Introduction to AC Circuits

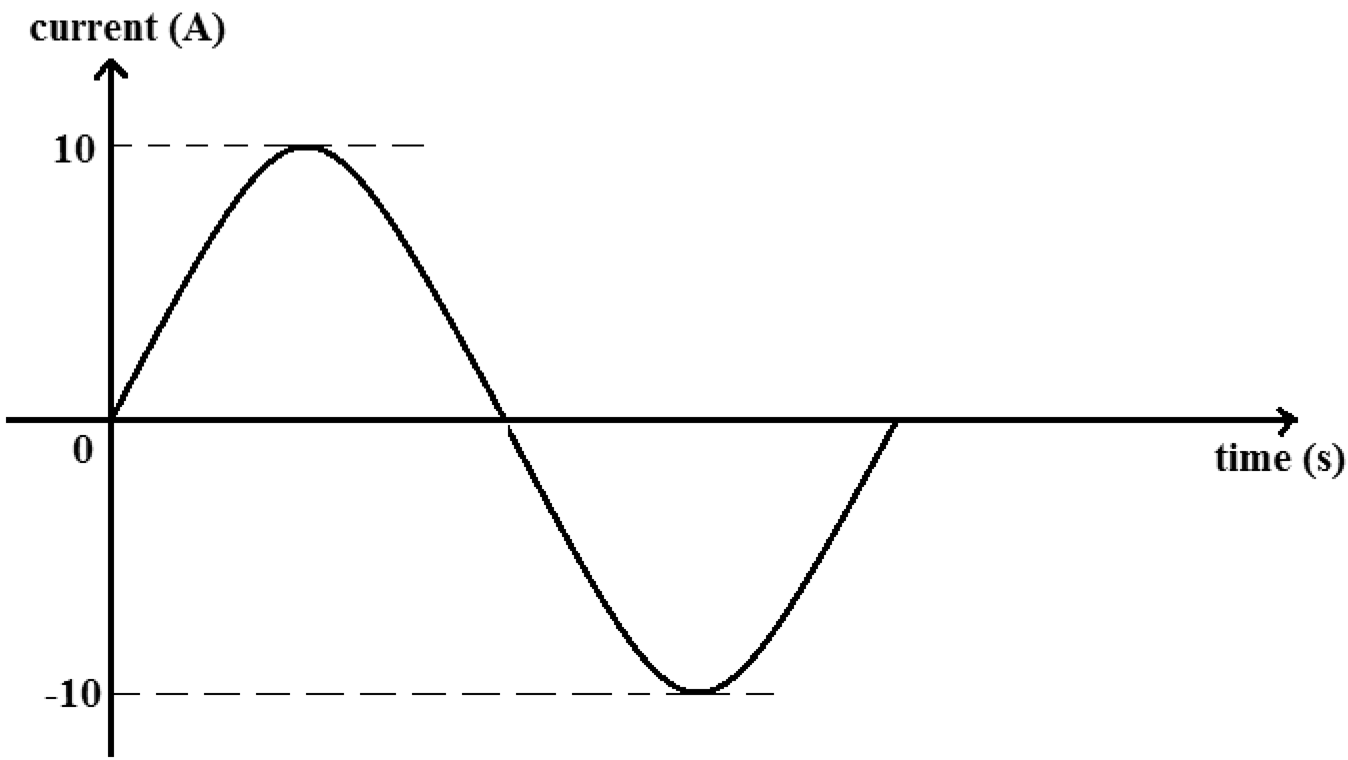
EEE 101: "Basic Electrical Technology"

Civil Engineering Department

Fall 2021

Alternating current

An alternating current goes through a series of different values, **both positive and negative**, in a period of time, after which it continuously repeats the same series of values in a cyclic manner.



Oscillating current

An oscillating current alternately increases and decreases in magnitude with respect to time according to some definite law.

Periodic current

A periodic current is an oscillating current the values of which occur at equal intervals of time.

An alternating current is a periodic current whose average value over a period is zero.

Period and cycle

The period of an alternating current is the smallest value of time which separates recurring value of the alternating current.

One complete set of positive and negative values of an alternating current is called a cycle.

