Different combinations of resistances

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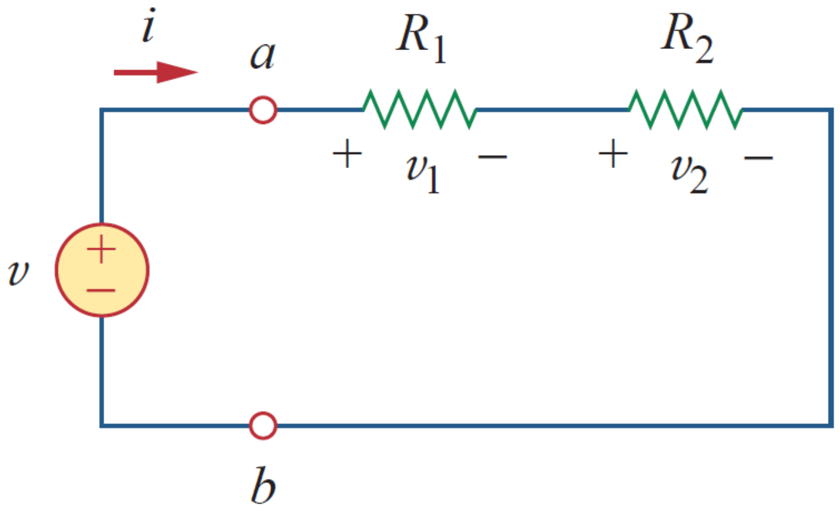
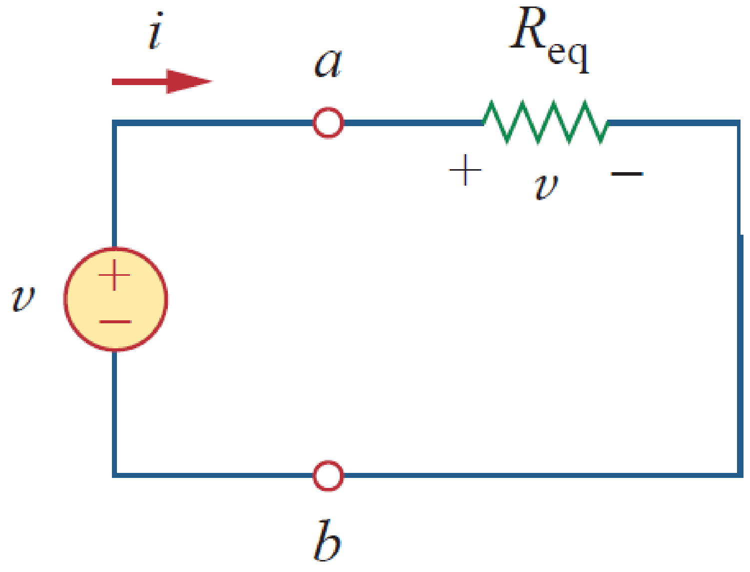
Series Combination of Resistors



$$v_1 = iR_1, \qquad v_2 = iR_2$$

$$v = v_1 + v_2 = i(R_1 + R_2) \qquad i = \frac{v}{R_1 + R_2}$$



$$R_{eq} = R_1 + R_2$$

$$v = iR_{eq}$$

For $V = 10\ V$; $R_1 = 7\ \Omega$, $R_2 = 3\Omega$, The value of i ?

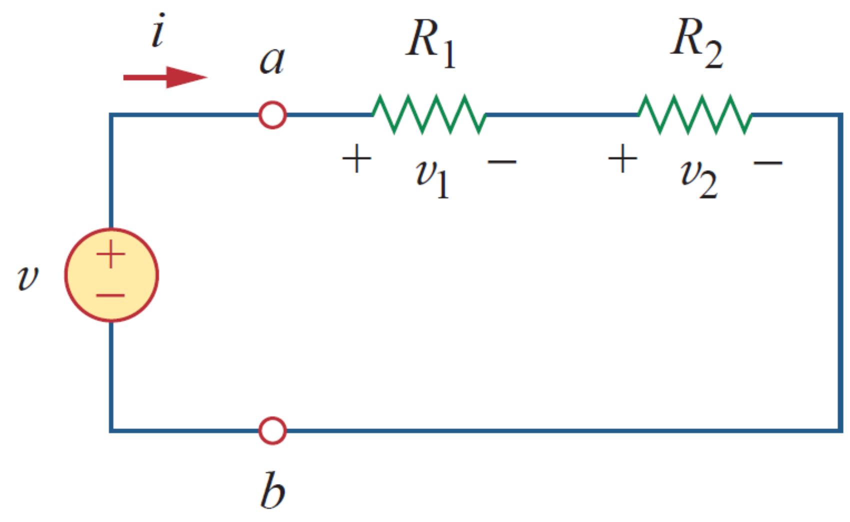
i= $1\ A$

For $V = 10\ V$; $R_{eq} = R_1 + R_2 = (7 + 3)\ \Omega$ = 10 Ω, The value of I ?

i= $1\ A$

Series Combination of Resistors

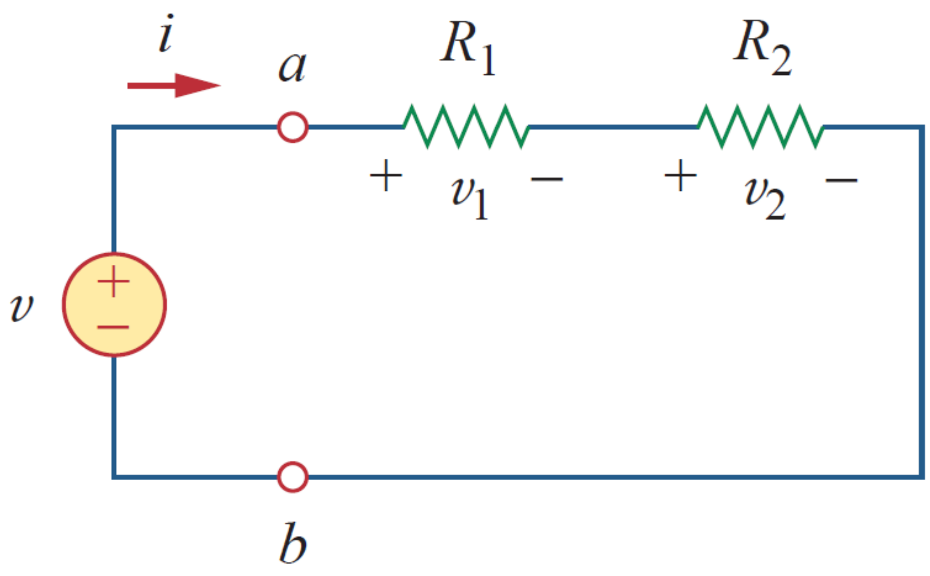
The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.



