**Measures of Central Tendency or Location**

In a representative sample, the value of a series of data have a tendency to cluster around a certain point usually at the center of the series is called central tendency. And its numerical measures are called the measures of central location.

* The purpose of a measure of central tendency is to point out the center of a set of data values or a distribution.

**Different Measures of Central Tendency**

The following are the important measures of central tendency:

1. Mean
2. Arithmetic mean or simply “mean”
3. Geometric mean
4. Harmonic Mean
5. Median
6. Mode

**Arithmetic Mean (AM) (for ungrouped data):** Let  be a random variable and the values of  are given by. Now, the arithmetic mean for n ungrouped observations is given by:



where,

is the arithmetic mean

*n* is the total number of observations.

*xi* is a particular value

** indicates the operation of adding

**Example-1:**

b) The thickness of printed circuit board is very important characteristics. A sample of 10 boards had the following thickness (in thousands of an inch):

63 61 65 62 61 64 60 66 68 61

Find out the mean/average/arithmetic mean of the given data.

**Solution:**

Let thickness be denoted by X

By using calculator, ∑631

Therefore the mean is,

Hence, the average thickness (in thousands of an inch) is 63.1.

**Arithmetic Mean (for grouped data):** Let  be a random variable and the mid-values of  are given by and the respective frequencies are  . Now, the arithmetic mean for n grouped observations is given by:



where,

*n* = 

**Example-2:**

The number of students present in a class in 20 working days:

20 20 26 30 22 **15** 17 34 23 35 23 36 28

34 31 19 **38** 32 37 24

Find out the mean/average/arithmetic mean of the given data.

**Solution:**

We know that,



Where,

k=number of classes

n=number of observations

* There are 20 observations so n=20.
* Two raised to the fifth power is 32.
* Therefore, we should have **at least** 5 classes, i.e., k=5.

Now to determine the class interval or width we have,

***.***

Table-1: Table of the number of students present in a class in 20 working days

|  |  |  |
| --- | --- | --- |
| Students present | Mid-point ( | Frequency, *f* |
| 15-19 | 17 | 3 |
| 20-24 | 22 | 6 |
| 25-29 | 27 | 2 |
| 30-34 | 32 | 5 |
| 35-39 | 37 | 4 |
| Total |  | n=20 |

We know that,



By using calculator,

=450

**Merits of AM**

1. It is easy to understand,
2. It is easy to calculate,
3. It takes all the observations into account.

**Demerits of AM**

1. It is affected by extreme values
2. It cannot be computed from a distribution that has open-end class interval

**Geometric Mean (GM) (for ungrouped data):** The geometric mean of a set of n values (ungrouped) can be defined as

**GM =** 

The calculation may sometime be simplified by taking logarithm, that is

Log G.M=

**=**

**G.M=Antilog ()**

**Example:**

From the previous **example-1** we can calculate the G.M.

We know that,

**G.M=Antilog ()**

Let us consider observation by X

Now by using calculator we get,

=13.043 and n=10

G.M=Antilog () =Antilog (1.7997) =63.052.

**Geometric Mean (GM) (for grouped data):** The geometric mean of a set of n values (grouped) can be defined as

**GM =** 

where,

*n* = 

**Example:**

From the table-1 of previous **example-2** we can calculate the G.M.

**Solution**

Calculation of geometric mean

|  |  |  |
| --- | --- | --- |
| Students present | Mid-point ( | Frequency, *f* |
| 15-19 | 17 | 3 |
| 20-24 | 22 | 6 |
| 25-29 | 27 | 2 |
| 30-34 | 32 | 5 |
| 35-39 | 37 | 4 |
| Total |  | n=20 |

We know that,

**G.M=Antilog ()**

From the given data, by using calculator we get

=28.407 and n=20

G.M=Antilog =Antilog (1.420) =26.303

**Harmonic Mean (for ungrouped data):** The harmonic mean of a set of n values (ungrouped) can be define as

**HM =** =

**Example:**

From the previous **example-1** we can calculate the H.M.

We know that,

**HM=** 

Let, the observations denoted by X.

By using calculator we get,

=0.159 and n=10

H.M==62.893.

**Harmonic Mean (for grouped data):** The harmonic mean of a set of n values (grouped) can be define as

**H.M =** 

where,

*n*= 

**Example:**

From the table-1 of previous **example-2** we can calculate the H.M.

**Solution**

Calculation of geometric mean

|  |  |  |
| --- | --- | --- |
| Students present | Mid-point ( | Frequency, *f* |
| 15-19 | 17 | 3 |
| 20-24 | 22 | 6 |
| 25-29 | 27 | 2 |
| 30-34 | 32 | 5 |
| 35-39 | 37 | 4 |
| Total |  | n=20 |

We know that,

**H.M =** 

By using calculator we get,

=0.788 and n==20

H.M==25.301

**Median for ungrouped data:** The Median is defined as the middle most observation when the observations are arranged in order of magnitude (in ascending or descending order).