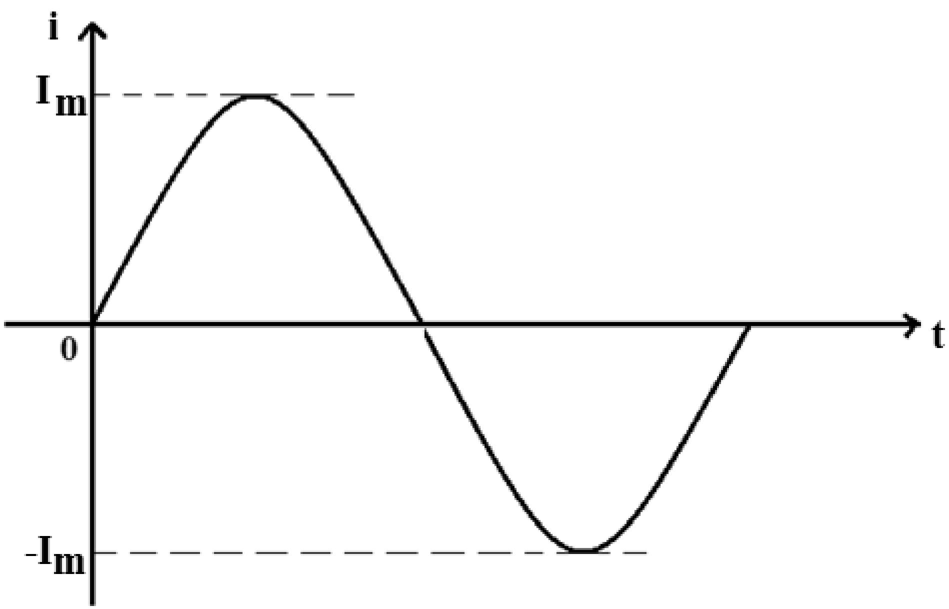
Frequency

Frequency is the number of cycles per second.

Waveform

The shape of the curve obtained by plotting the values of an alternating quantity against time is called waveform or wave shape.

Alternating quantities are usually represented by sine waves.



$$i = I_m \sin \omega t$$

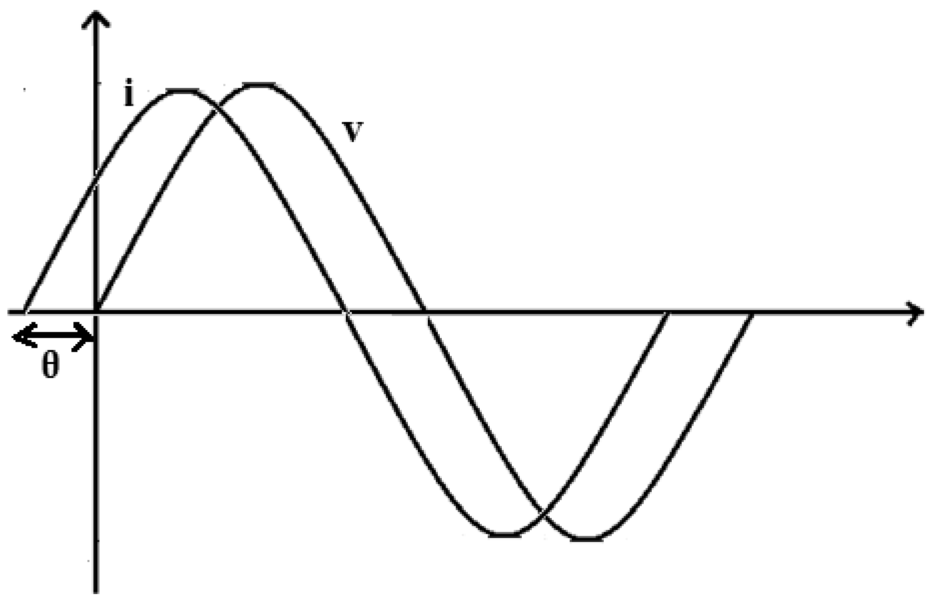
ω is the angular velocity/ angular frequency

Phase difference

The phase angle is a very important tool for properly locating different alternating quantities with respect to one another.

e.g. if applied voltage is $\mathbf{v = V_m \sin \omega t}$ and it is known from the nature and magnitude of circuit parameters that current comes to a **corresponding point** on its wave before the voltage wave by θ degrees, then current can be expressed as $\mathbf{i = I_m \sin (\omega t + \theta)}$.

