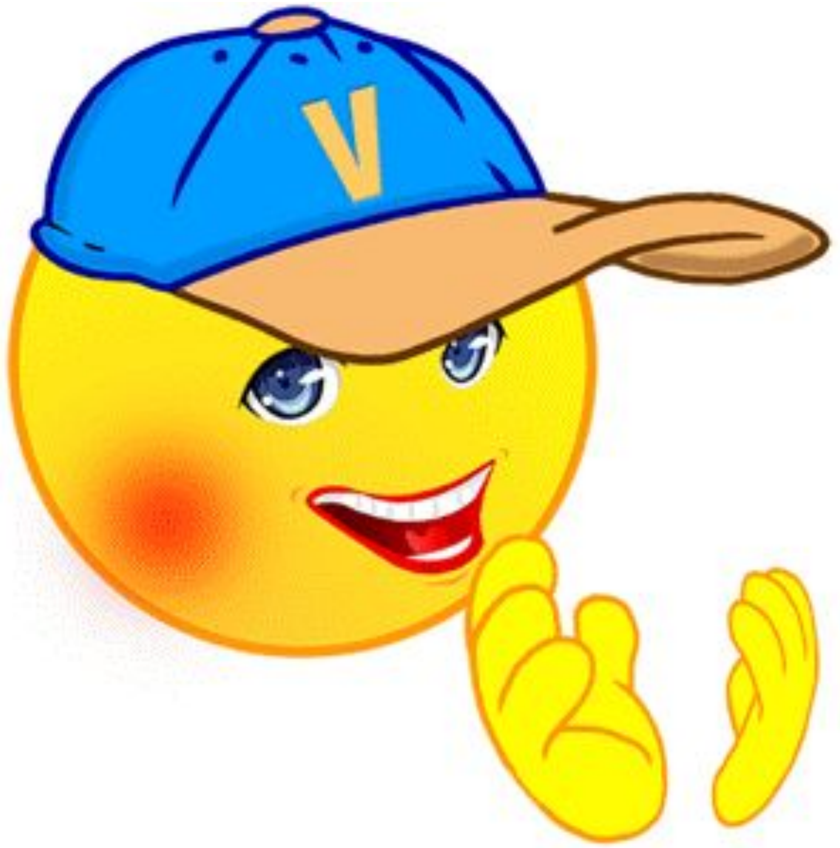