$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$R_{eq} = R_1 + R_2$$

Voltage Division Rule



$$v_1 = iR_1, \qquad v_2 = iR_2$$

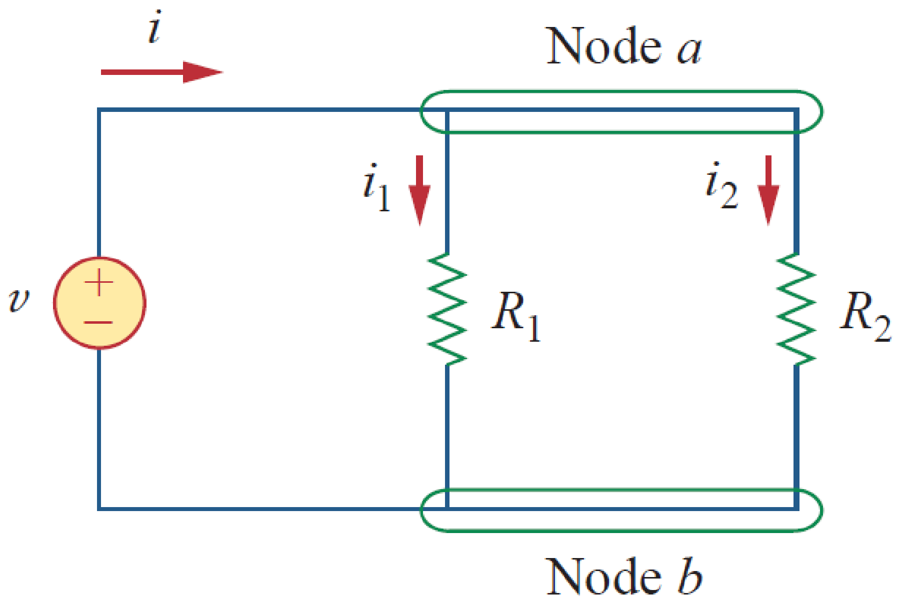
$$i = \frac{v}{R_1 + R_2}$$

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

For $V = 10\ V$; $R_1 = 7\ \Omega$, $R_2 = 3\Omega$, The value of v_1 and v_2 ?

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Parallel Resistors



$$v = i_1 R_1 = i_2 R_2 \qquad i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$i = i_1 + i_2$$

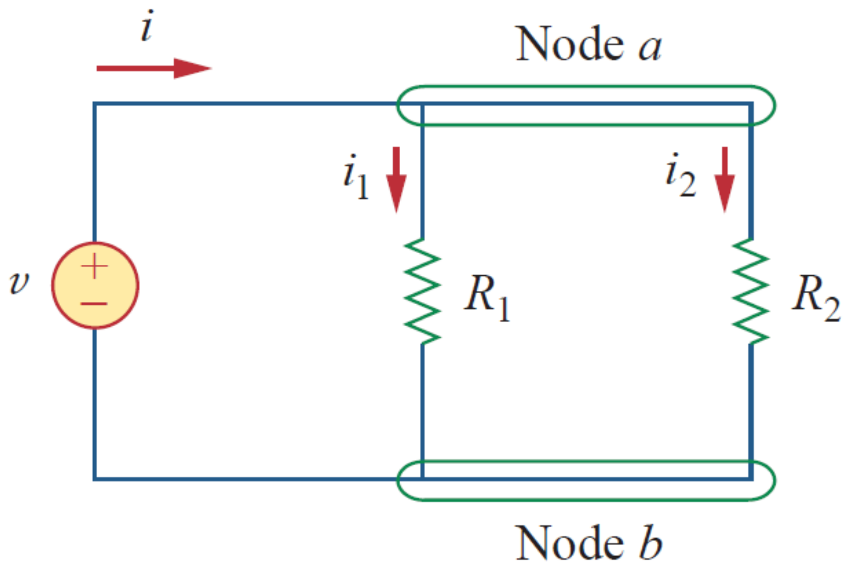
$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

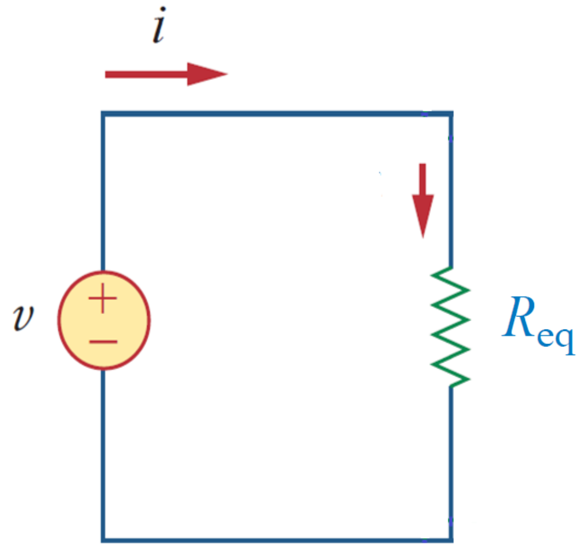
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For $V = 10\ V$; $R_1 = 7\ \Omega$, $R_2 = 3\Omega$, The value of i ?