Here i leads v by θ degrees, or v lags i by θ degrees.



Impedance

Impedance relates the voltage and current in an AC circuit. It is expressed in the form

$$Z \angle \text{angle}$$

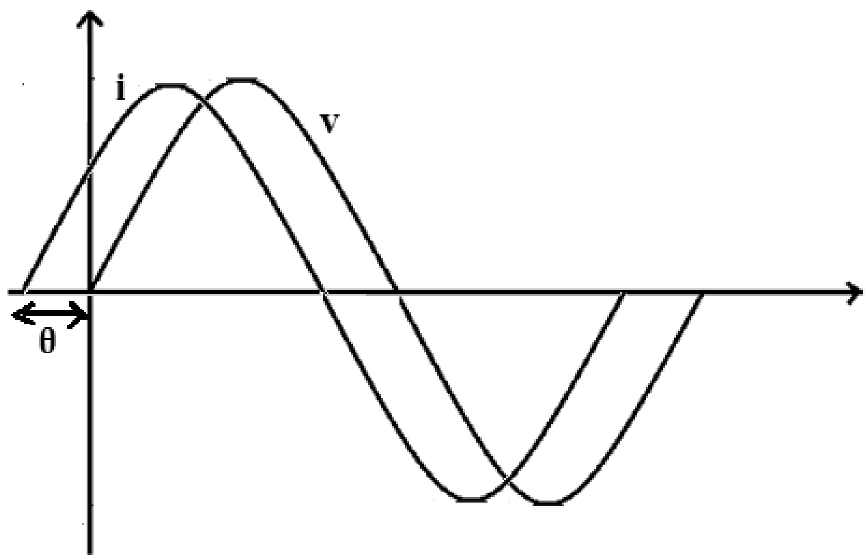
where

Z = ratio of V_m and I_m = V_m/I_m

$\angle$ angle = phase angle between voltage and current waves

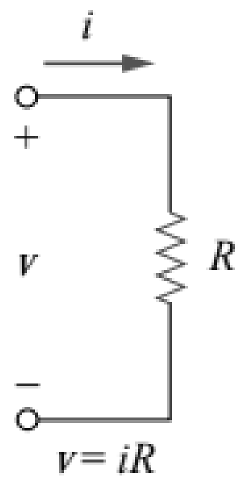
Positive angle means voltage leads current

Negative angle means voltage lags current (current leads voltage)



Impedance for R branch

A voltage $v(t) = V_m \sin \omega t$ is applied to a branch of resistance



Equation for dynamic equilibrium,

$$v = iR$$

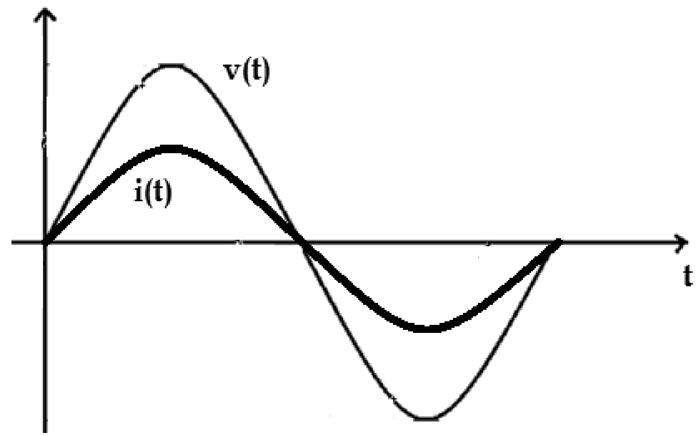
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

Impedance:

$$Z = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{R}} = R$$

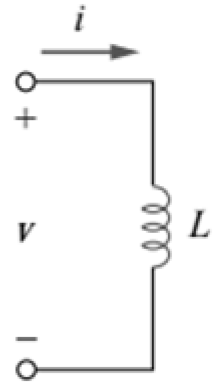
Angle: *v(t)* is in phase with *i(t)*, so angle = 0°

