# Mesh Analysis

What is Mesh Analysis ?

- *Mesh Analysis or Loop analysis* is a process that uses KVL to determine loop currents.

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero. So, Sum of voltage drops = Sum of voltage rises
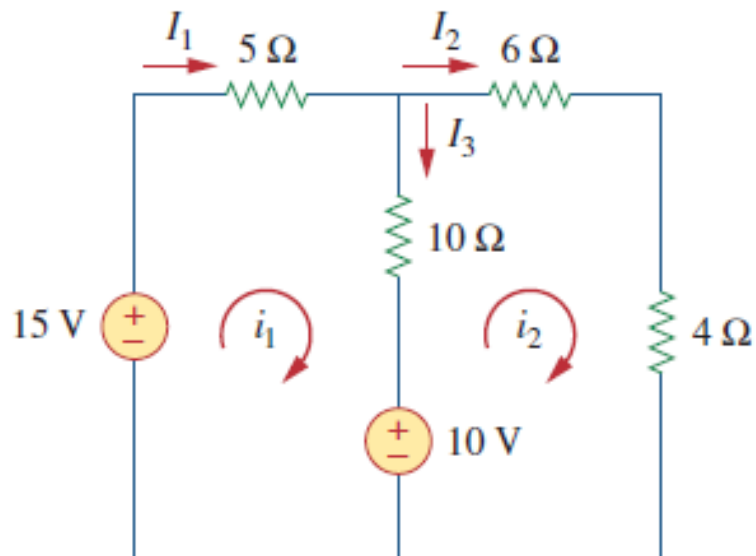
- $\sum_{m=1}^{M} v_m = 0$

- *Mesh* currents are assigned to all independent loops.

- To solve for all loop currents, write a KVL equation for all independent loops and then solve.

