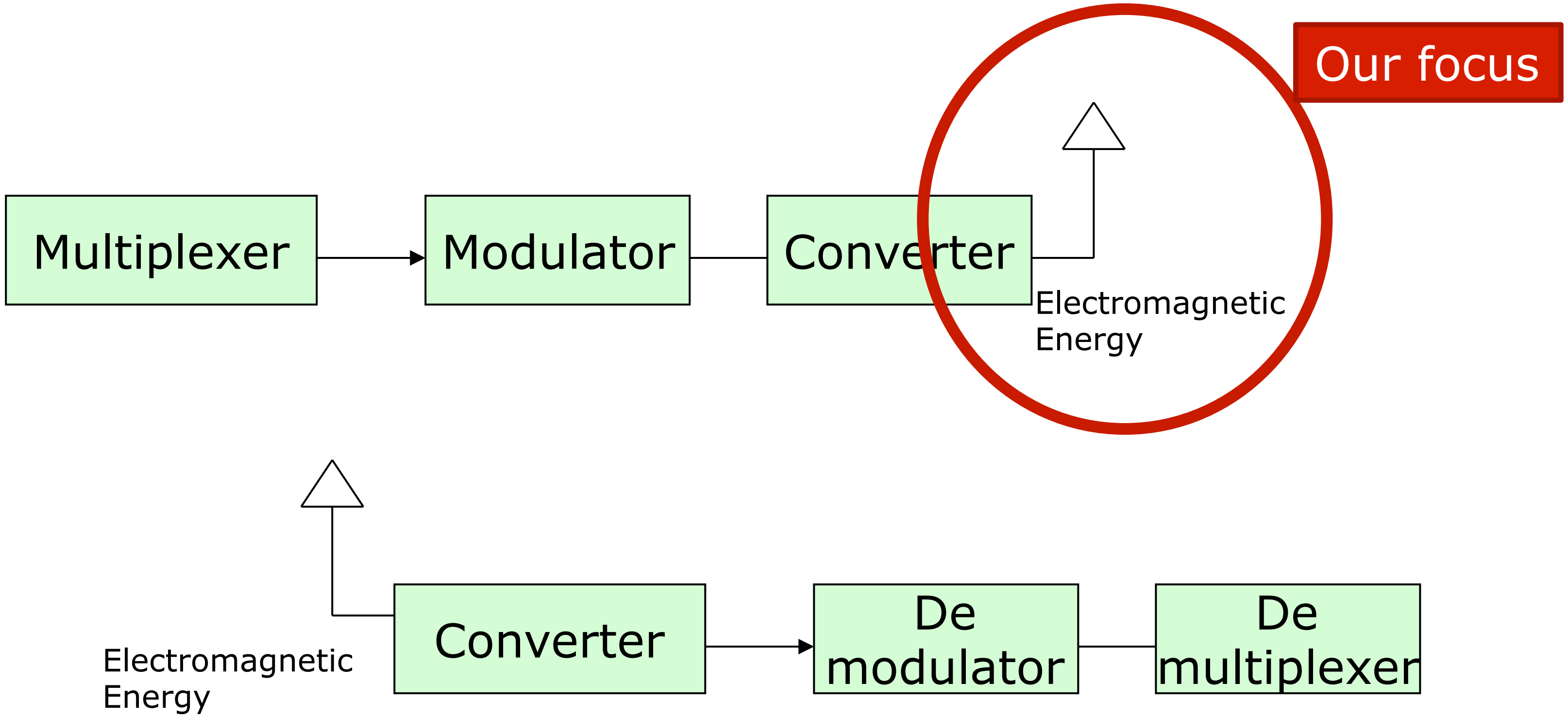
Antennas and Propagation

updated: 09/29/2014

Wireless Communication Systems



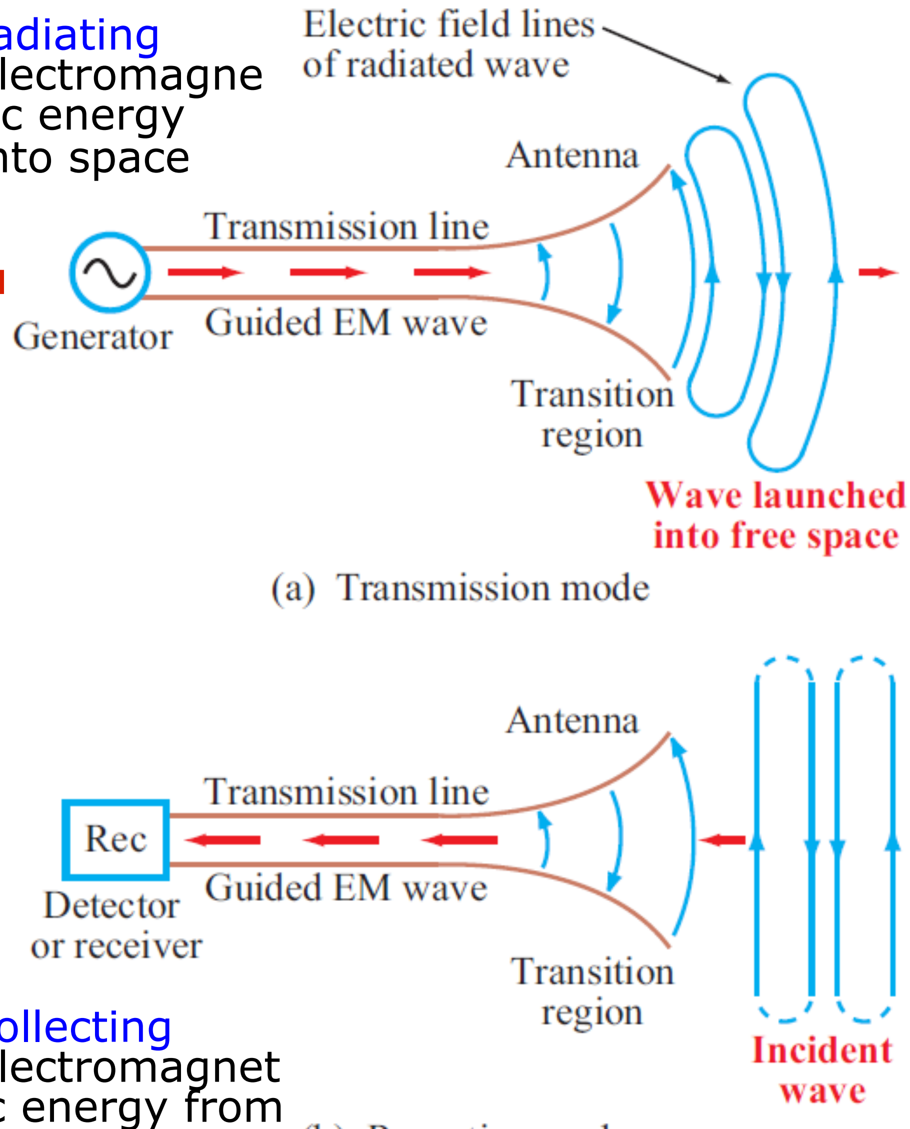
Antenna Characteristics

- Radiation patterns
- Radiated power
- Half-power beam width of the antenna
- Antenna position, shape, and length
- Antenna gain with respect to an ideal case

Remember: Antenna Properties

- An antenna is an electrical conductor (transducer) or system of conductors
  - They carry time-varying currents and, consequently, accelerating electrons
  - → A Transmission Antenna radiates electromagnetic energy into space
  - → A Reception Antenna collects electromagnetic energy from space