i$= 4.76\ A$

Parallel Resistors





$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

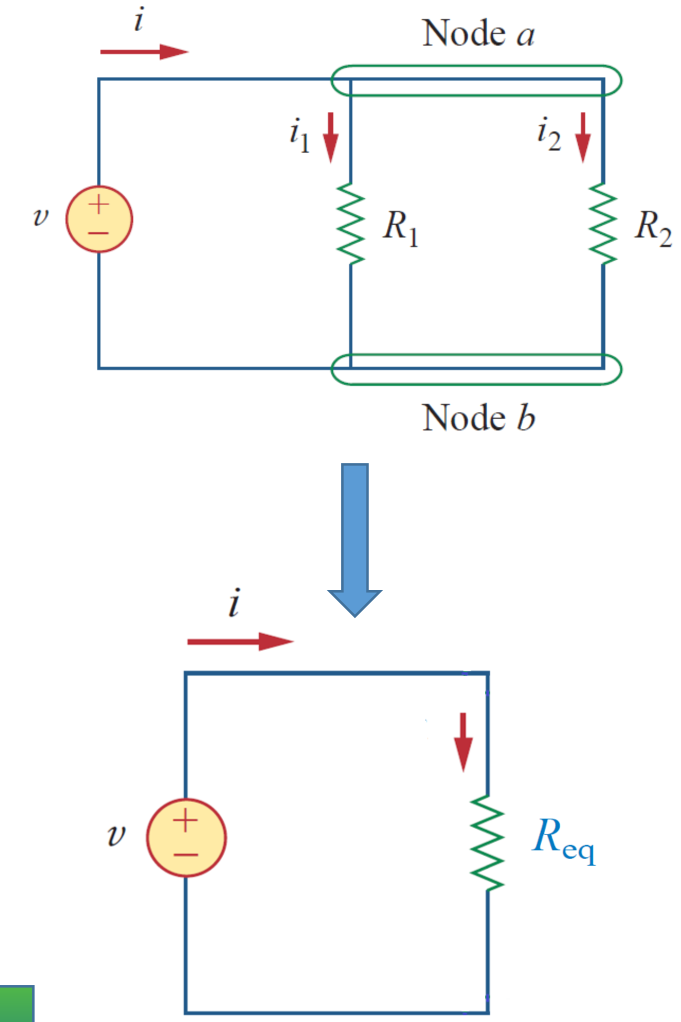
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

For $V = 10\ V$; $R_1 = 7\ \Omega$, $R_2 = 3\Omega$, The value of R_{eq} ? $R_{eq} = 2.1\ \Omega$

The value of i ? i= $4.76\ A$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Current Division Rule



$$v = i_1 R_1 = i_2 R_2 \qquad i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$v = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

Voltage & Current Division Rule

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

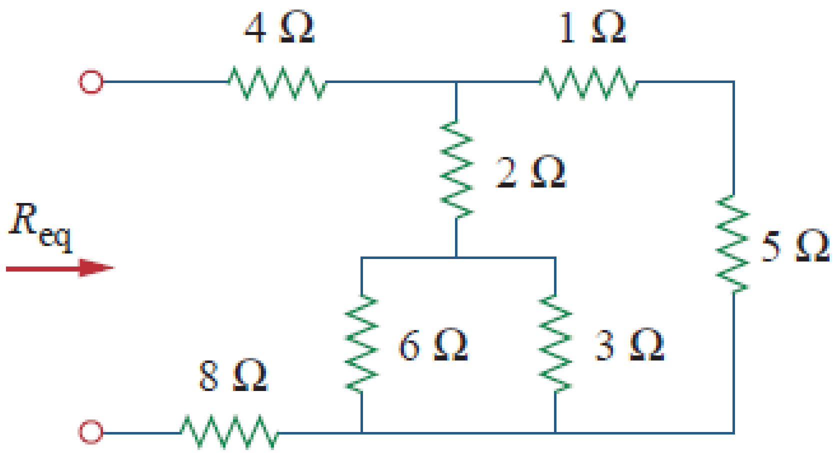
$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} i \Rightarrow i_1 = \frac{R_2\, i}{R_1 + R_2},$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

$$i_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} i \Rightarrow i_2 = \frac{R_1\, i}{R_1 + R_2}$$

Mathematical Examples

Find R_{eq} for the circuit shown in Fig.



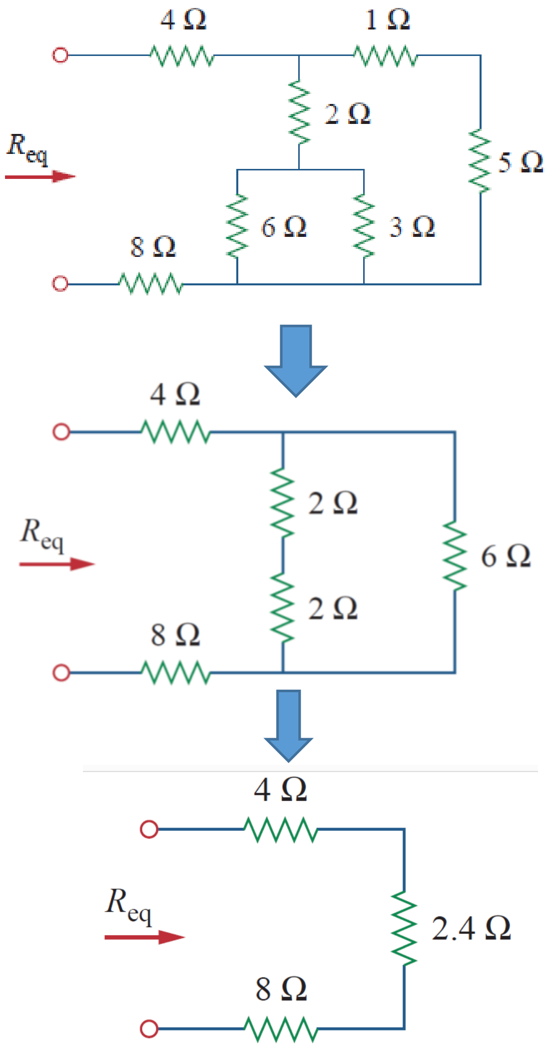
$$6\,\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