$R\,(I_2 - I_1)$
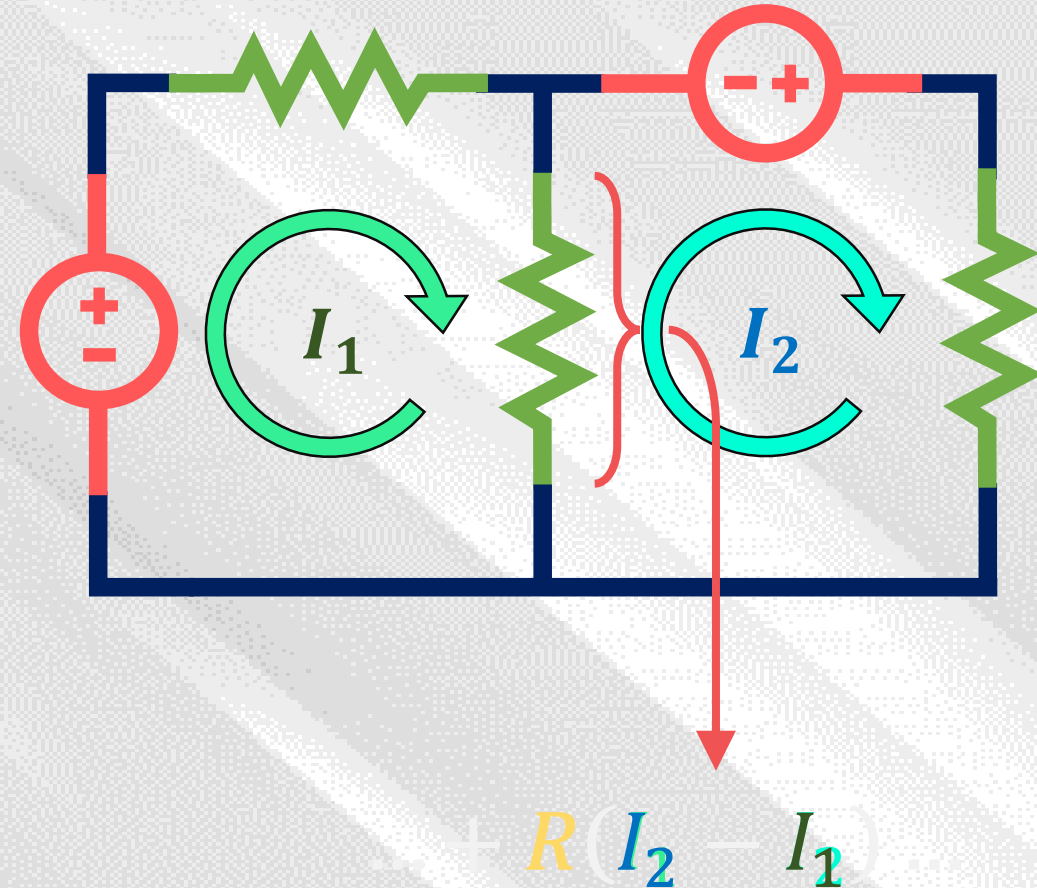
Mathematical Problem-1

Determine I_1, I_2, I_3

Equation-for Mesh-1

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$3i_1 - 2i_2 = 1$$

- $-15 + 5i_1 + 10i_1 + 10 - 10i_2 = 0$
- $15i_1 - 10i_2 = 5$
- $3i_1 - 2i_2 = 1$

Answer

$$i_1 = \mathbf{1\ A}$$
$$i_2 = \mathbf{1\ A}$$
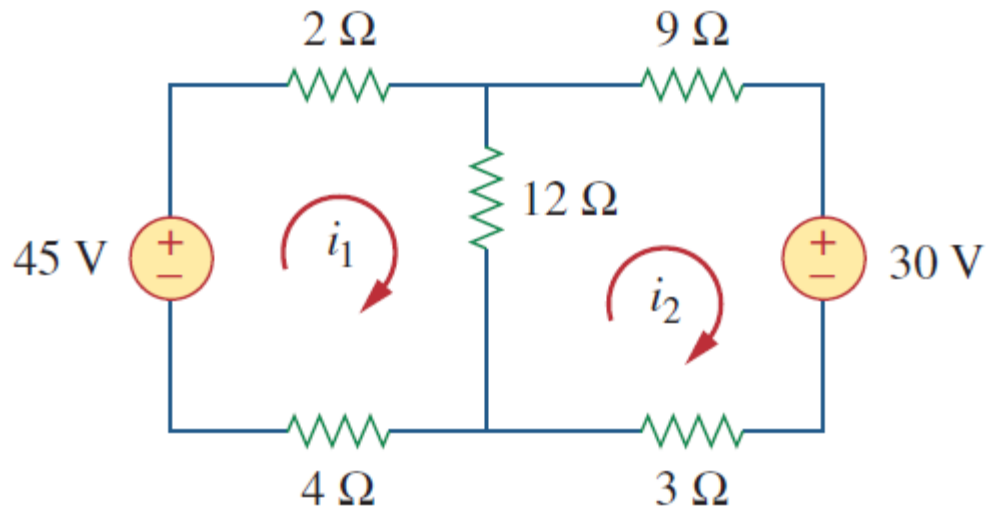$$I_1 = i_1 = 1A,$$
$$I_2 = i_2 = 1A,$$
$$I_3 = i_1 - i_2 = 0$$

Equation-for Mesh-2

- $-10 + 10i_2 + 6i_2 + 4i_2 - 10i_1 = 0$
- $20i_2 - 10i_1 = 10$
- $-i_1 + 2i_2 = 1$

Mathematical Problem-2

Determine i_1 , i_2

Equation-for Mesh-1

- $-45 + 2i_1 + 12i_1 + 4i_1 - 12i_2 = 0$
- $(2 + 12 + 4)i_1 - 12i_2 - 45 = 0$
- $18i_1 - 12i_2 = 45$
- $6i_1 - 4i_2 = 15$

Equation-for Mesh-2

- $(12 + 9 + 3)i_2 - 12i_1 + 30 = 0$
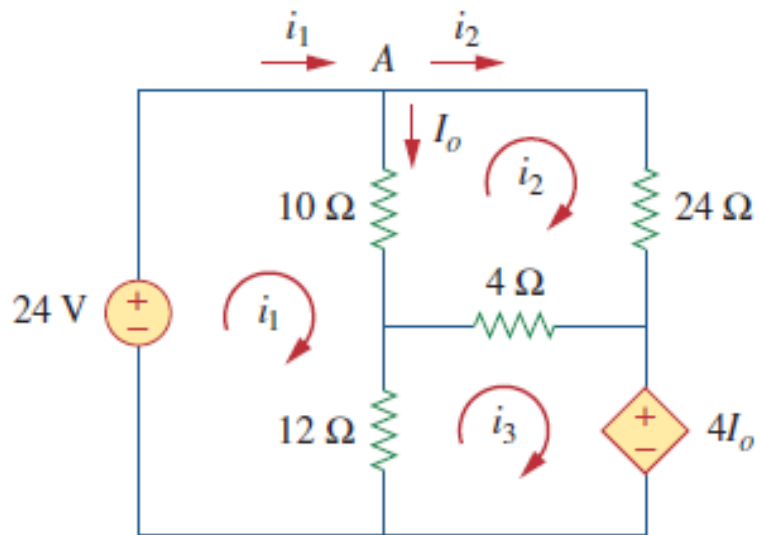- $24i_2 - 12i_1 = -30$
- $-2i_1 + 4i_2 = -5$