radiating electromagnetic energy into space

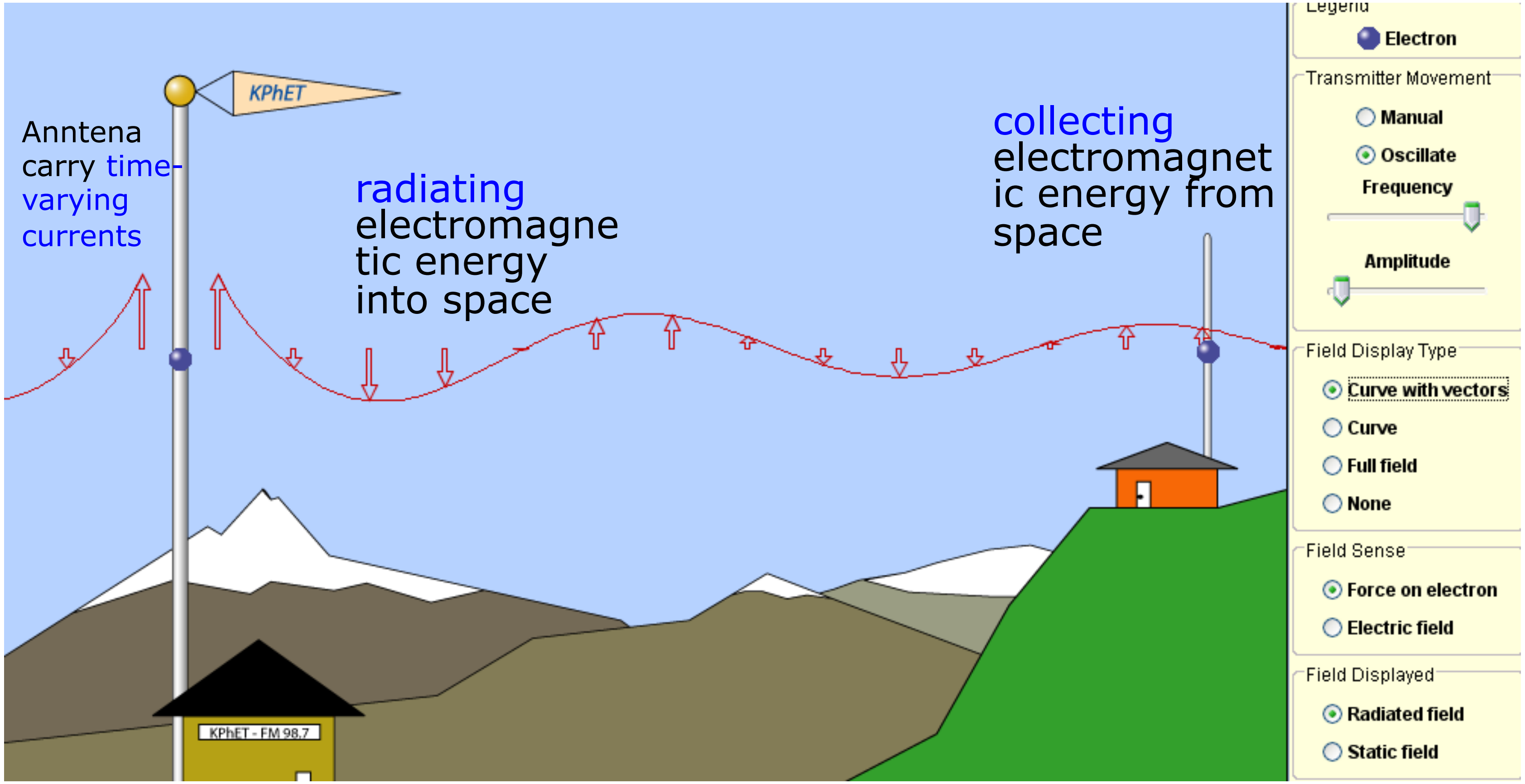


(a) Transmission mode

(b) Reception mode

collecting electromagnetic energy from space

Remember:
Waves and Propagation - Demo

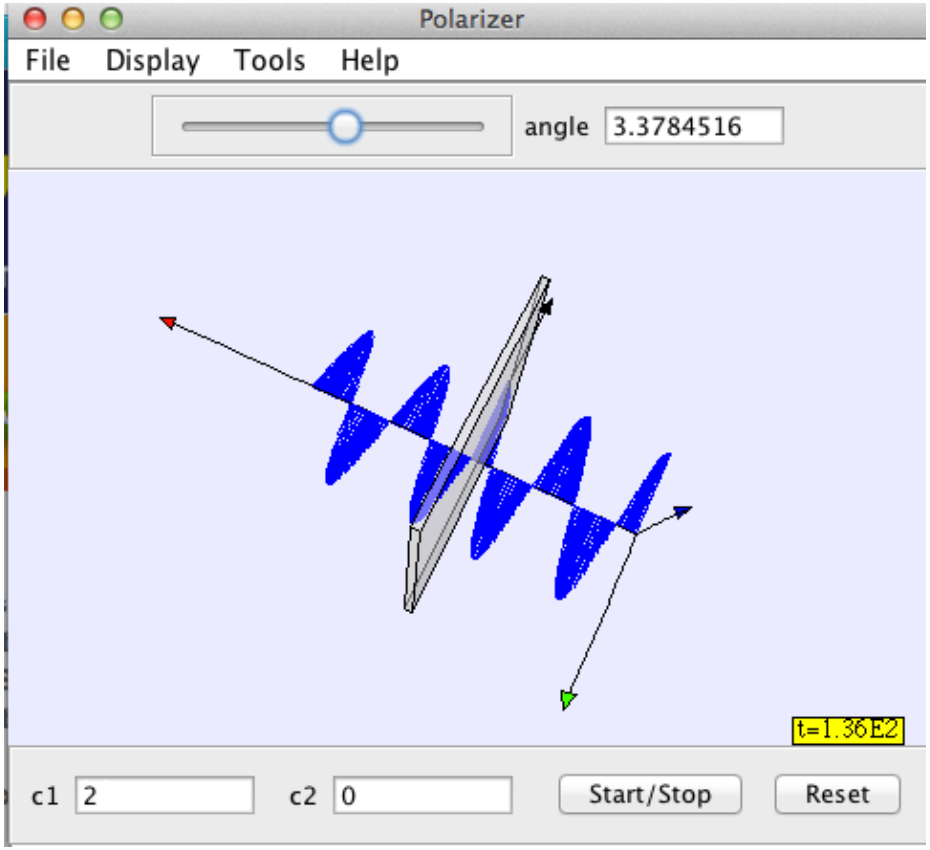


http://phet.colorado.edu/simulations/sims.php?sim=Radio_Waves_and_Electromagnetic_Fields

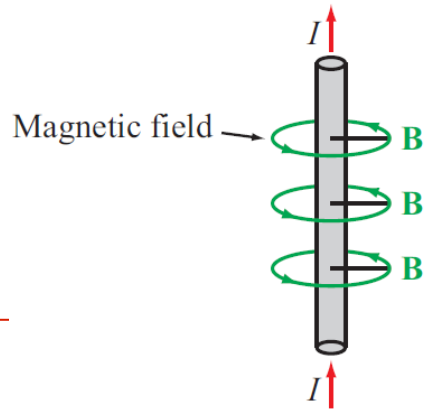
Remember: Antenna Properties
Reciprocal Devices

- Most antennas are reciprocal devices,
  - That is they are exhibiting the same radiation pattern for transmission as for reception
- When operating in the receiving mode, the antenna captures the incident wave
  - Only that component of the wave whose electric field matches the antenna polarization state is detected
- In two-way communication, the same antenna can be used for transmission and reception
  - Antenna characteristics are the same for transmitting or receiving electromagnetic energy
- The antenna can receive on one frequency and transmit on another

Polarization



http://www.cabrillo.edu/~jmccullough/Applets/optics.html



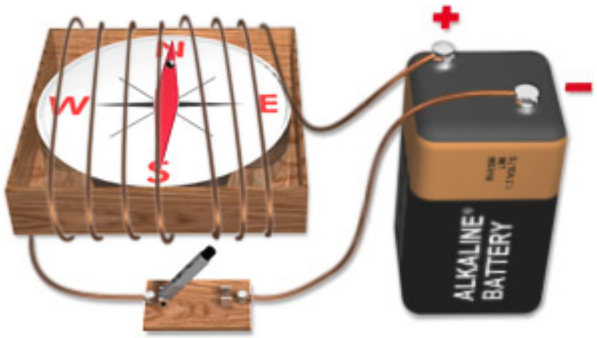
Start With Basics:

- We know:
  - Q (static charges) → E field– capacitor example[1]
  - I (moving charges) → H field– compass example[2]
  - Thus, in the presence of time-varying current → we will obtain interdependent EM fields
- A time-varying current **(I)** along a wire generates rings of Electromagnetic field **(B)** around the wire
- Similarly the current passing through a coil generates Electromagnetic field in the Z axis

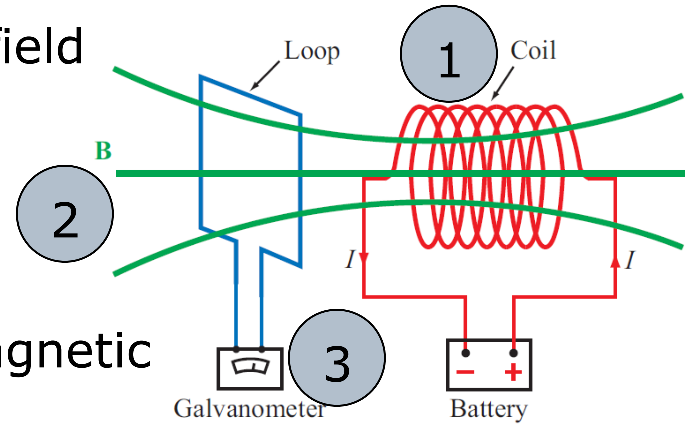
[2] http://micro.magnet.fsu.edu/electromag/java/compass/index.html

[1] http://micro.magnet.fsu.edu/electromag/electricity/capacitance.html

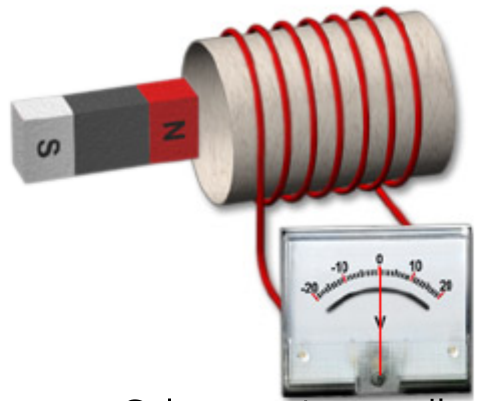
Simple Experiments



An induced magnetic field



Electromagnetic field (B)



Galvanometer needle moves

Faraday confirmed that a moving magnetic field is necessary in order for electromagnetic induction to occur

Faraday's Law: Electromotive force (voltage) induced by time-varying magnetic flux:

Maxwell Equations

- Gauss's Law
- Faraday's Law
- Gauss's Law for Magnetism
- Ampere's Law

Relationships between charges, current, electrostatic, electromagnetic, electromotive force!