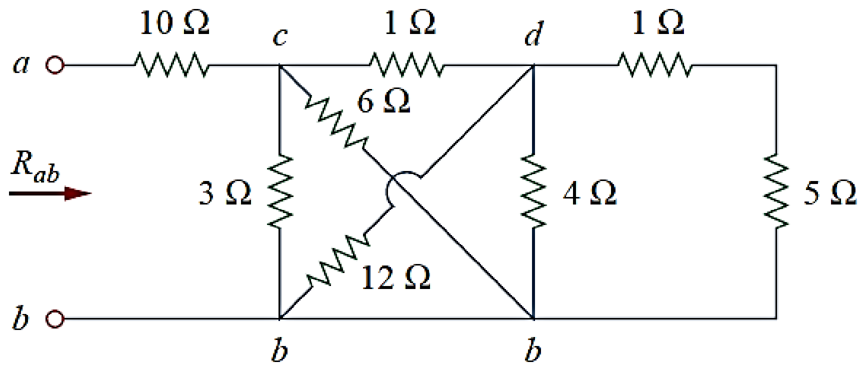
$$4\,\Omega \parallel 6\,\Omega = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

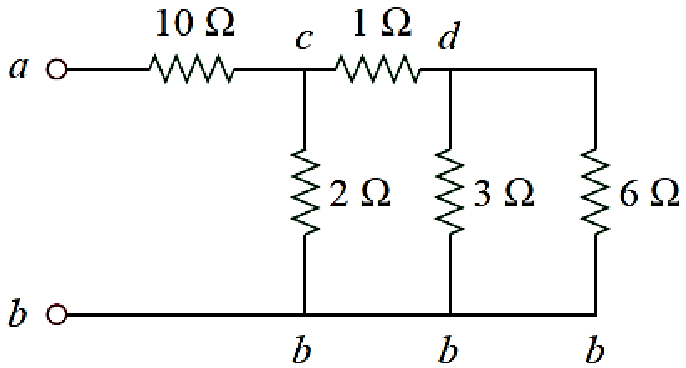
$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

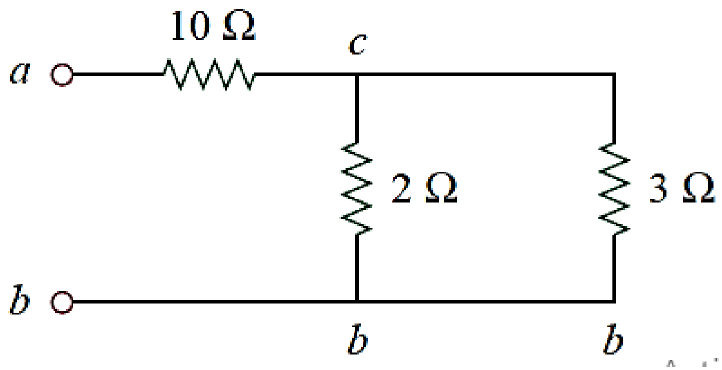


Mathematical Examples

Calculate the equivalent resistance R_{ab} in the circuit







$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega$$

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

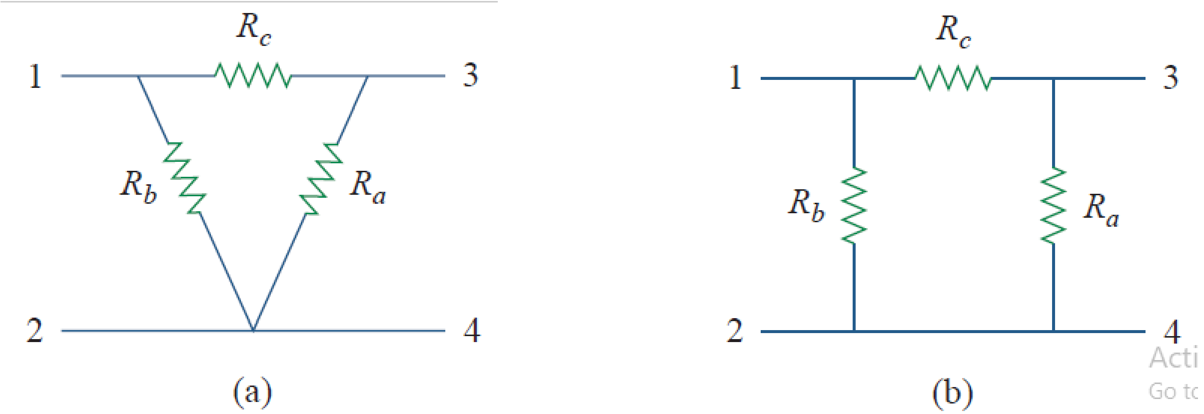
$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

$$1\ \Omega + 2\ \Omega = 3\ \Omega$$

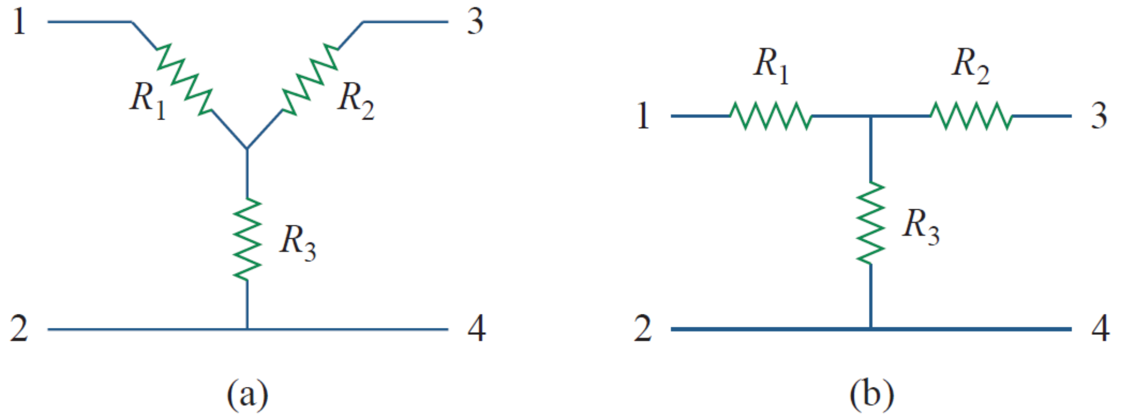
$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

Wye-Delta Transformations



Two forms of the same network: (a)
(b) Π.



