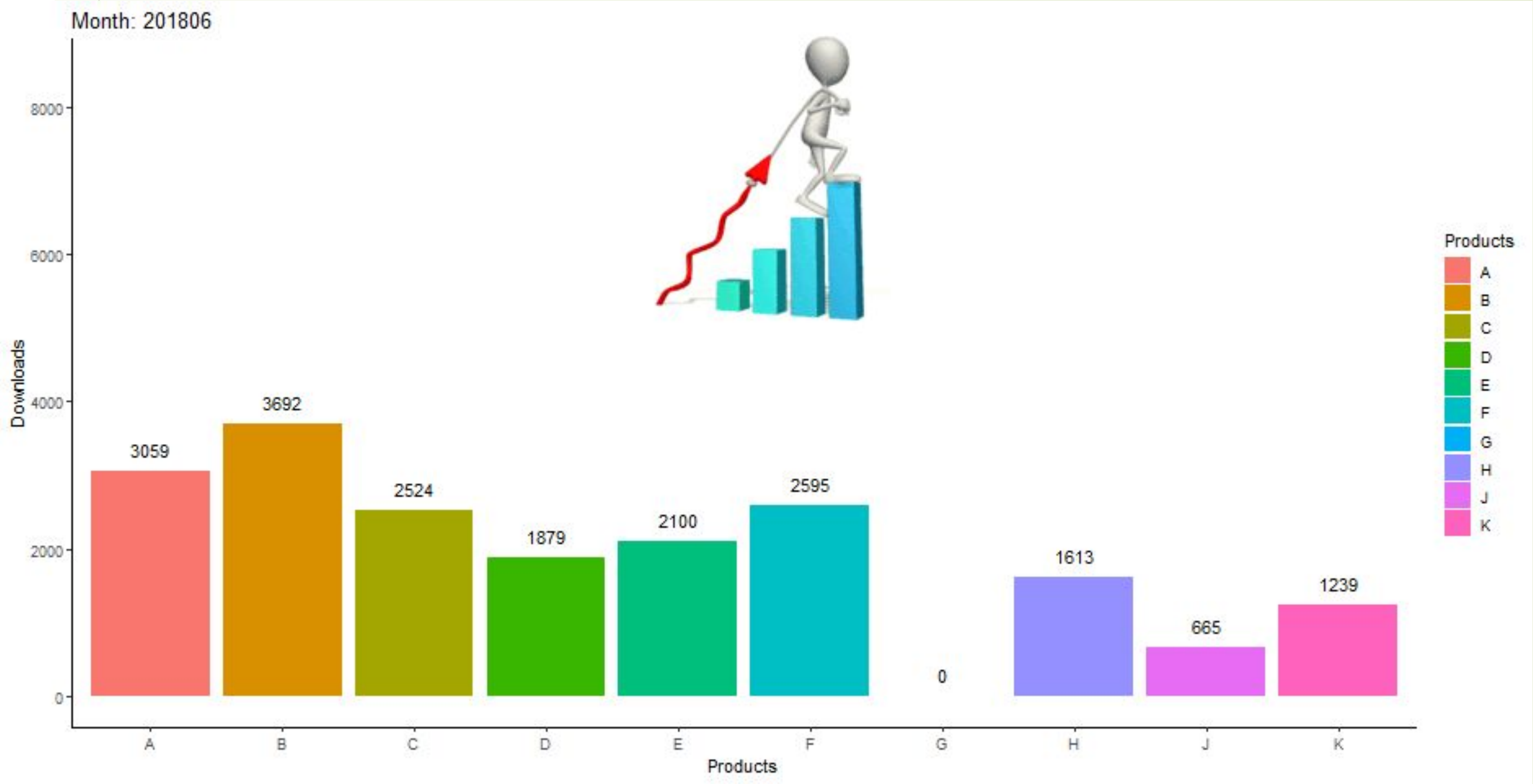

WELCOME



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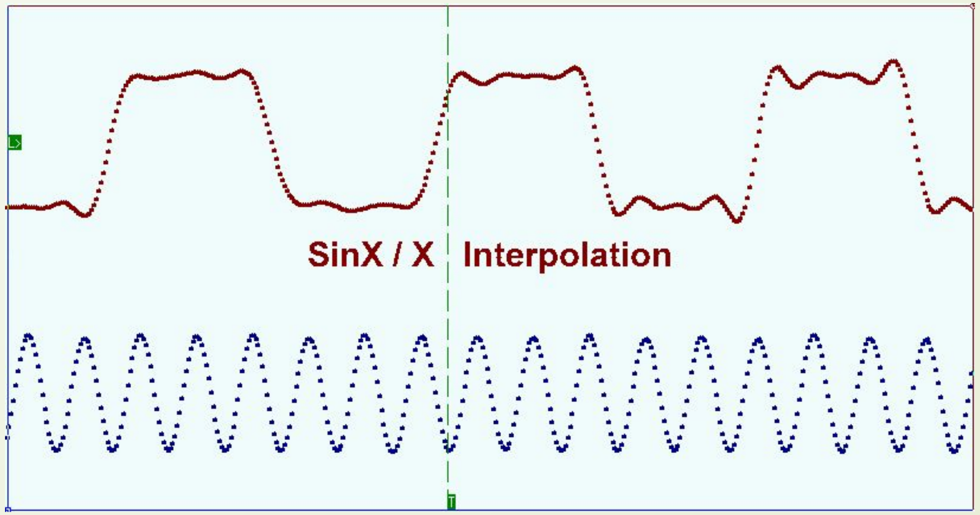
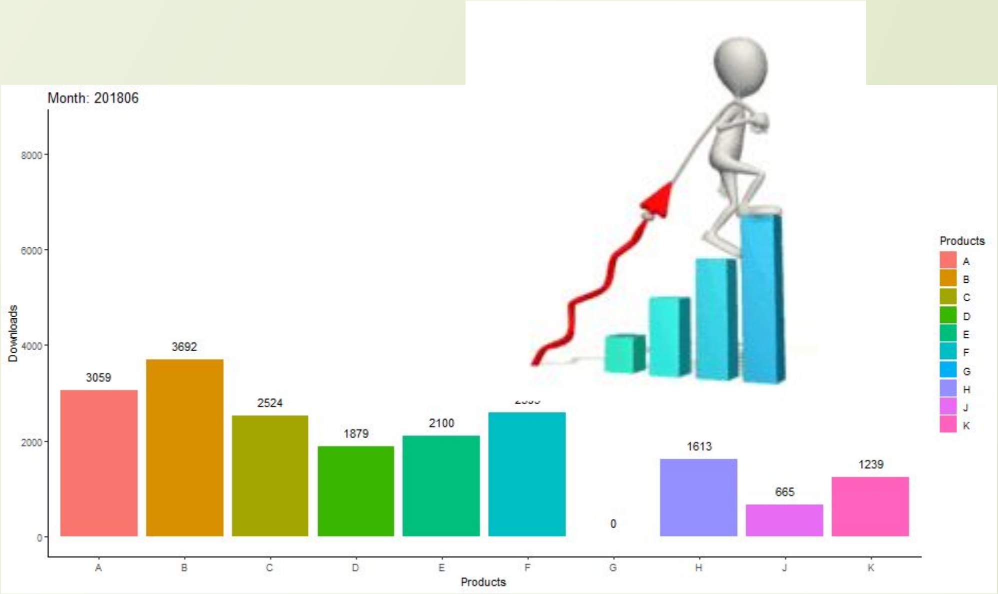
Interpolation





Interpolation ?

- In the mathematical field of numerical analysis, **interpolation** is a method of constructing new data points within the range of a discrete set of known data points.

- In our daily life we are sometimes confronted with the problem where we become interested in finding some unknown values with help of a given set of observations.

Example:

If we are find out the population of Bangladesh in 1978 when we know the population of Bangladesh in the year 1971, 1975, 1979, 1984, 1988, 1992 and so on.

That is :

The figure of population are available for **1971, 1975, 1979, 1984, 1988, 1992 etc.,** then the process of finding the population of 1978 is known as interpolation.

x	1971	1975	1979	1984	1988	1992
y	25	26	27	58	78	86

If X= 1978 , then what is the population ,y ?

Types of Interpolation

- Piecewise constant interpolation
- Linear interpolation
- **Polynomial interpolation ; Y= f (X)**
- Spline interpolation

Mathematical Definition of Interpolation

Mathematically, let a function $y = f(x)$ be

$f(a), f(a+h), f(a+2h) \ldots \ldots \ldots \ldots f(a+nh)$

for $x = a, a+h, a+2h, \ldots \ldots a+nh$ respectively.

Then the method of finding $f(x)$ for $x = \alpha$, where α lies in the range a and $a+nh$ is known as **interpolation**

and if the value of $x = \alpha$ lies outside this range it is called **extrapolation.**

For example, let as suppose we are given the following data.

x	0	3	6	9	12	15
f(x)	1	2	7	12	17	22

Then the method of finding f(10) or f(4) with help of the given data will be called **interpolation ,**

and that for f(27) or f(-2) will be known as **extrapolation**.