* when n is odd, the middle most observation that is  th observation will be the median in the series.
* when n is even, the median will be the arithmetic mean of  th and  th observations in the series.

**Example:**

Considering example-1 we will calculate the median.

The data were,

63 61 65 62 61 64 60 66 68 61

**Solution:**

For calculating median the data have to arrange either in ascending or descending order. Here the data has been arranged in ascending order.

60 61 61 61 **62** **63** 64 65 66 68

We know that, when n is even, the median will be the arithmetic mean of  th and th observations in the series.

We have, n=10

n/2 th observation=10/2=5th observation

Observation===6th observation

Median= (5th observation+6th observation)/2= (62+63)/2=62.5

Hence, the median thickness (in thousands of an inch) is 62.5.

**Median for grouped data:**

For grouped frequency distribution the Median is given by 

where,

 = The lower limit of the median class (Median class is that class which contains  th observation of the series)

 = Cumulative frequency of the class just preceding the median class

 = Frequency of the median class

*c*  = Length of the median class

**Example:**

|  |  |  |
| --- | --- | --- |
| Students present | Frequency, *f* | Cumulative frequency |
| 15-19 | 3 | 3 |
| 20-24 | 6 | 9 |
| **25-29** | 2 | 11 |
| 30-34 | 5 | 16 |
| 35-39 | 4 | 20 |
| Total | n=20 |  |

Median=Size of n/2 th observation =20/2=10 th observation

Hence, the median lies in the class 25-29

Median=L+= 25+=25+2.5=27.50

Hence, the median is 27.50.

**Mode for ungrouped data:**

Mode is that observation of a data set for which the frequency is maximum.

**Example:**

Considering example-1 now we will calculate the mode.

The data were,

63 61 65 62 61 64 60 66 68 61

**Solution:**

Since the observation 61 occurs 3 times (i.e. with highest frequency), therefore 61 is the mode value for this data set.

∴ Mode= 61.

**Mode of Grouped Data:**

For grouped frequency distribution the Mode is given by 

where,

 = The lower limit of the modal class (Modal class is that class for which the frequency is maximum)

= The difference between the frequency of the modal class and pre-modal class

= The difference between the frequency of the modal class and post-modal class

*c* = The length of the modal class.

**Example:**

|  |  |
| --- | --- |
| Students present | Frequency, *fi* |
| 15-19 | 3 |
| 20-24 | **6** |
| 25-29 | 2 |
| 30-34 | 5 |
| 35-39 | 4 |
| Total | n=20 |

Calculate the mode.

**Solution**

Since the maximum frequency 6 is in the class 25-29, therefore 25-29 is the modal class.

Mode=

L=20, (6-3) =3, (6-2) =4, c=5

Mode==20+=22.14≈22

Hence, the mode of the given data is 22.

**Weighted Mean:**

The weighted mean is a special case of arithmetic mean. It occurs when there are several observations of the same value.

The Weighted mean of the positive real numbers with their weight is defined to be.

**Example**

Suppose that, the nearby Wendy’s Restaurant sold medium, large and Biggie-sized soft drinks for $0.90, $1.25 and $1.50 respectively of the last 10 drinks sold, 3 were medium, 4 were large and 3 were Biggie-sized. Find the mean price of the last 10 drinks sold.

**Solution**

We know that,



===$1.22

**Properties of Arithmetic Mean:**

Property 1: The algebraic sum of the deviations of a set of n values from their arithmetic mean is zero. Mathematically, 

Property 2: The arithmetic mean of a set of n constant observation A is A.

Property 3: The sum of the squares of the deviations of a set of values is minimum when taken about mean. Mathematically, 

Property 4: The arithmetic mean depends on the origin and scale of measurement. Mathematically, 

**Characteristics to be a good measure of central tendency:**

1. Should be easy to compute.
2. Should be easily understandable.
3. Should be amenable to mathematical calculation.
4. Should be based on all observation.
5. Should be unaffected by the presence of extreme values
6. Should not have sampling variability**.**

**Compare Mean, Median and Mode:**

1. Mean, median and mode are easy to understand and easy to calculate.
2. Mean is based upon all observations but median and mode are not.
3. Mean is amenable to mathematical calculation but median and mode are not.
4. Mean cannot be calculated from the distribution with open class but median and mode can be calculated from the distribution with classes.
5. Mean is affected very much by extreme values but median and mode are not affected by the extreme values.