POINT FORM	INTEGRAL FORM
$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$ (Ampère's law)
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S}$ (Faraday's law; S fixed)
$\nabla \cdot \mathbf{D} = \rho$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho \, dv$ (Gauss's law)
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ (nonexistence of monopole)

Maxwell's Equations – Free Space Set

- We assume there are **no charges** in free space and $J_c = \sigma E = 0$

POINT FORM	INTEGRAL FORM
$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S}$
$\nabla \cdot \mathbf{D} = 0$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = 0$
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

Time-varying E and H cannot exist independently!
If dE/dt non-zero→ dD/dt is non-zero →Curl of H is non-zero → D is non-zero (Amp. Law)

If H is a function of time → E must exist! (Faraday's Law)

Interrelating magnetic and electric fields!

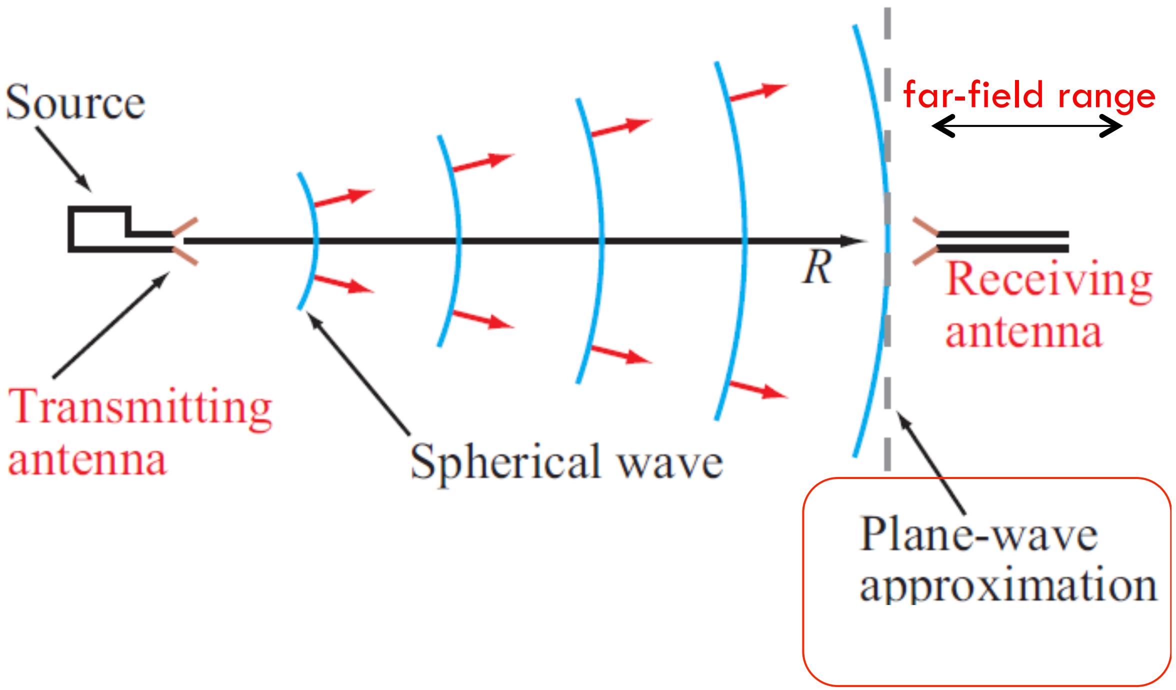
Our Focus: Far-Field Approximation

$d_{far\_field}=(2*l^2)/\lambda$

1. In close proximity to a radiating source, the wave is spherical in shape, but at a far distance, it becomes approximately a plane wave as seen by a receiving antenna.

2. The far-field approximation simplifies the math.

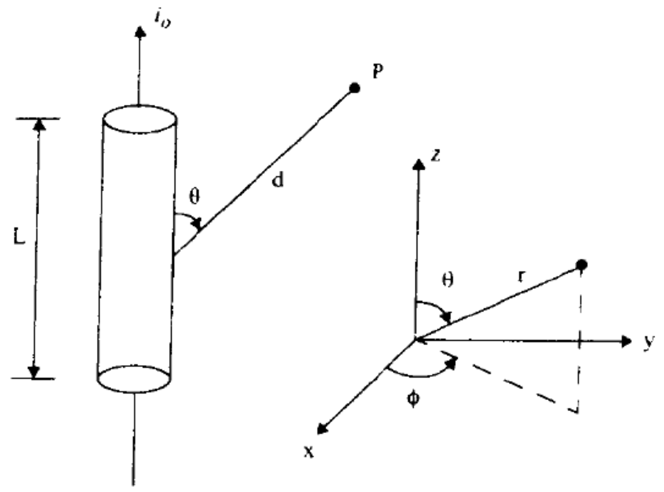
3. The distance beyond which the far-field approximation is valid is called the far-field range (will be defined later).

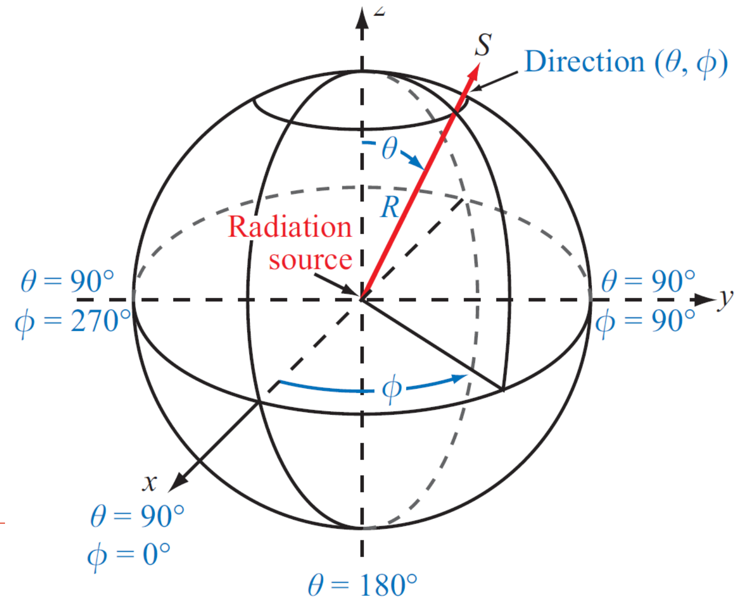


Very similar to throwing a stone in water!

What is the power radiated?
A far field approximation

- Assuming the alternating current travels in Z direction→ radiated power must be in Z
- Antenna patterns are represented in a spherical coordinate system
- Thus, variables R, θ, φ →
  - range,
  - zenith angle (elevation),
  - azimuth angle





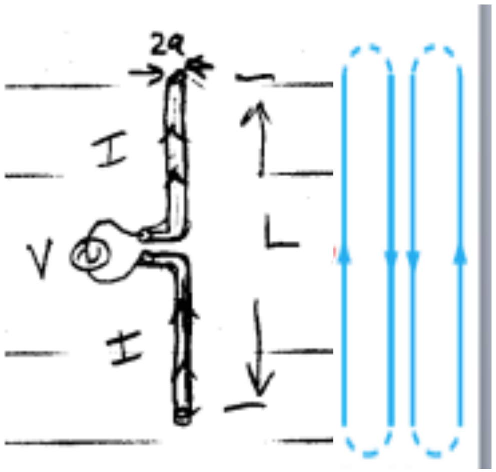
http://www.flashandmath.com/mathlets/multicalc/coords/shilmay23fin.html

Hertzian Dipole Antenna

- Using Ampere's Law

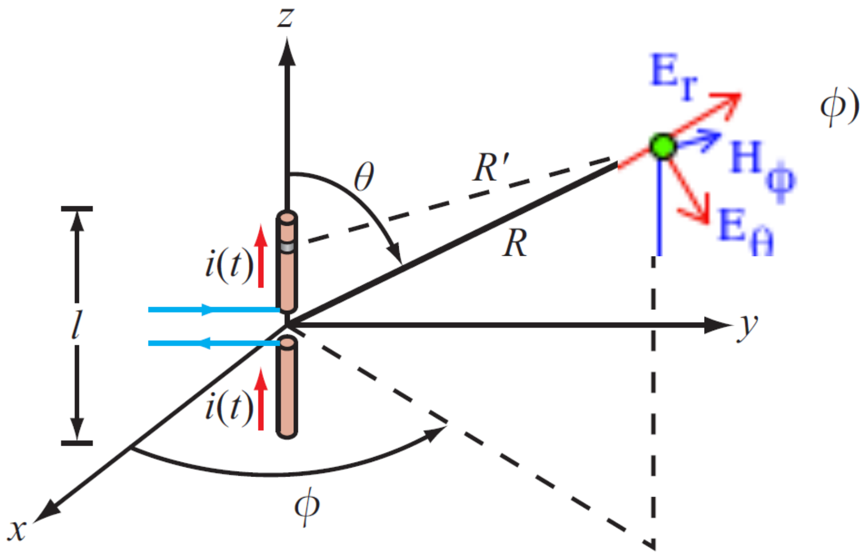
$$\bar{E}(\bar{r}) = \frac{1}{j\omega\varepsilon}\nabla \times \bar{H}(\bar{r})$$