Answer

$i_1 = \mathbf{2.5A}$

$i_2 = \mathbf{0A}$

Mathematical Problem-3

Use mesh analysis to find current I_o in the circuit.

Equation-for Mesh-1

- $(10 + 12)i_1\text{-}10i_2-12i_3-24 = 0$
- $22i_1\text{-}10i_2-12i_3 = 24$

Equation-for Mesh-2

- $(10 + 4 + 24)i_2\text{-}10i_1-4i_3 = 0$
- $38i_2\text{-}10i_1-4i_3 = 0$

Equation-for Mesh-3

- $(12 + 4)i_3\text{-}4i_2-12i_1 + 4I_o = 0$
- $16i_3\text{-}4i_2-12i_1 + 4(i_1-i_2) = 0$

- $I_o = (i_1-i_2)$

Answer

$i_1 = \mathbf{2.25A}$
$i_2 = \mathbf{0.75A}$
$i_3 = \mathbf{1.5A}$
$I_o = \mathbf{1.5A}$

Mathematical Problem-4

Use mesh analysis to find current I_o in the circuit.
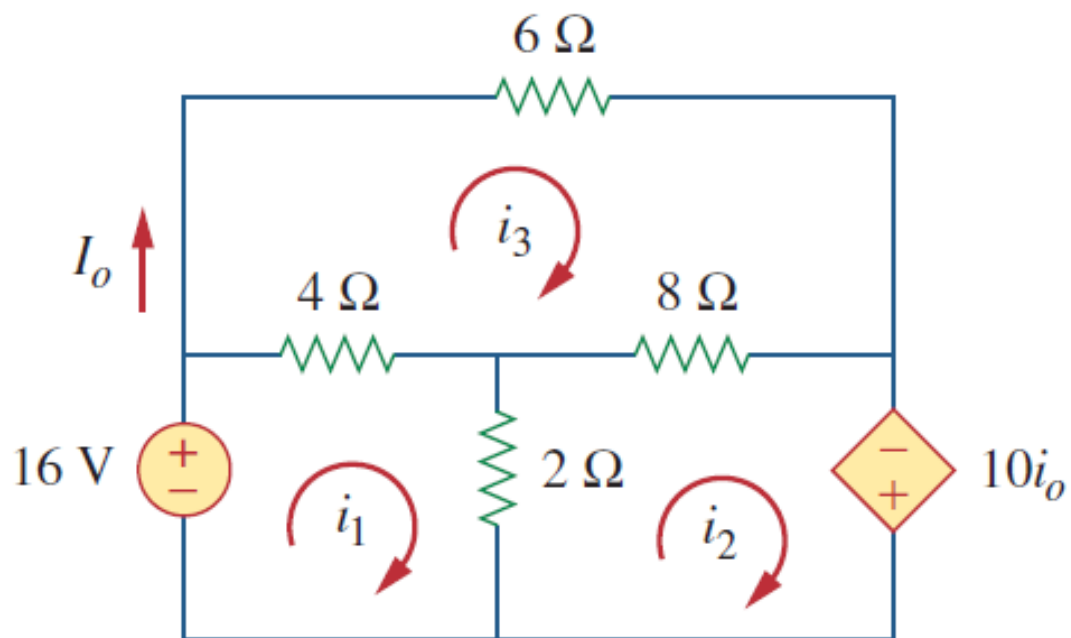
Equation-for Mesh-1