$$\boldsymbol{Z_R = R \angle 0}$$



Impedance for L branch

A voltage $v(t) = V_m \sin \omega t$ is applied to a branch of inductance



Equation for dynamic equilibrium,

$$v = V_m \sin \omega t = L \frac{di}{dt}$$

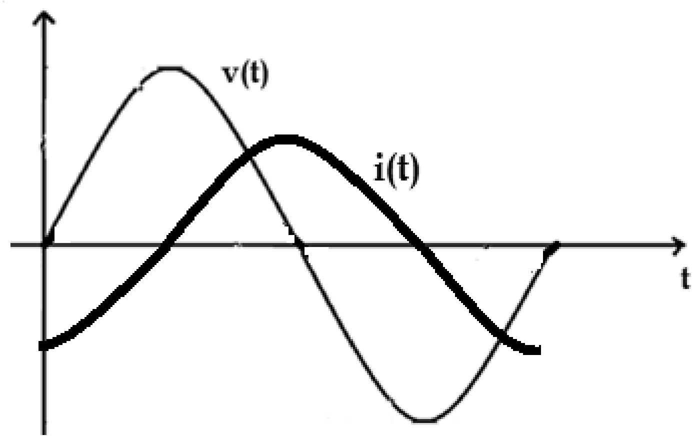
$$i = -\frac{V_m}{\omega L} \cos \omega t = -\frac{V_m}{\omega L} \sin(90° - \omega t) = \frac{V_m}{\omega L} \sin(\omega t - 90°) = I_m \sin(\omega t - 90°)$$

Impedance:

$$Z = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$$

Angle: *i(t)* lags *v(t)* by 90°, so *v(t)* leads *i(t)* by 90°

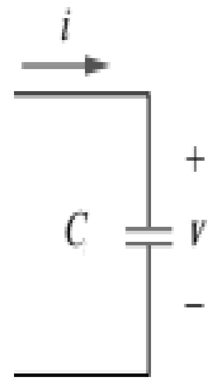
$$\boldsymbol{Z_L = \omega L \angle 90°}$$



Impedance for C branch

A voltage $v(t) = V_m \sin \omega t$ is applied to a branch of inductance

Equation for dynamic equilibrium,



$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t)$$

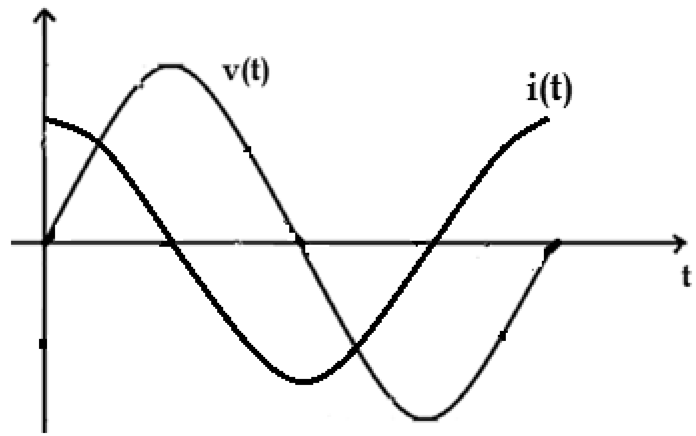
$$i = V_m \omega C \cos \omega t = V_m \omega C \sin(\omega t + 90°) = I_m \sin(\omega t + 90°)$$

Impedance:

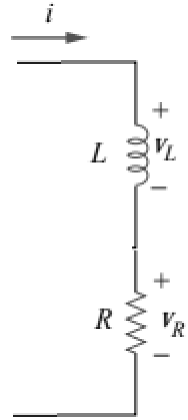
$$Z = \frac{V_m}{I_m} = \frac{V_m}{V_m \omega C} = \frac{1}{\omega C}$$