- But how is the current I(r) distributed on the antenna?
- One way to approximate this rather difficult problem is to use thin-wire dipole antenna approximation
  - Dipole because we have two poles (wires)
  - a<<L
- We only consider the case when L is very very short (Hertzian Dipole)
  - Infinitesimally short
  - Uniform current distribution



http://whites.sdsmt.edu/classes/ee382/notes/382Lecture32.pdf

Hertzian Dipole (Differential Antenna) Radiation Properties

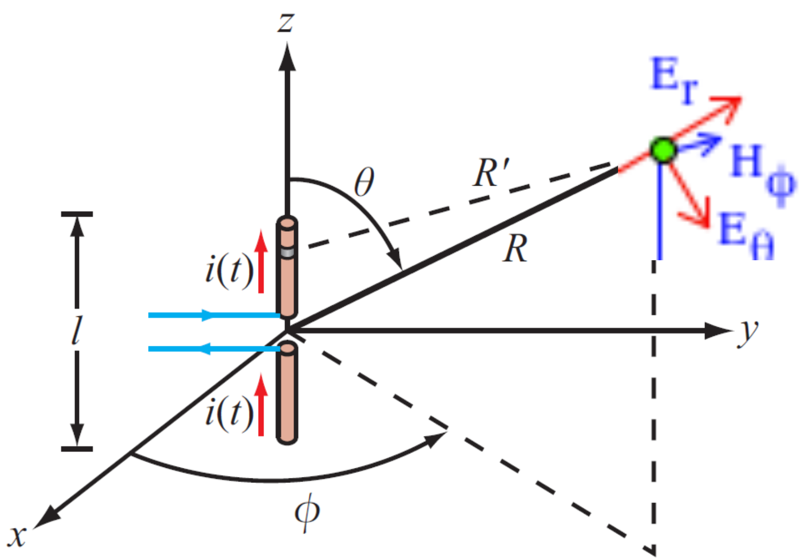


$$i(t) = I_0 \cos \omega t = \mathfrak{Re}[I_0 e^{j\omega t}]$$

- Very thin, short ($l<\lambda/50$) linear conductor
- Observation point is somewhere in the space

http://www.amanogawa.com/archive/Antenna1/Antenna1-2.html

Hertzian Dipole (Differential Antenna) Radiation Properties



$$\widetilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2}\right]\sin\theta,$$

$$\widetilde{E}_R = \frac{2 I_0 l k^2}{4\pi} \eta_0 e^{-jkR}\left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\cos\theta,$$

$$\widetilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta$$

$$\eta_0 = \sqrt{\mu_0 / \varepsilon_0} = 120\pi;$$

ε is the free space permittivity (about 1)
μ is magnetic Permeability
η is the intrinsic impedance in Ohm

Note that:

$$\omega = 2\pi f;$$

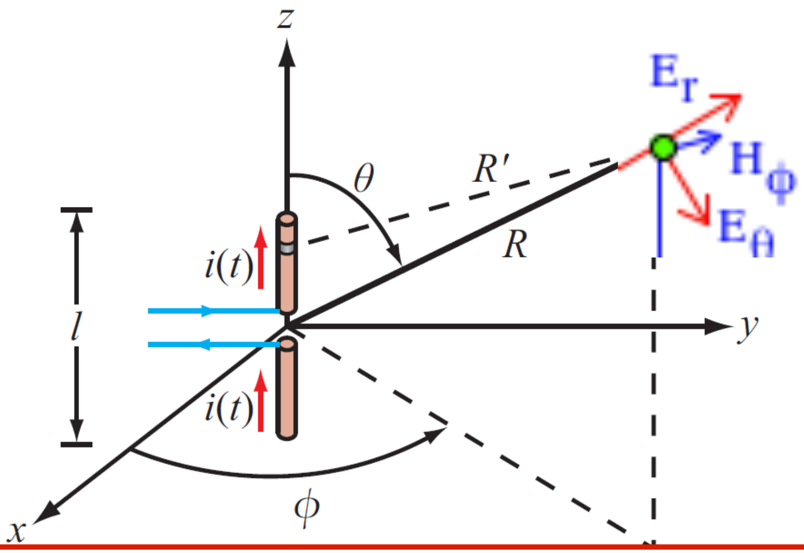
$$k = \omega / c = 2\pi f / c = 2\pi / \lambda;$$

K=wave number

$$l << \lambda / 50;$$

$$R \approx R' = \Gamma;$$

Hertzian Dipole (Differential Antenna) Radiation Properties

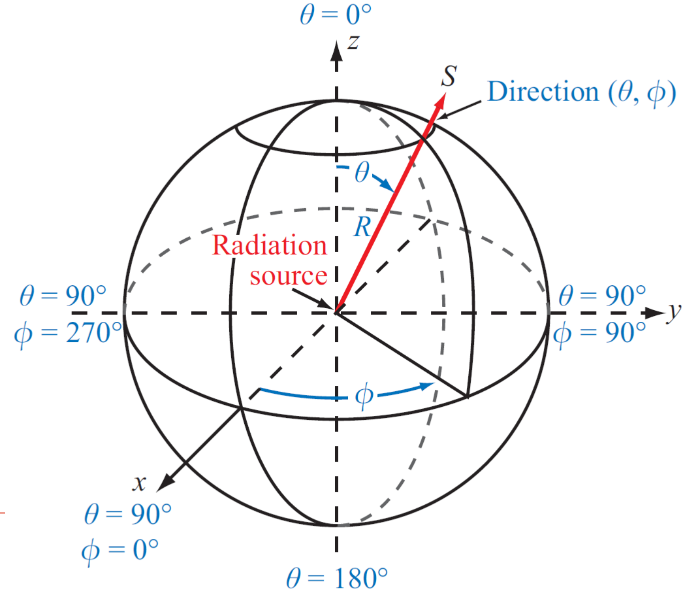


$$\widetilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2}\right]\sin\theta, \text{ A/m}$$

$$\widetilde{E}_R = \frac{2I_0 l k^2}{4\pi}\eta_0 e^{-jkR}\left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\cos\theta,$$

$$\widetilde{E}_\theta = \frac{I_0 l k^2}{4\pi}\eta_0 e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta \text{ V/m}$$

In this case:
1/R→ radiation field components
1/R^2→induction field components
1/R^3→elecrostatic field components
Far-field → only radiation field!
→ Only Eθ and Hφ will be significant!



R= Range
θ=Zenith (elevation) – side view
φ=Azimuth – top view

Hertzian Dipole (Differential Antenna) Radiation Properties

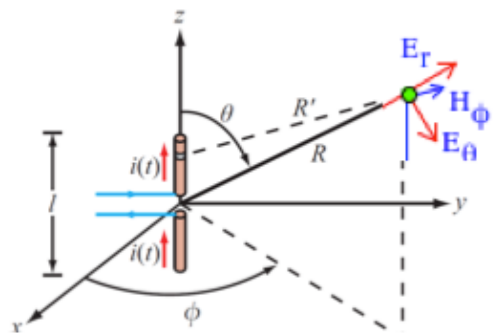
$$\widetilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2}\right] \sin\theta, \text{ A/m}$$

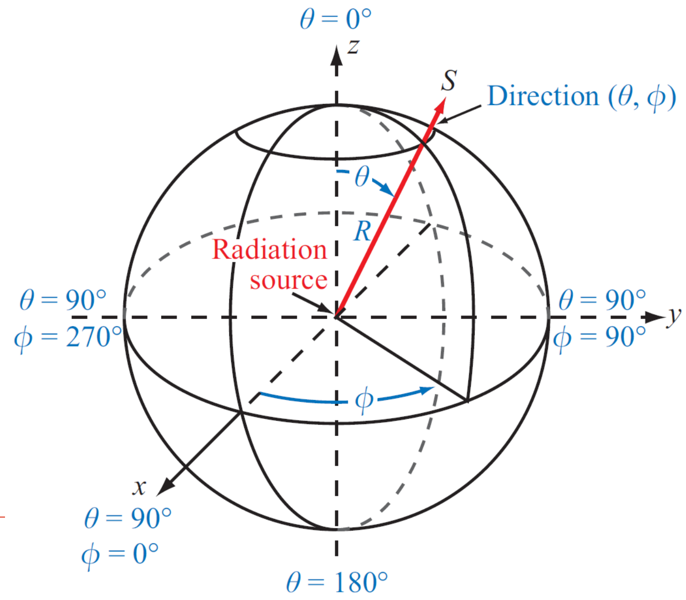
$$\widetilde{E}_R = \frac{2 I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right] \cos\theta,$$

$$\widetilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right] \sin\theta \text{ V/m}$$