Forward Difference

If $y_0, y_1, y_2, \ldots., y_n$ denote a set of values of y,

then $y_1 - y_0, y_2 - y_1, \ldots, y_n - y_{n-1}$ are called the differences of y.

Denoting the differences by $\Delta y_0, \Delta y_1, \ldots, \Delta y_{n-1}$ respectively,

we have, $\Delta y_0 = y_1 - y_0,$

$\Delta y_1 = y_2 - y_1,$

....

....

... $\Delta y_{n-1} = y_n - y_{n-1}$

Where Δ is called the forward difference operator

and $\Delta y_0, \Delta y_1, \ldots, \Delta y_{n-1}$ are called first forward differences.

Forward Difference (Remain Part)

The differences of the first forward differences are called second forward differences

and

denoted by $\Delta^2 y_0, \Delta^2 y_1, \ldots, \Delta^2 y_{n-1}$

So, $\Delta^2 y_0 = \Delta y_1 - \Delta y_0,$

$\Delta^2 y_1 = \Delta y_2 - \Delta y_1,$

.................................,

....

$\Delta^2 y_{n-2} = \Delta y_{n-1} - \Delta y_{n-2}$

Similarly, third, fourth, fifth etc. forward differences can be obtained.

Prove That, $\Delta^2 y_2 = y_4 - 2y_3 + y_2$

$$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$$
$$= y_4 - y_3 - (y_3 - y_2)$$
$$= y_4 - y_3 - y_3 + y_2$$
$$= y_4 - 2y_3 - + y_2$$

$\Delta y_3 = y_4 - y_3$

$\Delta y_2 = y_3 - y_2$

Prove That, $\Delta^3 y_3 = y_6 - 3y_5 + 3y_4 - y_3$

Forward Difference Table

x	y						
	y_0						
		$\Delta y_0 = y_1 - y_0$					
	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$				
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$			
	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$		
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$		$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$	
	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$		$\Delta^6 y_0 = \Delta^5 y_1 - \Delta^5 y$
		$\Delta y_3 = y_4 - y_3$		$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$		$\Delta^5 y_1 = \Delta^4 y_2 - \Delta^4 y_1$	
	y_4		$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$		$\Delta^4 y_2 = \Delta^3 y_3 - \Delta^3 y_2$		
		$\Delta y_4 = y_5 - y_4$		$\Delta^3 y_3 = \Delta^2 y_4 - \Delta^2 y_3$			
	y_5		$\Delta^2 y_4 = \Delta y_5 - \Delta y_4$				
		$\Delta y_5 = y_6 - y_4$					
	y_6						

Example : Forward Difference Table

Example: Find the Forward Difference table by the following data is :

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

x	y			
		$\Delta y_0 = 0.0487$		
			$\Delta^2 y_0 = -0.0010$	
		0.0477		$\Delta^3 y_0 = -0.0003$
			-0.0013	
		0.0464		