Angle: *i(t)* leads *v(t)* by 90°, so *v(t)* lags *i(t)* by 90°

$$\boldsymbol{Z_C = \frac{1}{\omega C} \angle -90°}$$



Impedance for RL branch



A current $i(t) = I_m \sin \omega t$ flows through the RL branch

Equation for dynamic equilibrium,

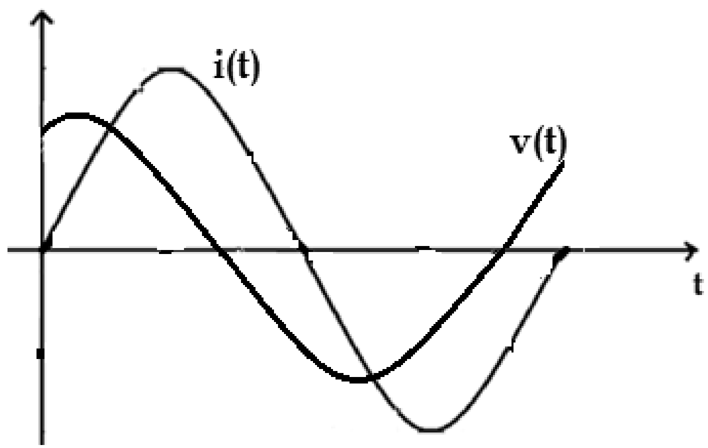
$$v = v_R + v_L = iR + L\frac{di}{dt}$$

$$v = R(I_m \sin \omega t) + L(\omega I_m \cos \omega t) = I_m(R \sin \omega t + \omega L \cos \omega t)$$

Impedance:

$$Z = \frac{V_m}{I_m} = \sqrt{R^2 + (\omega L)^2}$$

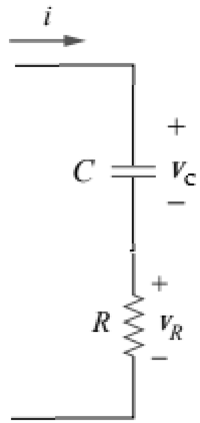
Angle: *v(t)* leads *i(t)* by $\tan^{-1}\left(\frac{\omega L}{R}\right)$



$$\boldsymbol{Z_{RL} = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

Impedance for RC branch

A current $i(t) = I_m \sin \omega t$ flows through the RC branch



Equation for dynamic equilibrium,

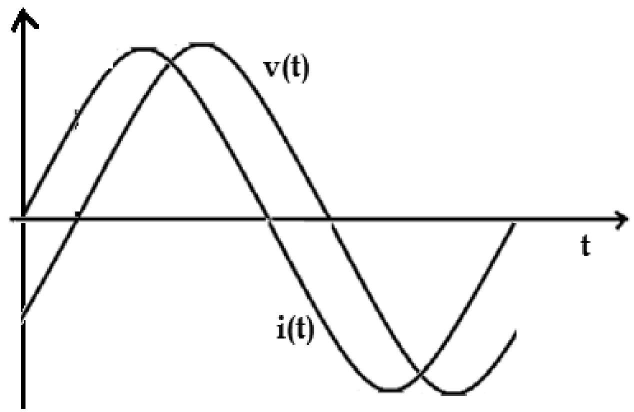
$$v = v_R + v_C = iR + \frac{q}{C} = iR + \frac{\int i\,dt}{C}$$

$$v = R(I_m \sin \omega t) - \frac{I_m}{\omega C} \cos \omega t = I_m \left(R \sin \omega t - \frac{1}{\omega C} \cos \omega t \right)$$

Impedance:

$$Z = \frac{V_m}{I_m} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

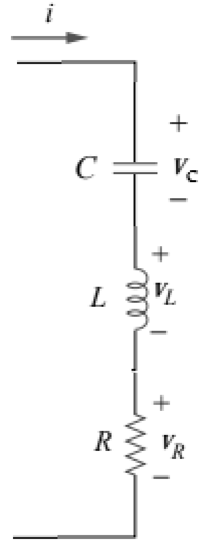
Angle: *v(t)* leads *i(t)* by $\tan^{-1}\left(-\frac{\frac{1}{\omega C}}{R}\right)$



$$\boldsymbol{Z_{RC} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \angle \tan^{-1}\left(\frac{-\frac{1}{\omega C}}{R}\right)}$$

Impedance for RLC branch

A current $i(t) = I_m \sin \omega t$ flows through the RL branch



Equation for dynamic equilibrium,

$$v = v_R + v_L + v_C = iR + L\frac{di}{dt} + \frac{q}{c}$$

$$v = R(I_m \sin \omega t) + L(\omega I_m \cos \omega t) - \frac{I_m}{\omega C}\cos \omega t$$

$$v = I_m \left[R \sin \omega t + \left(\omega L - \frac{1}{\omega C} \right) \cos \omega t \right]$$

Impedance:

$$Z = \frac{V_m}{I_m} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

Angle: *v(t)* leads *i(t)* by $\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

$$\boldsymbol{Z_{RL} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \angle \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$$