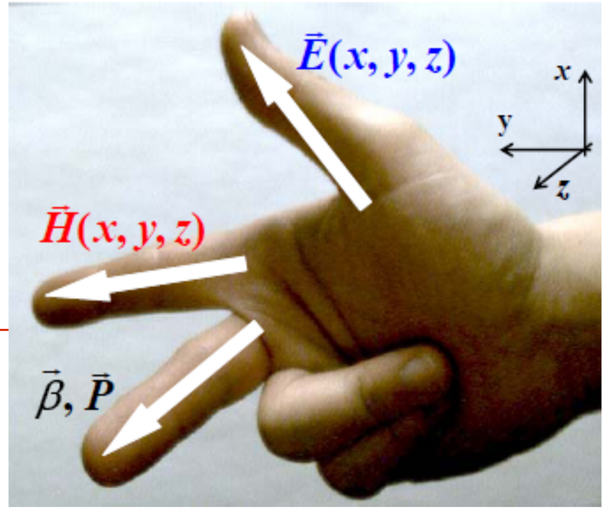
$$\widetilde{E}_\theta = \frac{j I_0 l k \eta_0}{4\pi} \left(\frac{e^{-jkR}}{R}\right) \sin\theta \quad \text{(V/m)},$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0} \qquad \text{(A/m)},$$





Radiated Power Flux Density



E(0, Eθ,0)

H(0,y, Hϕ)

$P(P_R,0,z)$

Electric and Magnetic Intensity Fields (E_R ~ 0)

$$\widetilde{E}_\theta = \frac{jI_0 l k\eta_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta \quad \text{(V/m)},$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0} \quad \text{(A/m)},$$

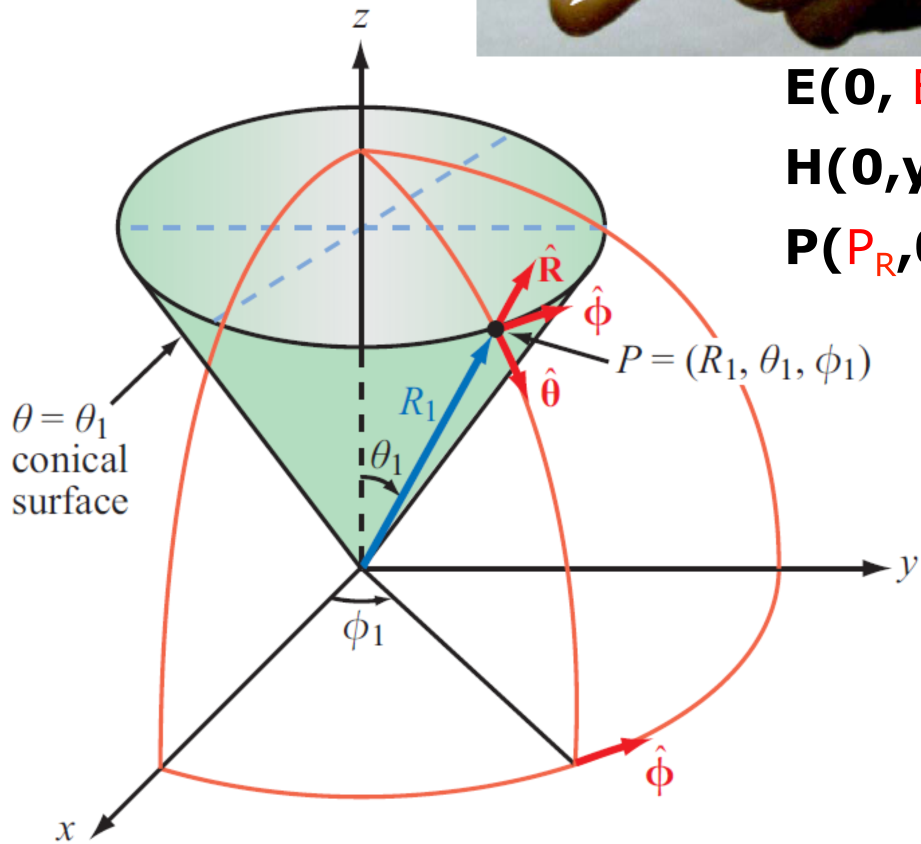
Average Power flux Density

$$\mathbf{S}_{av} = \frac{1}{2}\mathfrak{Re}\left(\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}^*\right) \quad (\text{W/m}^2).$$

$$\mathbf{S}_{av} = \hat{\mathbf{R}}\, S(R, \theta),$$

Cross product of E and H

$$S(R,\theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2}\right)\sin^2\theta$$
$$= S_0 \sin^2\theta \quad (\text{W/m}^2).$$



Normalized Radiation Intensity(F)

Normalized Radiation Intensity

→How much radiation in each direction?

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{max}} \qquad \text{(dimensionless)}$$

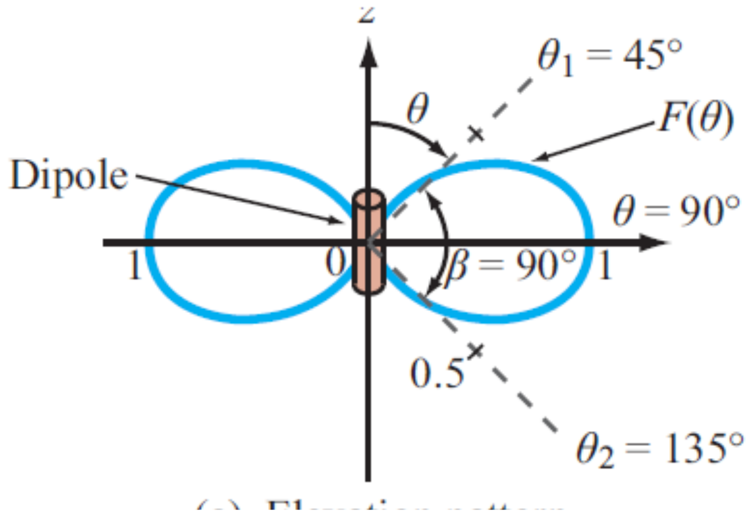
$$F(\theta, \phi) = F(\theta) = \sin^2 \theta$$

$$S(R, \theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} \right) \sin^2 \theta$$

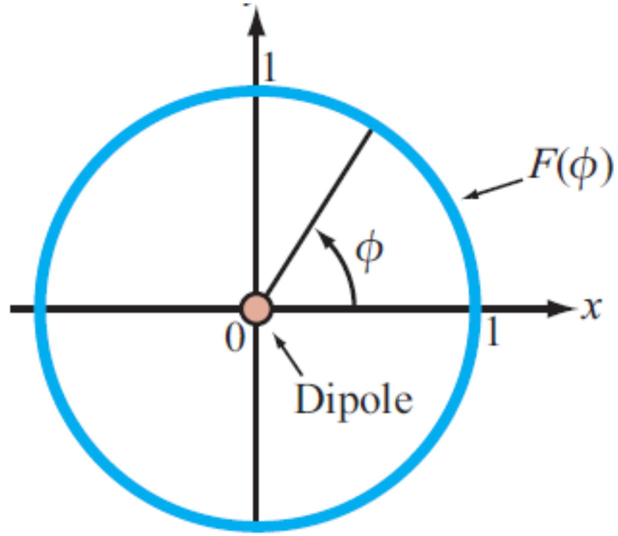
$$= S_0 \sin^2 \theta \qquad (\text{W/m}^2).$$

So=Smax= Max. Power Density

R= Range
θ=Zenith (elevation) – side view
ϕ=Azimuth – top view



Elevation Pattern
(side view)



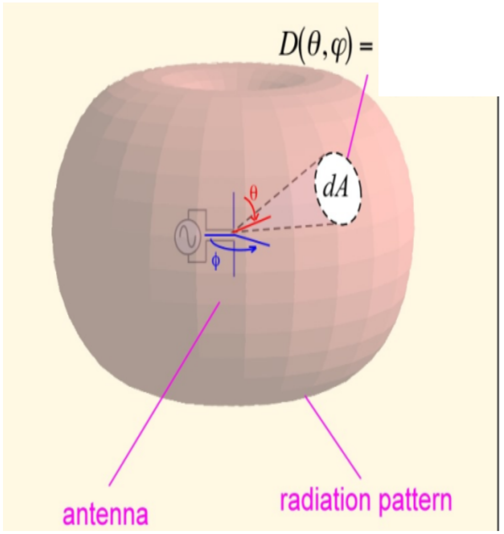
Azimuth Pattern
(top view)