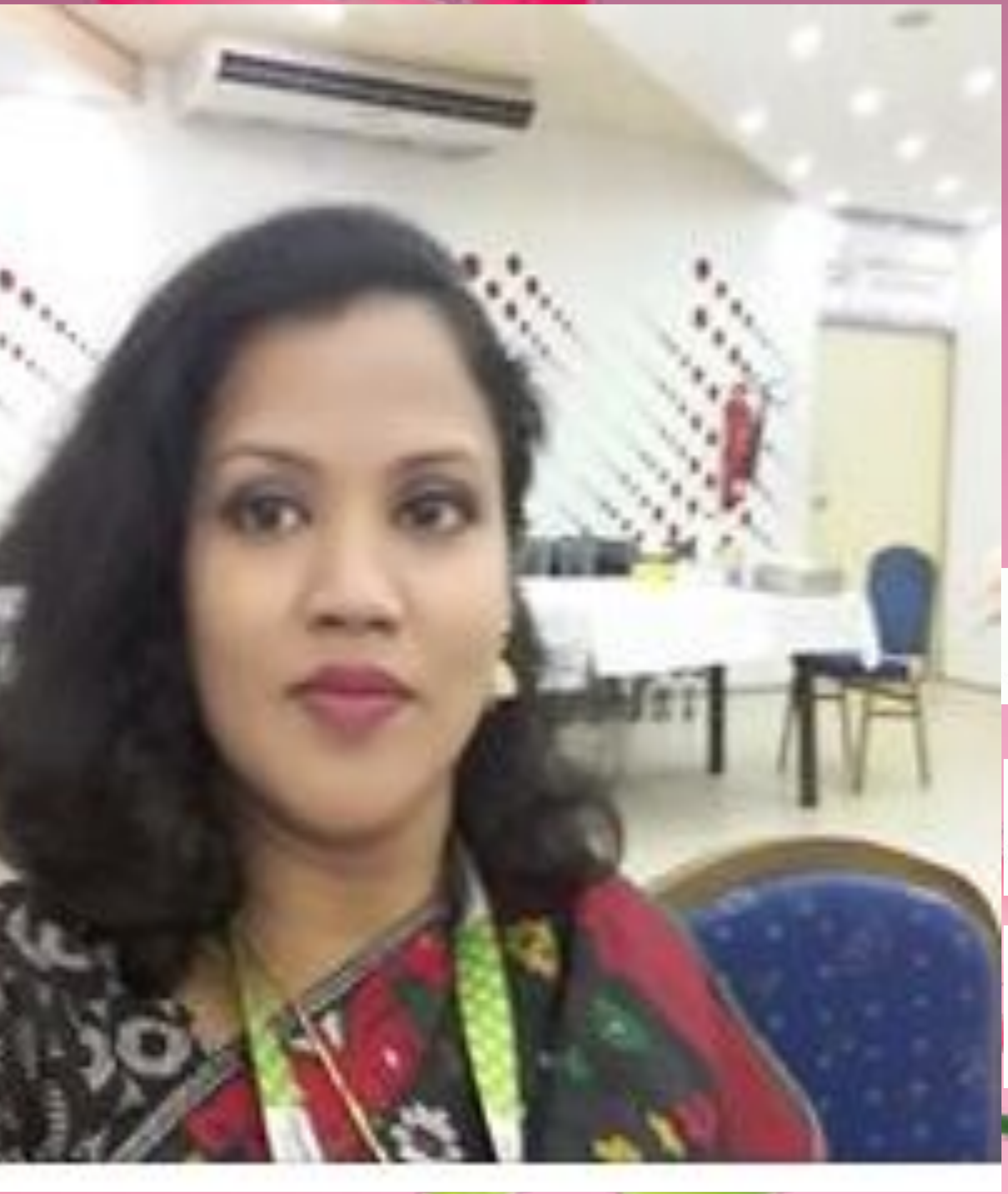
welcome