- $6i_1 - 2i_2 - 4i_3 = 16$

Equation-for Mesh-2

- $10i_2 - 2i_1 - 8i_3 = 10I_o$
- $i_3 = I_o$
- $10i_2 - 2i_1 - 18i_3 = 0$

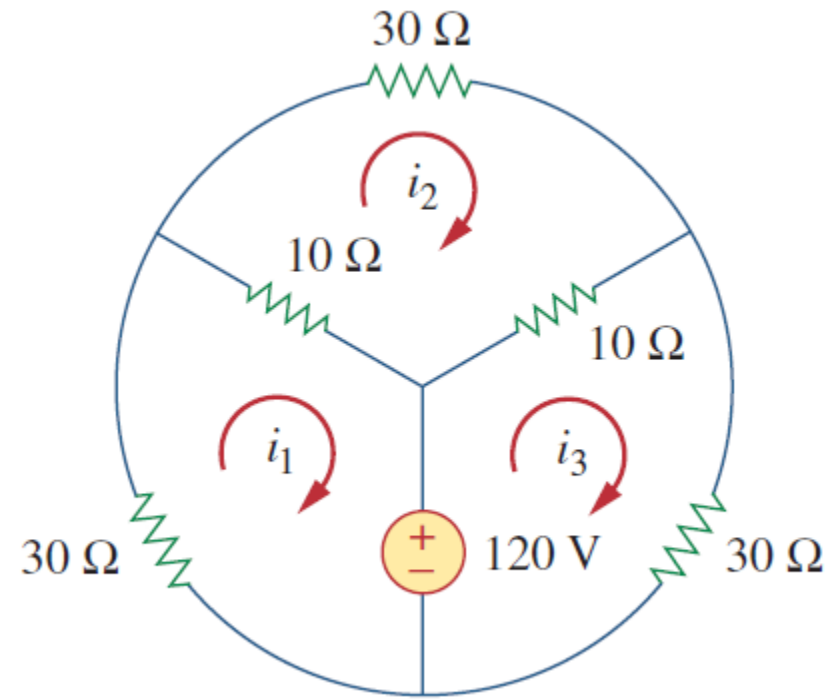## Equation-for Mesh-3

- $18i_3 - 4i_1 - 8i_2 = 0$

Try Yourself

Answer

$I_o = -4\text{A}$

Assignment Problem
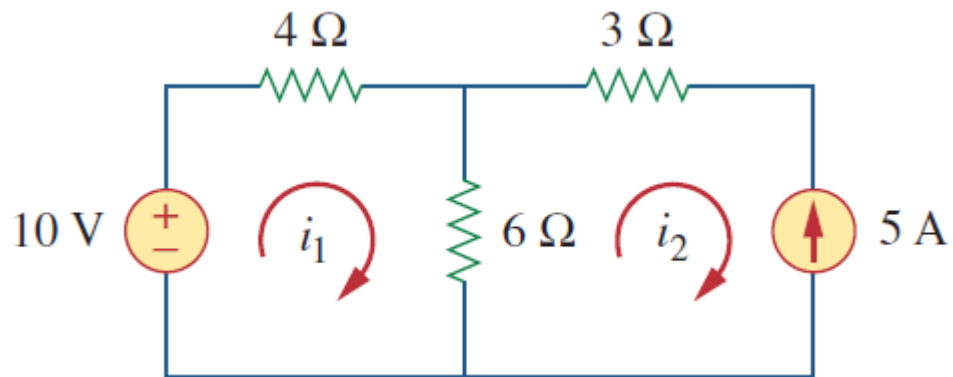
Use mesh analysis to find current i_1, i_2, i_3 in the circuit.

Mathematical Problem-5

Use mesh analysis with current source

Use mesh analysis to find current i_1 and i_2 in the circuit.