$\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1$

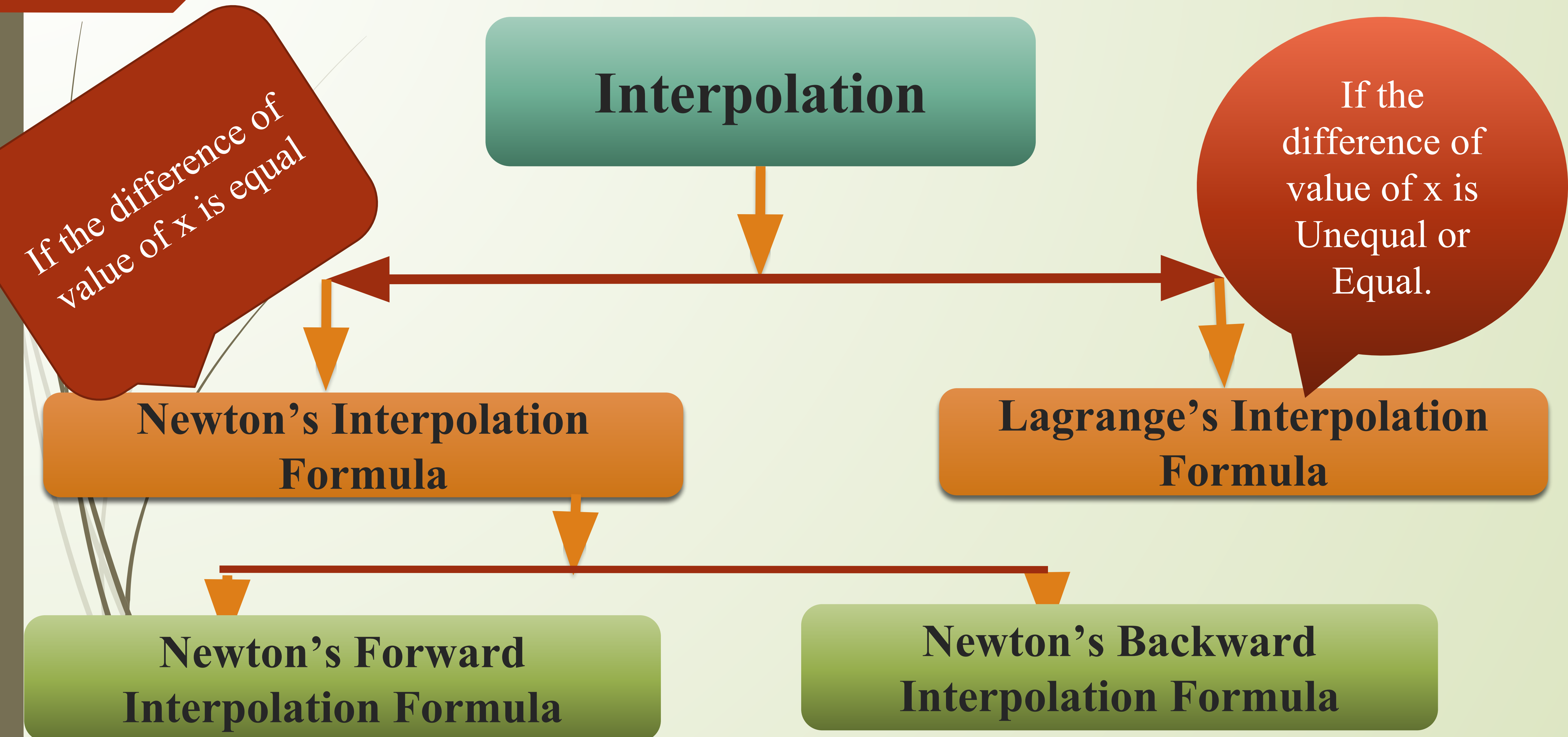
$\Delta y_2 = y_3 - y_2$

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$\Delta^2 y_1 = \Delta y_2 - \Delta 1$

$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

Types of Interpolation (According to Method)



Newton's Interpolation Formula

Newton's Forward Interpolation Formula

Find the value of y at x=21 from the following data:

x	20	23	26	29
y	0.342	0.3907	0.4384	0.4848

Newton's Backward Interpolation Formula

Find the value of y at x=28 from the following data:

x	20	23	26	29
y	0.342	0.3907	0.4384	0.4848

Newton's Forward Interpolation

- If the given data is,

				…	
				…	

- Then the **Newton's forward interpolation formula** will be,

- $$y(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots \ldots\ldots\ldots\ldots\ldots\ldots + \frac{u(u-1)(u-2)\ldots(u-n+1)}{n!}\Delta^n y_0$$

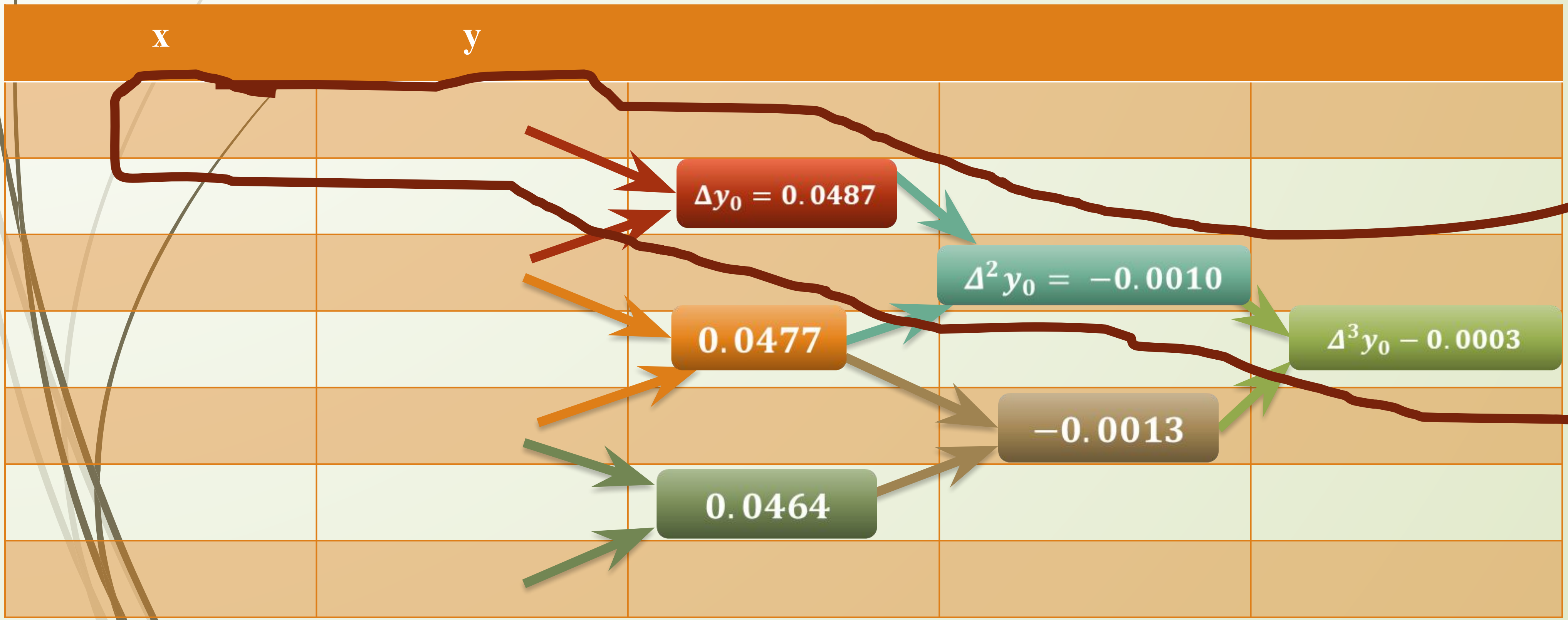
where, $u = \frac{x - x_0}{h}$; h is the difference of x values in the chart.