About Myself

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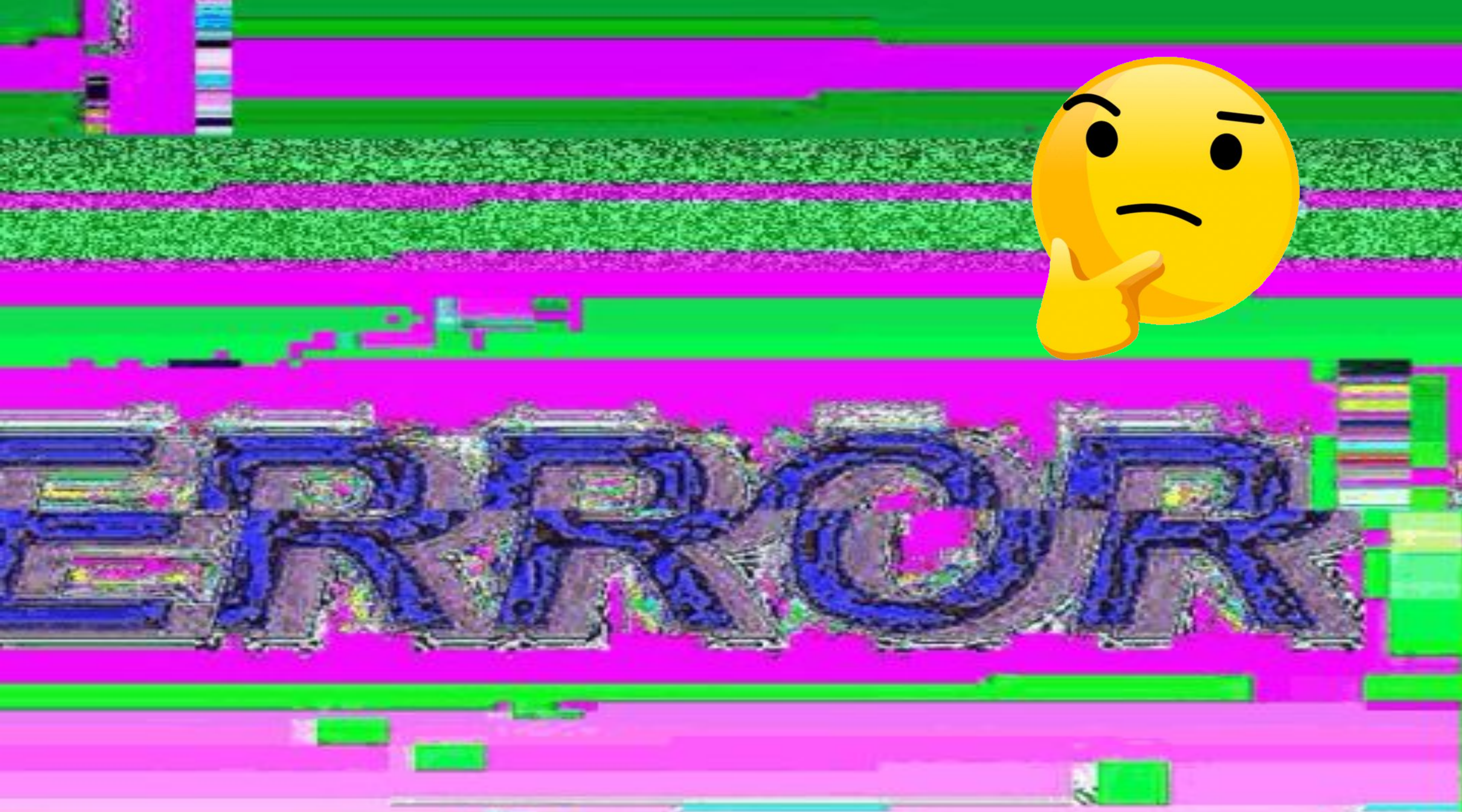
ERROR

Chapter 01

Error Analysis

Outcomes:

After reading this chapter, you should be able to:

1. To determine of Error, exact, relative, and percentage error.
2. To calculate of the absolute, relative, and approximate error to the number of significant digits.
3. To determine of error in numerical methods – round-off and truncation error.
4. To calculate about the difference between round-off and truncation error.

***Numerical Error

If we want to solve,

$$\frac{2}{5}+\frac{25}{10}=2.9$$ → **Exact Value**

$$\frac{25}{36}+\frac{12}{13}=1.61752136752$$ → **For our convenience we may write 1.6175**

Approximate value

So in the second calculation, calculated value is 1.61752136752 (Exact value) but we write 1.6175 (Approximate value).

So the Error occurred here, 1.61752136752 - 1.6175 = 0.00002136752.

Finally,
The error of a quantity is the difference between it's true value and approximate.
It is denoted by E.
If the true value is X and approximate value is x

$$E = X - x$$

then the error of the quantity is given

Different Types of Error

- Absolute error
- Relative error
- Percentage error

The absolute error of a quantity is the **absolute value** of the **difference** between **the exact value** and **the approximate value.** It is denoted by E_A. If the **exact value is X** and **approximate value is x**, the absolute error is define by, $E_A = |X - x|$.