Equation-for Mesh-1

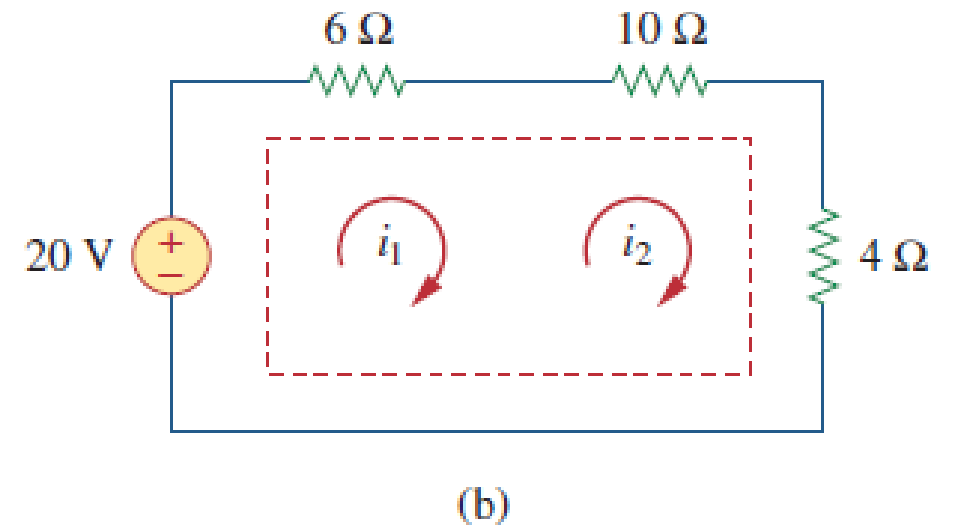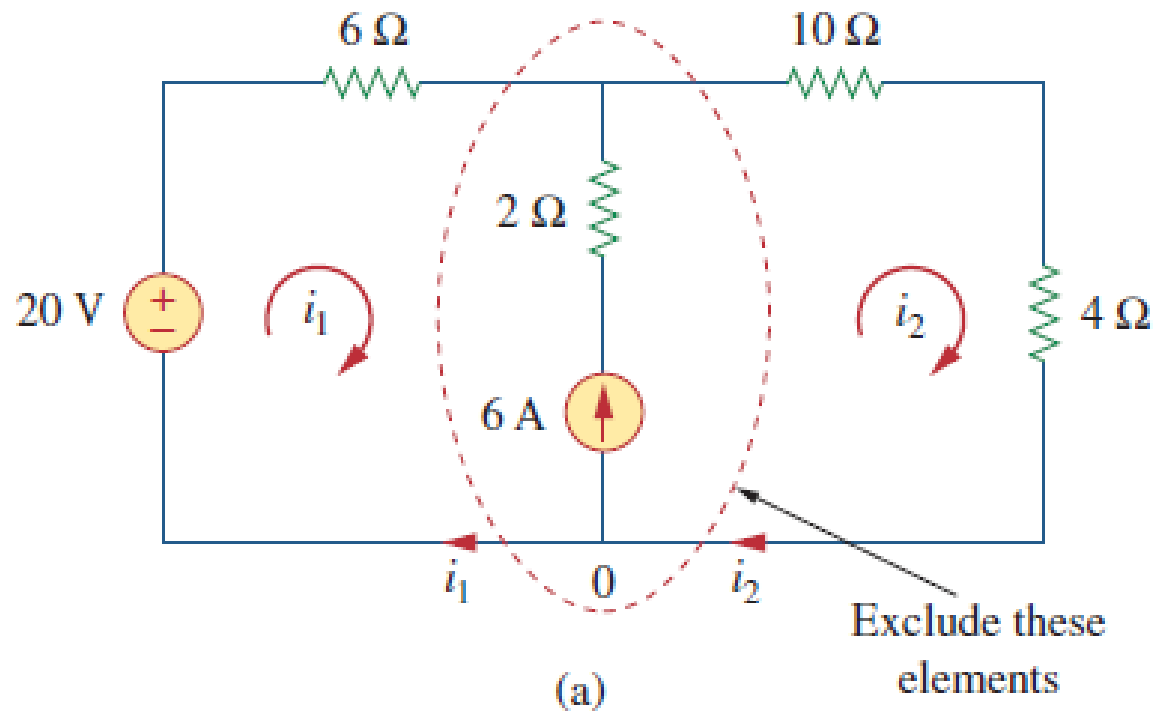- $10i_1\text{-}6i_2 - 10 = 0$