Radiation Pattern of Hertzian (short) Dipole

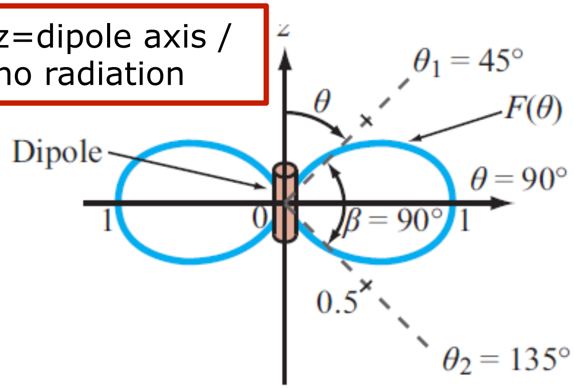
$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{\max}}$$

$$F(\theta, \phi) = F(\theta) = \sin^2 \theta$$

Note:
$F(\theta,\phi)=1$ for isotropic antenna

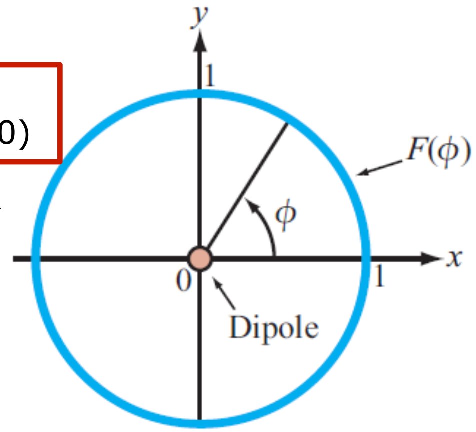


z=dipole axis /
no radiation



(a) Elevation pattern

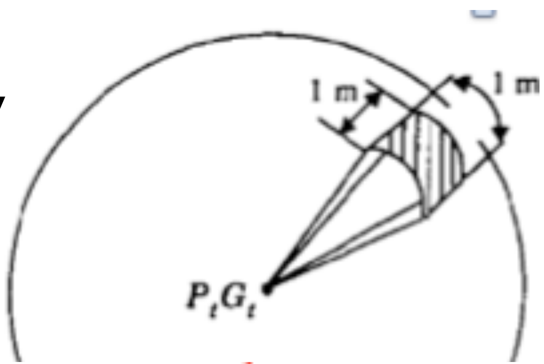
Maximum radiation
In the broadside direction (θ=90)



(b) Azimuth pattern

Doughnut-shaped radiation
pattern in θ–ϕ space

Connecting The Dots:
Radiated Power Flux Density



Electric and Magnetic Intensity Fields ($E_R \sim 0$)

$$\widetilde{E}_\theta = \frac{jI_0 l k \eta_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta \quad \text{(V/m)},$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0} \quad \text{(A/m)},$$

Antenna	Power Gain	Effective Area
Isotropic	1	$\lambda^2/(4\pi)$

Average Power flux Density

$$\mathbf{S}_{av} = \tfrac{1}{2}\mathfrak{Re}\left(\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}^*\right) \quad \text{(W/m}^2\text{)}.$$

For Isotropic Antenna We Obtain:

$$S_{av} = P_{den} = \frac{E^2}{\eta_0} = \frac{E^2}{120\pi} = \frac{EIRP}{4\pi \cdot d^2} \quad \text{W/m\^{}2}$$

$$\mathbf{S}_{av} = \hat{\mathbf{R}}\, S(R,\theta),$$

Cross product of E and H

$$P_r = P_{den} \cdot A_{er} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2} = \frac{E^2}{120\pi} \times A_{er} \quad \text{W}$$

$$S(R,\theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2}\right)\sin^2\theta = S_0 \sin^2\theta \quad \text{(W/m}^2\text{)}.$$

Note:
Sav is the average power radiated or pwr density radiated by an isotropic antenna
EIPR is the total power radiated
P_{den} is power flux density

Example

- Example A (Hertzian Dipole)
- Example B (Isotropic Antenna)

Antenna Directionality

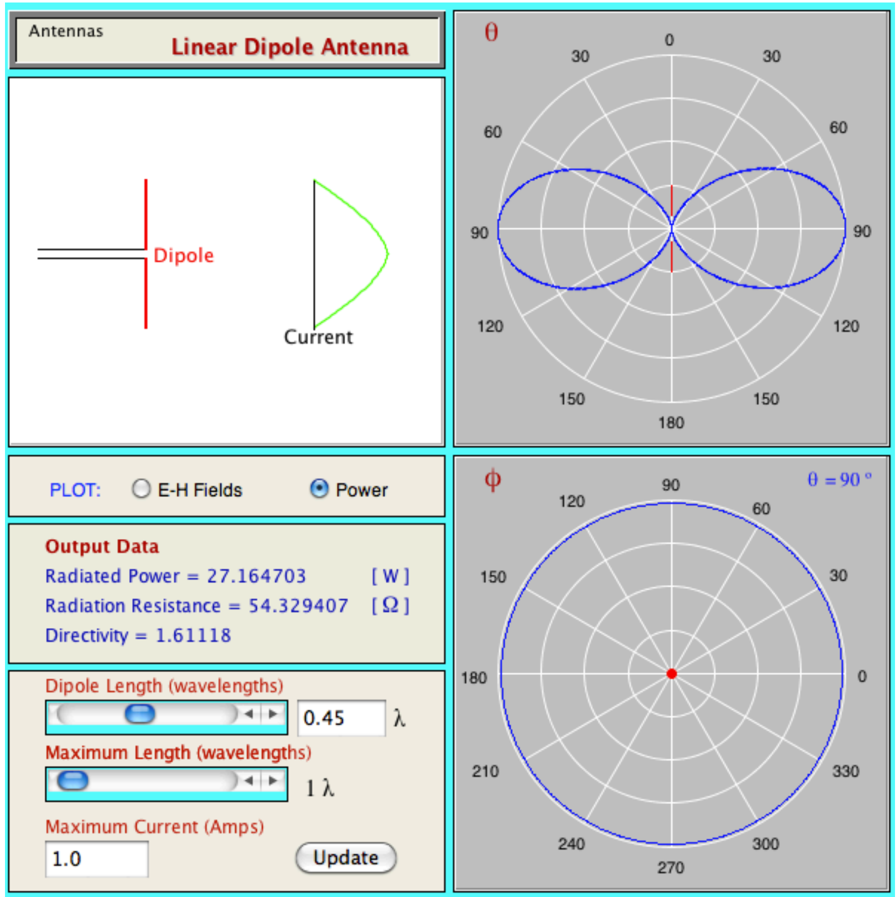
- Set the wavelength to 1
- Set Current to 1 A
- Plot Power
- Change the length
- Q1: What happens to the directivity when l changes?
- Q2: What happens to the power when L changes?

The magic is all here:

$$\widetilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2}\right]\sin\theta,$$

$$\widetilde{E}_R = \frac{2 I_0 l k^2}{4\pi} \eta_0 e^{-jkR}\left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\cos\theta,$$

$$\widetilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta$$



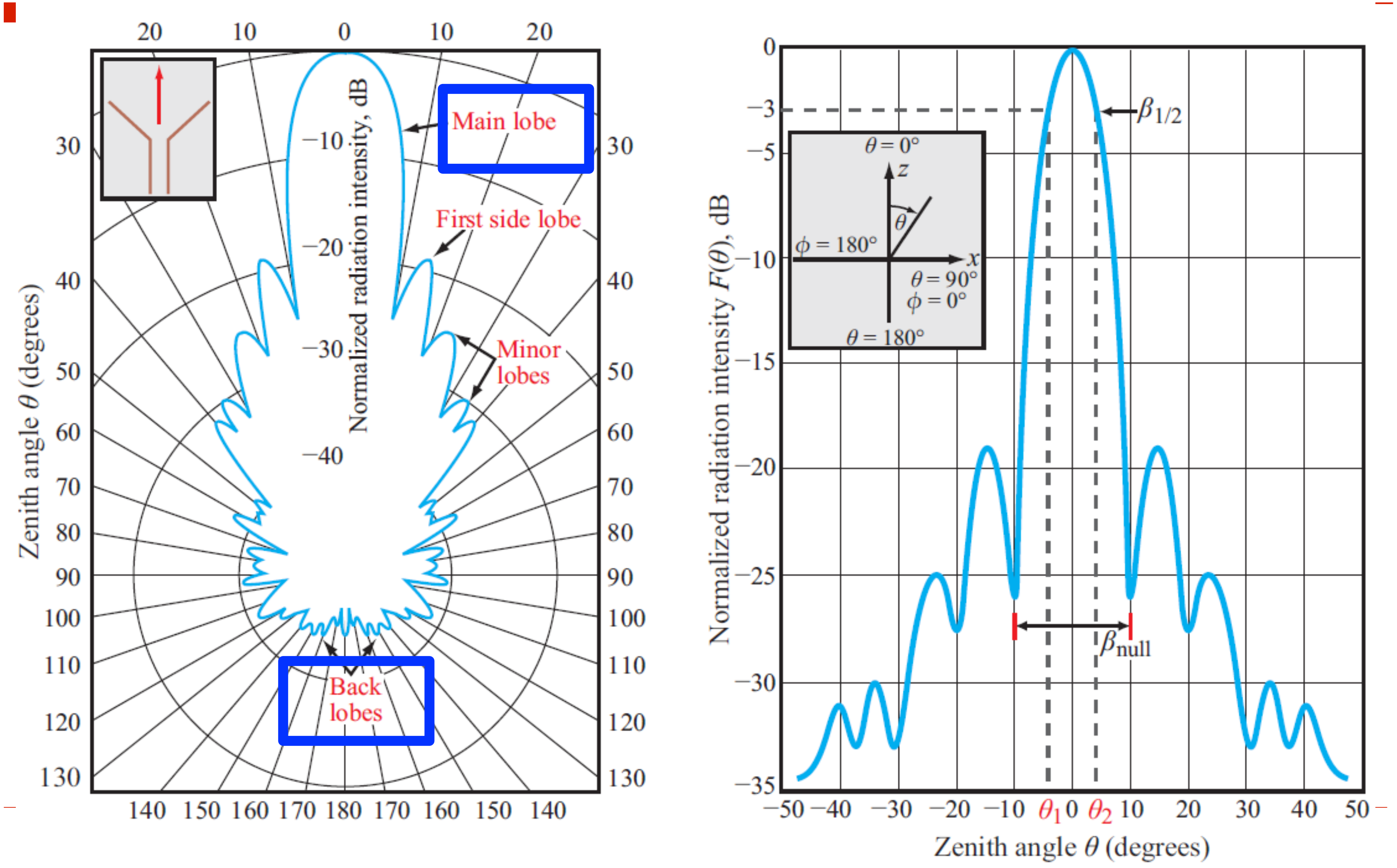
http://www.amanogawa.com/archive/DipoleAnt/DipoleAnt-2.html