The relative error of a quantity is the ratio of its absolute error to its exact value. It is denoted by E_R, $E_R = \frac{E_A}{X}$

The percentage error of a quantity is **100 times of its relative error.** It is denoted by E_P, that is, $E_P = 100E_R$

$$E_A = |X - x| = |3.1415926 - 3.1428571|$$

Problem: An approximate value of π is 3.1428571 and true value is 3.1415926. Find the absolute, relative and percentage errors

Solution: We have, true value $X = 3.1415926$

and approximate value $x = 3.1428571$

The absolute error is, $E_A = |X - x|$

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$$= |3.1415926 - 3.1428571|$$

$$= |-0.0012645| = 0.0012645$$

The relative error is, $E_R = \frac{E_A}{X} = \frac{0.0012645}{3.1415926} = 0.000402$

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The percentage error is, $E_p = 100 E_R$

$= 100 \times 0.000402$

$= 0.0402$

** *Round off Error:

When a calculator or digital computer is used to perform numerical calculations, an unavoidable error, called round-off error, must be considered. A round-off error, also called rounding error, is the difference between the calculated approximation of a number and its exact mathematical value due to rounding

For example, a number like $\frac{1}{3}$ may be represented as 0.333333 on a PC.

Then the round off error in this case is $\frac{1}{3} - 0.333333 = 0.00000033\bar{3}$.

Then there are other numbers that cannot be represented exactly.

For example, π and $\sqrt{2}$ are numbers that need to be approximated in computer calculations.

Formula: If the number P is rounded to N decimal place then $E_A = \frac{1}{2}(10^{-N})$

Problem: Find the absolute, relative and percentage errors of the number 8.2546 if this number rounded up to two decimal places.

Solution: The given number is $P = 8.2546$

Since the given number rounded up to two decimal places so $N = 2$

The absolute error is, $E_A = \frac{1}{2}(10^{-N})$

$$= \frac{1}{2}(10^{-2})$$

$$= 0.005$$

The given number , $P = 8.2546$

The absolute error is, $E_A = 0.005$

Here , Exact value $X = 8.25$

Given number is $P = 8.2546$, if we write this number as, 8.2541 then error will be,
E=8.2546-8.2549 = 0.0005
From the error we can say, as long as there is zero there is no mistake that is exact.

The relative error is, $E_R = \frac{E_A}{X} = \frac{0.005}{8.25}$

$$= 0.00060606061$$

The percentage error is, $E_P = 100E_R$

$$= 100 \times 0.00060606061$$

$$= 0.060606061\%$$

Significant Digits:

5.21432 → 5 decimal places

But the significant digits are 6

Example:
456 has 3 significant digits
68.29 has 4 significant digits

Example:
0.067 has 2 significant digits
0.000008 has 1 significant digit

Some rules to identify the significant numbers

Example:
12.2300 has six significant figures
0.000122300 still has only six significant figures

Example:
5.609 has 4 significant digits.
700.0879 has 7 significant digits.

- Every non-zero digit is significant
- Zeros before non-zero digits are never significant
- Zeros behind non-zero digits are sometimes significant
- Zeros between non-zero digits are always significant

Problem: Evaluate the sum $S = \sqrt{2} + \sqrt{3} + \sqrt{5}$ to 4 significant digits and also find its absolute, relative and percentage errors.

Solution: For the 4 significant digits we may write

$\sqrt{2} = 1.414,$ → **Rounded up to 3 decimal places, so for N=3**

$\sqrt{3} = 1.732,$ → **Rounded up to 3 decimal places, so for N=3**

$\sqrt{5} = 2.236,$ → **Rounded up to 3 decimal places, so for N=3**

So, sum $\boldsymbol{S = 1.414 + 1.732 + 2.236 = 5.382}$

Since the values of $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$ all are rounded to three decimal places, so for all values $\boldsymbol{N = 3}$.

Now the absolute error is, $\boldsymbol{E_A = \frac{1}{2}(10^{-N}) + \frac{1}{2}(10^{-N}) + \frac{1}{2}(10^{-N})}$

$$\boldsymbol{= \frac{1}{2}(10^{-3}) + \frac{1}{2}(10^{-3}) + \frac{1}{2}(10^{-3})}$$