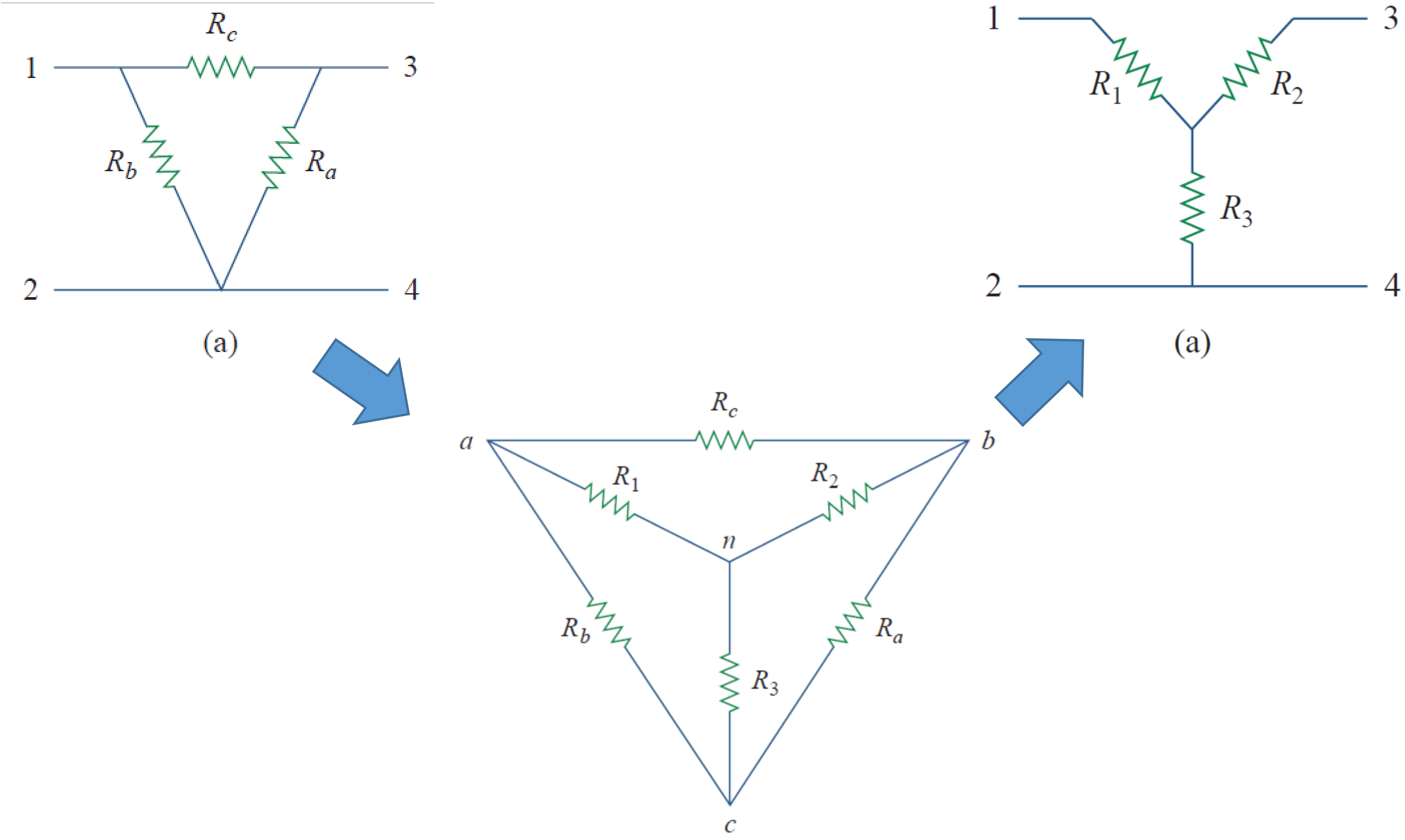
Two forms of the same network: (a) Y, (b) T.

Delta to Wye Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

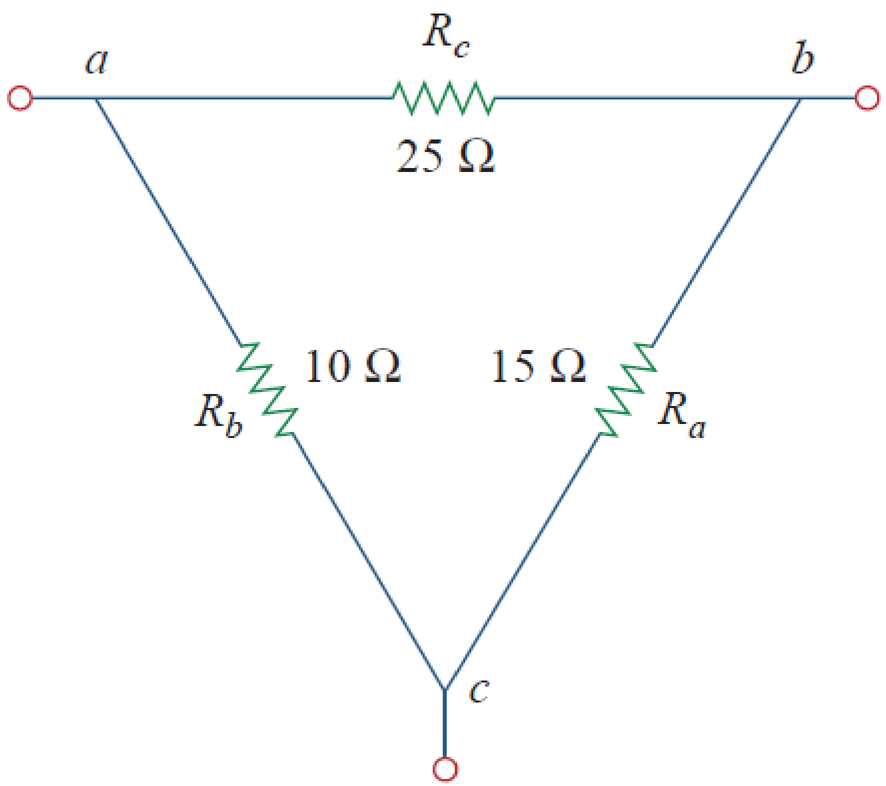
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