Equation-for Mesh-2

$$i_2 = -5 \text{ A}$$
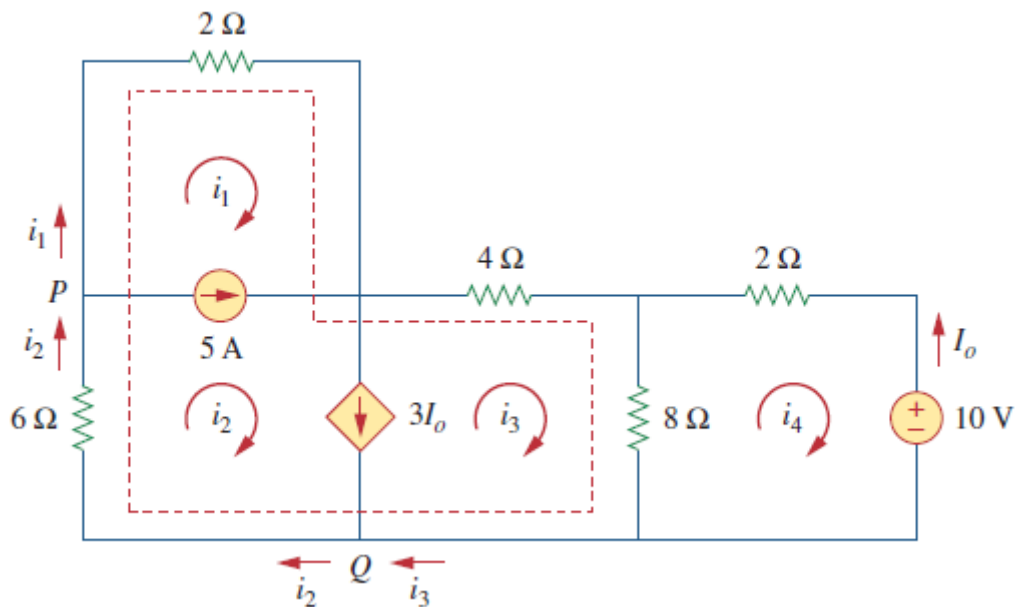
Answer

$$i_1 = -\mathbf{2A}$$

Supermesh

(a)

(b)

Mathematical Problem-6

Use mesh analysis to find current i_1 to i_4 in the circuit.

Equation-1

- $2i_1 + 6i_2 + 12i_3 - 8i_4 = 0$
- $i_1 + 3i_2 + 6i_3 - 4i_4 = 0$

Equation-2

- $i_2 - i_1 = 5$

Equation-3

- $i_2 - i_3 = 3I_o = 3(-i_4)$
- $i_2 - i_3 + 3i_4 = 0$

Equation-4

- $10i_4 - 8i_3 + 10 = 0$
- $5i_4 - 4i_3 = -5$

Answer
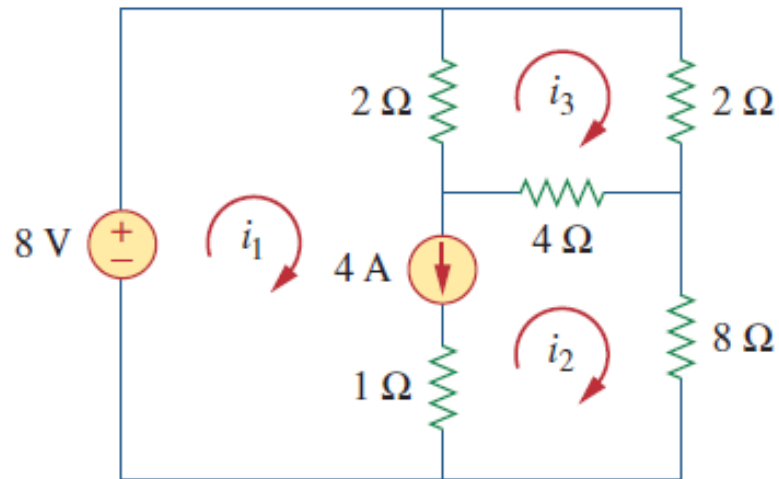
$i_1 = -7.5 \text{ A}$
$i_2 = -2.5 \text{ A}$
$i_3 = 3.93 \text{ A}$
$i_4 = 2.143 \text{A}$

Mathematical Problem-6

Find current i_1 to i_3 in the circuit.

Equation-1

- $2i_1 + 12i_2 - 2i_3 - 4i_3 - 8 = 0$
- $2i_1 + 12i_2 - 6i_3 = 8$
- $i_1 + 6i_2 - 3i_3 = 4$