$$\boldsymbol{= 0.0005 + 0.0005 + 0.0005 = 0.0015}$$

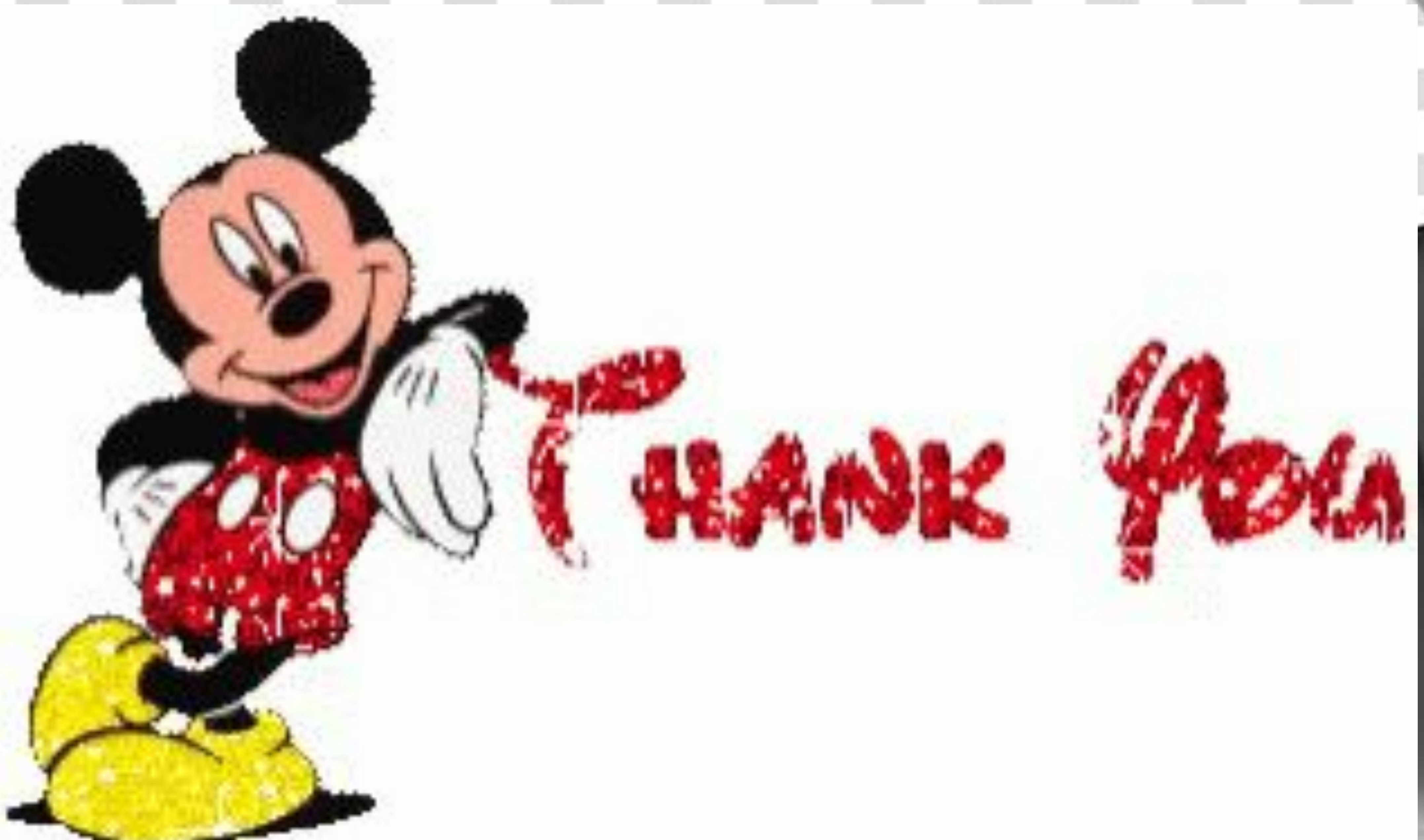
The absolute error shows that the sum is correct to two decimal places only, so we take exact value ,

$$X = 5.38$$

The relative error is, $E_R = \frac{E_A}{X} = \frac{0.0015}{5.38} = 0.00028$

The percentage error is, $E_P = 100E_R = 100 \times 0.00028$

$$= 0.028\%$$


Thank