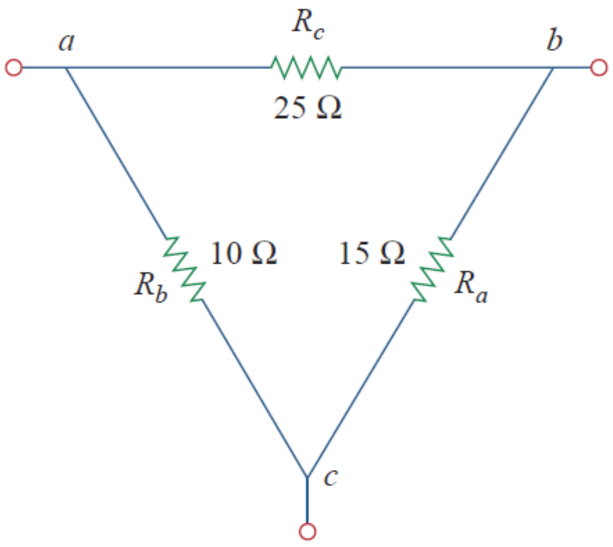
Delta to Wye Conversion

Convert the Δ network in Fig. to an equivalent Y network.

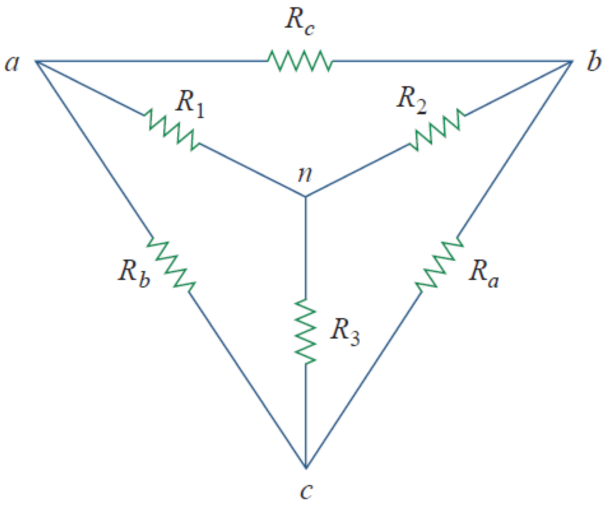


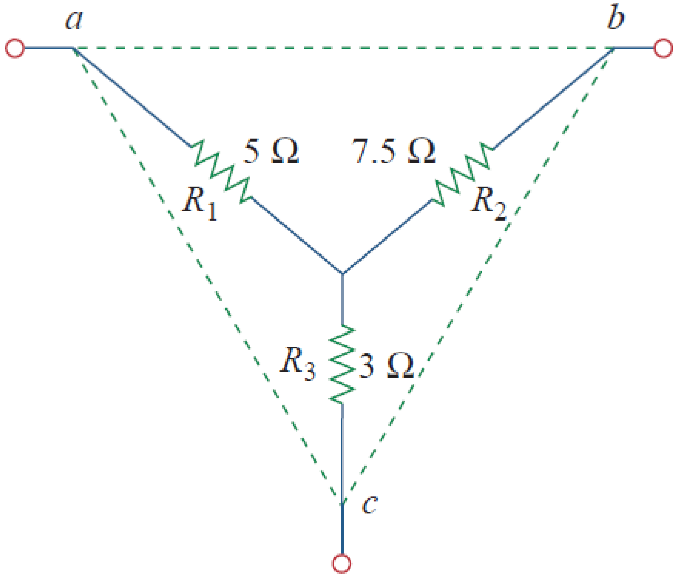
(a)

Delta to Wye Conversion



(a)





(b)

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

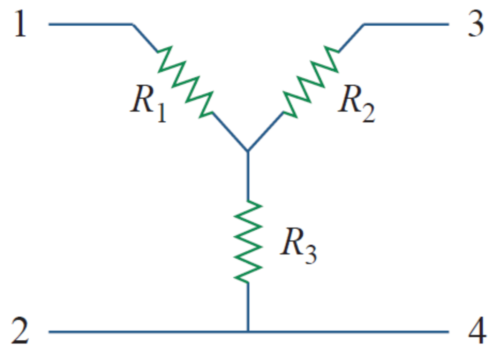
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Wye to Delta Conversion

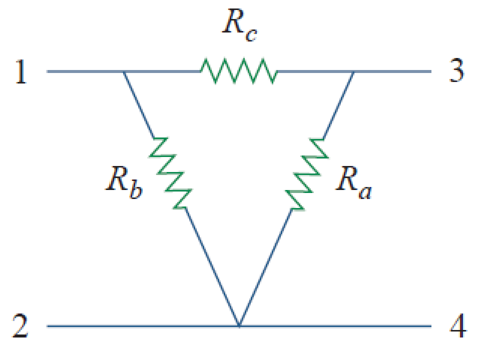
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