Equation-2

- $i_1 - i_2 = 4$

Equation-3

- $8i_3 - 4i_2 - 2i_1 = 0$
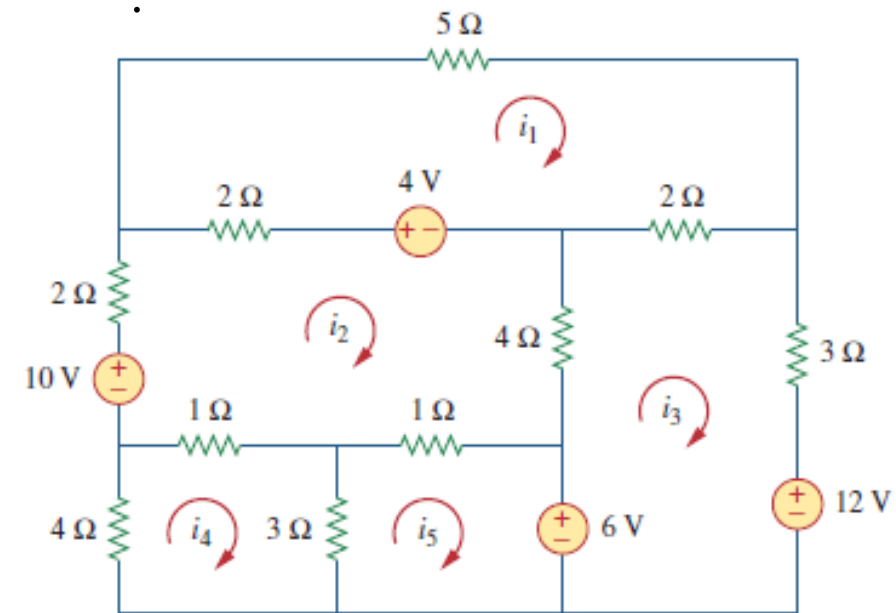- $4i_3 - 2i_2 - i_1 = 0$

Answer

$$i_1 = 4.632 \text{A}$$
$$i_2 = 631.6 \text{ mA}$$
$$i_3 = 1.4736 \text{ A}$$

Mathematical Problem-6

By inspection write the mesh current equations for the circuit

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Equation-1
- $9i_1 - 2i_2 - 2i_3 = 4$
- $9i_1 - 2i_2 - 2i_3 - 0.i_4 - 0.i_5 = 4$

Equation-2
- $10i_2 - 2i_1 - 4i_3 - i_4 - i_5 - 10 + 4 = 0$
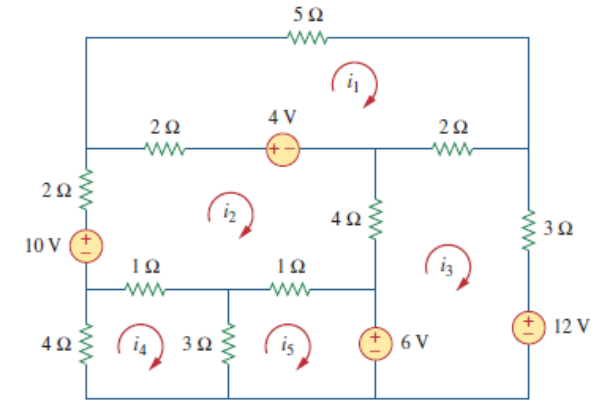- $-2i_1 + 10i_2 - 4i_3 - i_4 - i_5 = 6$

Equation-3
- $9i_3 - 2i_1 - 4i_2 - 6 + 12 = 0$
- $-2i_1 - 4i_2 + 9i_3 - 0.i_4 - 0.i_5 = 6$

Equation-4
- $8i_4 - 3i_5 - i_2 = 0$
- $0.i_1 - i_2 + 0.i_3 + 8.i_4 - 3.i_5 = 0$

Equation-5
- $4i_5 - 3i_4 - i_2 + 6 = 0$
- $0.i_1 - i_2 + 0.i_3 - 3.i_4 + 4.i_5 = 0$

Equation-1
- $9i_1 - 2i_2 - 2i_3 = 4$
- $\mathbf{9i_1 - 2i_2 - 2i_3 - 0.i_4 - 0.i_5 = 4}$

Equation-2
- $10i_2 - 2i_1 - 4i_3 - i_4 - i_5 - 10 + 4 = 0$
- $\mathbf{-2i_1 + 10i_2 - 4i_3 - i_4 - i_5 = 6}$

Equation-3
- $9i_3 - 2i_1 - 4i_2 - 6 + 12 = 0$
- $\mathbf{-2i_1 - 4i_2 + 9i_3 - 0.i_4 - 0.i_5 = -6}$

Equation-4
- $8i_4 - 3i_5 - i_2 = 0$
- $\mathbf{0.i_1 - i_2 + 0.i_3 + 8.i_4 - 3.i_5 = 0}$

Equation-5
- $4i_5 - 3i_4 - i_2 + 6 = 0$
- $\mathbf{0.i_1 - i_2 + 0.i_3 - 3.i_4 + 4.i_5 = -6}$

Thus, the mesh-current equations are:

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

Thank you