Other Antenna Properties

- We already looked at the radiation intensity and radiation pattern
- Other properties
  - Radiation Pattern Characteristics
  - Radiation Resistance

Radiation Pattern – Polar and Rectangular Plots

Principal planes:
1. ***Elevation plane* (x-z and y-z planes) - sides**
2. ***Azimuth plane* (x-y plane) - top**



(a) Polar diagram

(b) Rectangular plot

Radiation Pattern Beamwidth Dimensions

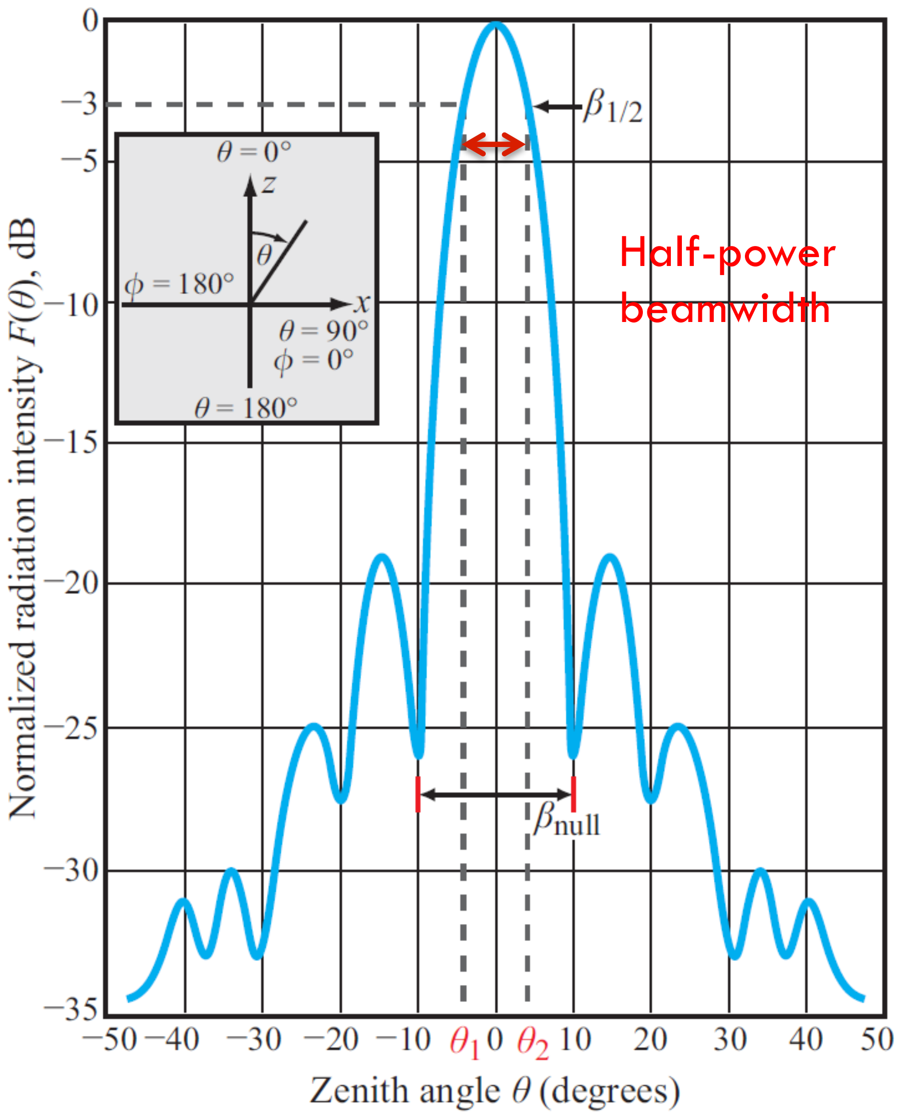
Null Bandwidth &

Half-power beamwidth

$$\beta = \theta_2 - \theta_1,$$

where θ_1 and θ_2 are the ***half-power angles*** at which $F(\theta, 0) = 0.5$ (with θ_2 denoting the larger value and θ_1 denoting the smaller one,

Since 0.5 corresponds to −3 dB, the half power beamwidth is also called the 3-dB beamwidth.



Half-power angles: theta1 and theta2

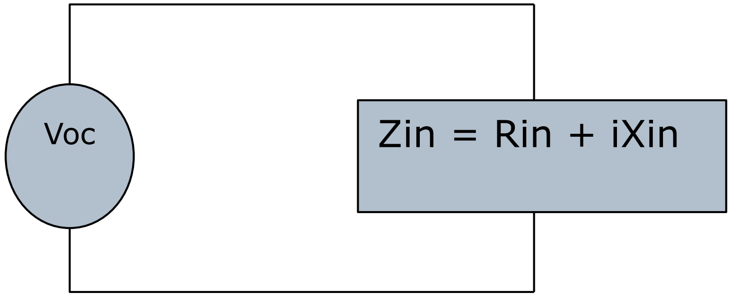
Antenna Radiation/Reception and Loss Resistances (Zin = R + jX)

In this case: $R_{in} = R_{rad} + R_{loss}$
Radiation resistance and loss resistance

We also have Pt = Prad + Ploss

$$P_{rad} = \frac{1}{2} I_0^2 R_{rad},$$

$$P_{loss} = \frac{1}{2} I_0^2 R_{loss},$$



Circuit model for TX Antenna

$$\eta_{eff\_receiver} = P_r / P_{total} = P_r / (P_r + P_{loss})$$

From the receiver point of view Pr is power received)

$$\eta_{eff\_transmitter} = P_{rad} / P_{total} = P_{rad} / (P_{rad} + P_{loss}) = R_{rad} / (R_{rad} + R_{loss})$$

Note:
We want R_loss to be minimized!
Also, when efficiency is 1 → there is no loss!

Note that Rrad determines **the impedance matching** between the TX antenna and the source OR the RX antenna and the LOAD

Gain, Directivity, Power Radiated & Rrad

For any antenna:

$$P_{\text{rad}} = \frac{1}{2} I_0^2 R_{\text{rad}}$$

Directionality & Gain

$$D = \frac{4\pi R^2 S_{\max}}{P_{rad}} = \frac{S_{\max}}{S_{av}}$$

$$G = \eta_{eff} . D$$

$$P_{rad} = \frac{4\pi R^2 S_{\max}}{D}$$

Antenna Gain and Effective Area

Antenna	Power Gain	Effective Area
Isotropic	1	$\lambda^2/(4\pi)$
Small Dipole or Loop	1.5	$(1.5\lambda^2)/(4\pi)$
Half-Wave Dipole	1.64	$(1.64\lambda^2)/(4\pi)$
Horn, mouth area *A*	$(10A)/\lambda^2$	$0.81A$
Parabola, face area *A*	$(7A)/\lambda^2$	$0.56A$
Turnstile	1.15	$(1.15\lambda^2)/(4\pi)$

Assuming there is no ohmic loss (lossless antenna)

For the Hertzian dipole:

S_{max}

$$P_{\text{rad}} = \frac{4\pi R^2}{1.5} \times \frac{15\pi I_0^2}{R^2} \left(\frac{l}{\lambda}\right)^2 = 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 .$$