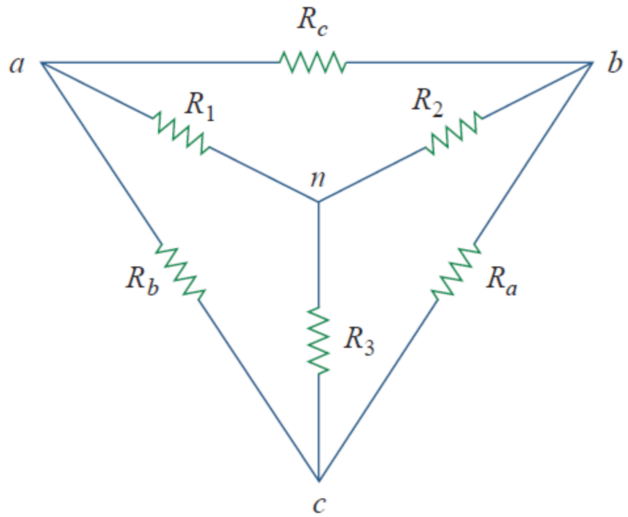
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



(a)

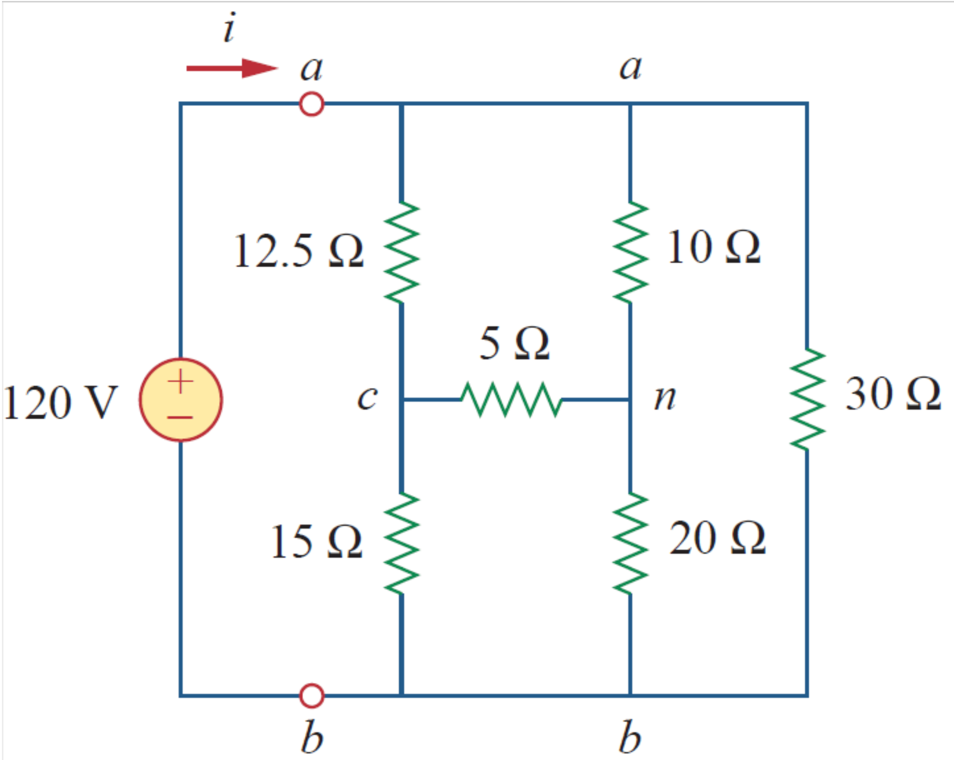


(a)



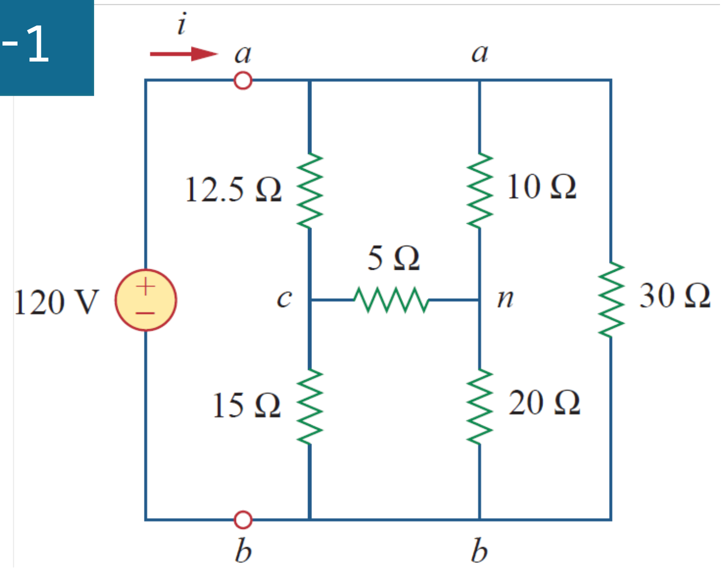
Circuit Solve

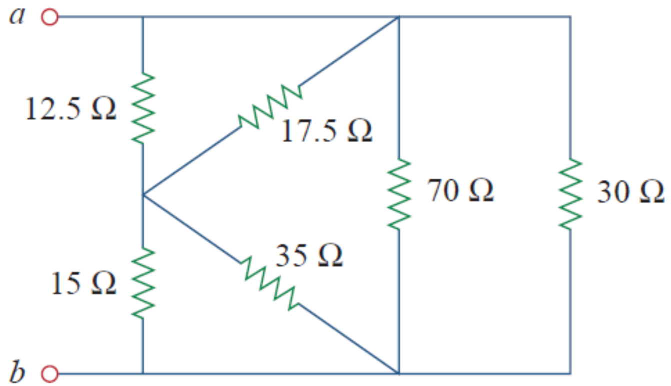
Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.



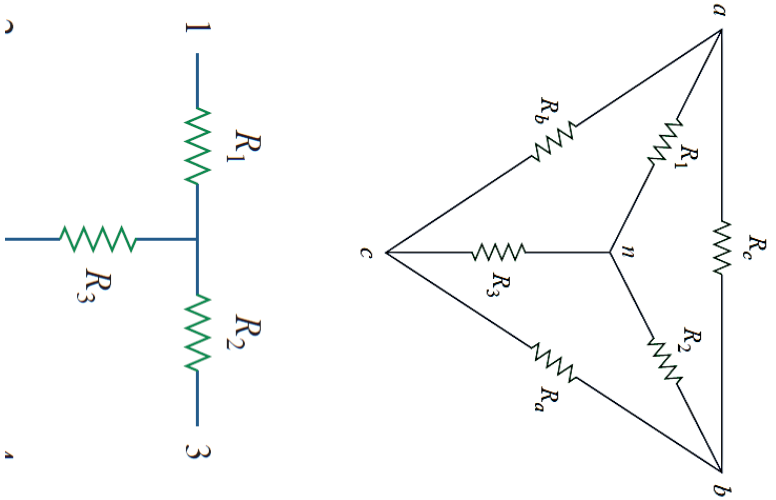
Circuit Solve

METHOD-1





(a)



$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

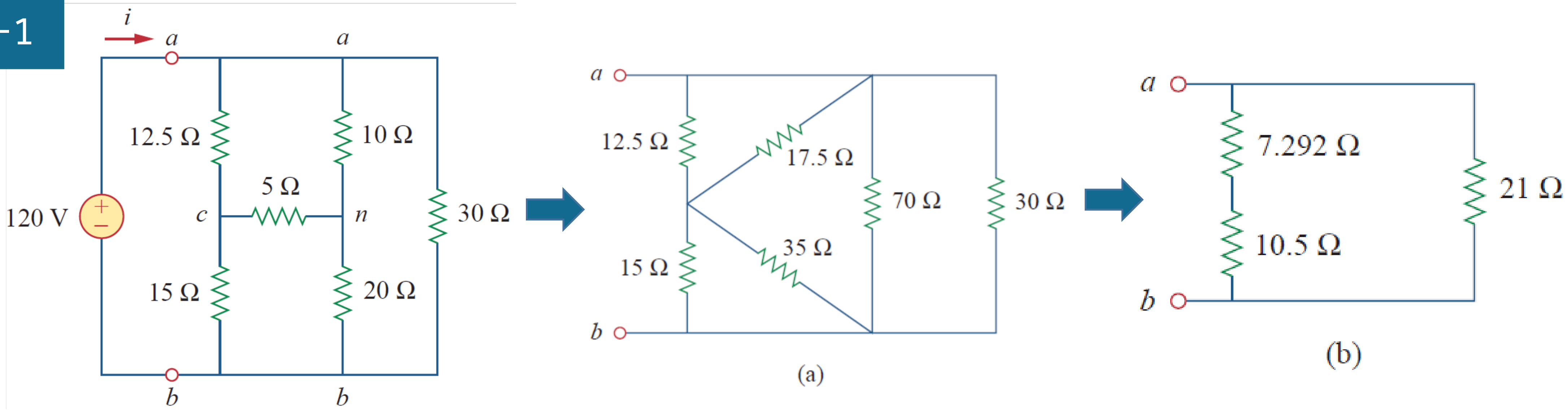
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10} = \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

Circuit Solve

METHOD-1



$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

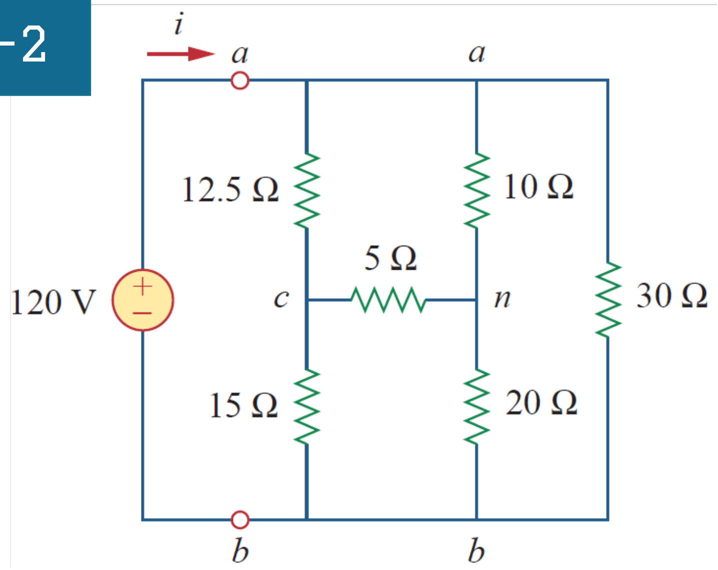
$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

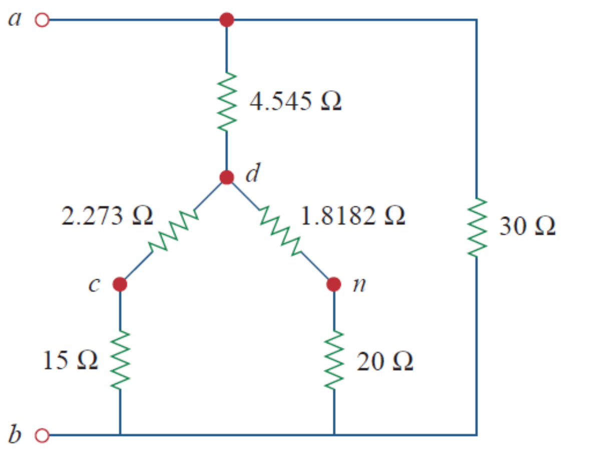
$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

Circuit Solve

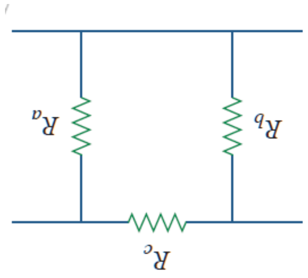
METHOD-2

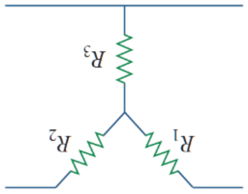




(c)

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$





$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = \mathbf{9.631\ \Omega}$$

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

Thank you