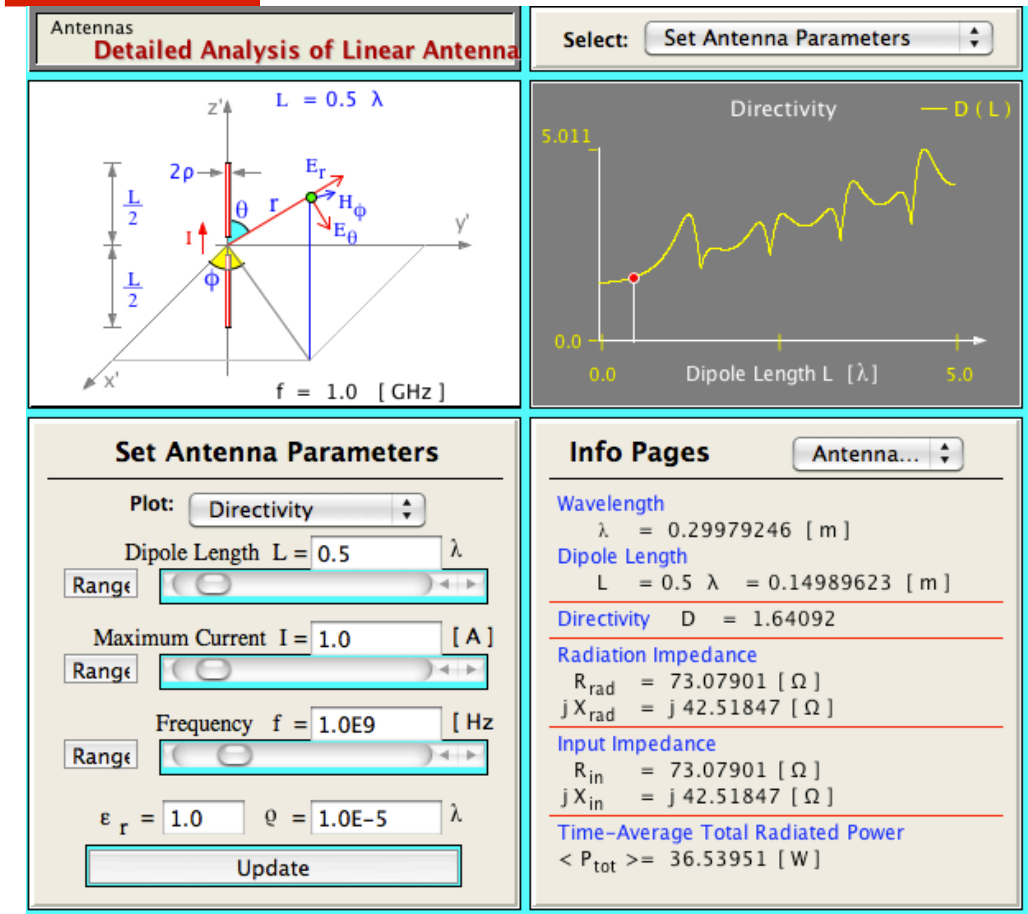
Power Gain = 1.5

Smax is the average power radiated or pwr density radiated by an antenna

$$R_{\text{rad}} = 80\pi^2 (l/\lambda)^2 \qquad (\Omega).$$

Analyze a Dipole Antenna

- Set the current to 1 A
- Set length of the dipole to half wave
- What is the Rrad?
- What is Rin resistance?
- What is Directionality?
- What is the total radiate power?
- Go to "Scan Fields and Power",
  - Set ϕ equal to 90
  - Determine at which angle of θ we get max power
  - Determine at which angle of θ we get half power
  - What is the 3dB BW for this dipole?
- Answer the above questions but this time set the length to ¼ of the wave length.



http://www.amanogawa.com/archive/Antenna1/Antenna1-2.html

Example

- Example C (Measuring the received power)
- Example D (Resistance loss in a short dipole)
- Example E (rewrite the average power density in terms of current, distance between the two antennas, length of the antenna, and frequency of operation for a short dipole (loop)

Review: Isotropic Antenna

- Radiated Power Approximation:
- Power Density (W/m^2) is
- For isotropic antennas Pt=Prad
- Note that the free space impedance is ratio of E and H fields

$$P_{rad} = EIPR = P_t \cdot G = \frac{4\pi d^2}{D} S_{\max}$$

$$S_{\max} = E^2 / \eta_o = \frac{V^2 / m^2}{\Omega} = W / m^2$$

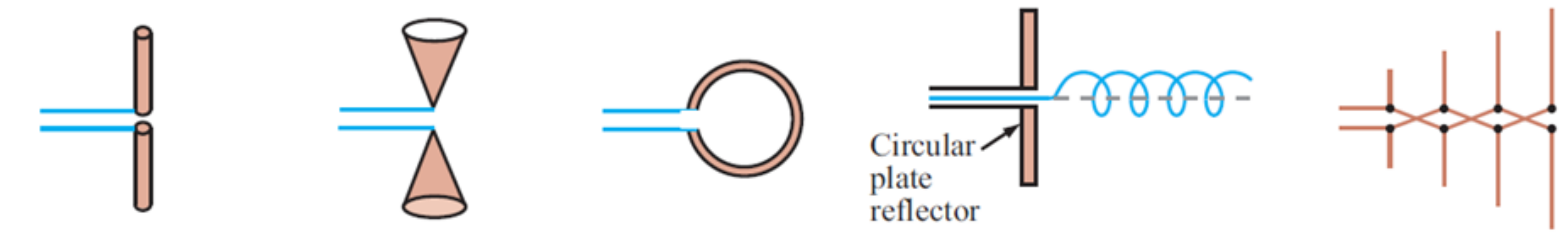
$$E = \frac{\sqrt{30 P_t}}{d}$$

$$H = \frac{\sqrt{P_t}}{68.8 d}$$

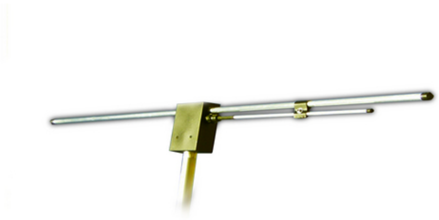
$$S_{\max} = P_{den} = E \cdot H = \frac{P_t}{4\pi d^2}$$

$$\eta = E / H$$

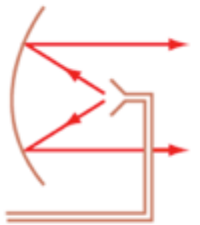
Different Antenna Types



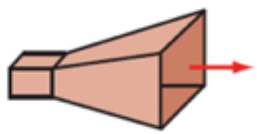
(a) Thin dipole (b) Biconical dipole (c) Loop (d) Helix (e) Log-periodic



Dipole Antenna



(f) Parabolic dish reflector



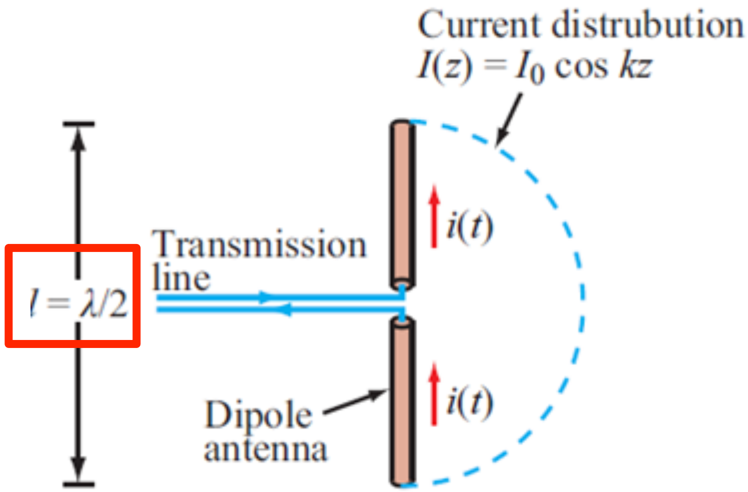
(g) Horn

Yagi Antenna

Fields in Half-Wave Dipole
Homework Assignment

$$\widetilde{E}_\theta = j\,60 I_0 \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\} \left(\frac{e^{-jkR}}{R} \right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}\,.$$



- **Example F:** Find the following:
- Find the expression for average power density
- Power Density (Smax)
- Normalized radiation intensity, F
- Directionality, D (from the table)
- P_radiated
- R_radiated
- Prove that 3dB BW is in fact 78 degrees (you can use substitution to prove!
- HINT: Use the applet to check your answers

All works must be shown!

Example:
Complete the Table Below

Example G

Antenna	Gt=D	Rrad	Prad	3dB BW	Eff. Area
Isotropic	----------	----------	---------	----------	----------
Short Dipole					
½ Wave					
¼ Wave	Later	Later	Later	Later	Later

Why dBi = dBd + 2.15dB? Explain!

References

- Ulaby, Fawwaz Tayssir, Eric Michielssen, and Umberto Ravaioli. *Fundamentals of Applied Electromagnetics: XE-AU...*. Prentice Hall, 2001, Chapter 9
- Black, Bruce A., et al. *Introduction to wireless systems*. Prentice Hall PTR, 2008, Chapter 2
- Rappaport, Theodore S. *Wireless communications: principles and practice*. Vol. 2. New Jersey: Prentice Hall PTR, 1996, Chapter 3
- Wheeler, Tom. *Electronic communications for technicians*. Prentice Hall, 2001. Section 12-1 & 12-2