$\Delta^0 y_0$	$u\,\Delta^1 y_0$	$\frac{u(u-1)}{2!}\Delta^2 y_0$	And So on.

Problem 1. Find the value of y at x=21 from the following data:

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

Solution: We construct the difference table for the given data is as follows:



$\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1$

$\Delta y_2 = y_3 - y_2$

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$\Delta^2 y_1 = \Delta y_2 - \Delta 1$

$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

We have to find y(21).

Here $u = \frac{x - x_0}{h}$

$= \frac{21-20}{3} = 0.333$

Since x=21 , x_0 = 20 , h = 3

h is the difference of x values in the chart. Here 23-20 =3 ; 26-23 = 3 , 29-26 = 3

Problem 1. Find the value of y at x=21 from the following data:

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

Since x=21 is nearer to the beginning of the table, so we use Newton's forward formula.

$$y(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

- $y(21) = 0.342 + 0.333 \times 0.0487 + \frac{0.333\,(0.333-1)}{2!}(-0.0010) + \frac{0.333\,(0.333-1)(\,0.333-2)}{3!}(-0.0003)$
- $= 0.3583$

x	y			
		$\Delta y_0 = 0.0487$		
			$\Delta^2 y_0 = -0.0010$	
		0.0477		$\Delta^3 y_0 = -0.0003$
			-0.0013	
		0.0464		

Problem 2. A second degree polynomial passes through (0,1),(1,3),(2,7), and (3,13). Find the polynomial.

Solution: We construct the difference table for the

x	y			
0	1			
		$\Delta y_0 = 3 - 1 = 2$		
1	3		$\Delta^2 y_0 = 4 - 2 = 2$	
		$\Delta y_1 = 7 - 3 = 4$		$\Delta^3 y_0 = 2 - 2 = 0$
2	7		$\Delta^2 y_1 = 6 - 4 = 2$	
		$\Delta y_2 = 13 - 7 = 6$		
3	13			

- **By Newton's forward interpolation formula, we get**
- $f(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$

where , $u = \frac{x - x_0}{h}$

- In this problem, $u = \frac{x-0}{1} = x$
- Therefore, $f(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0$
- $\Rightarrow f(x) = 1 + x \times 2 + \frac{x(x-1)}{2} . 2$
- $\Rightarrow f(x) = x^2 + x + 1,$
- Which is the required polynomial

$x_0 = 0$

$h = 1$

$y_0 = 1$

$\Delta y_0 = 2$

$\Delta^2 y_0 = 2$

$\Delta^3 y_0 = 0$

Practice Work

1. Construct a difference table to find the polynomial of the data $(1,1),(2,8),(3,27),(4,64)$ $,(5,125),(6,216),(7,343),(8,512)$, considering appropriate method. Also find s, where $(10,s)$ is given.

2. The population of a village in the last six censuses was recorded as follows. Estimate the population for the year 1945.

Year(x)	1941	1951	1961	1971	1981	1991
Population(y)	2500	2800	3200	3700	4350	5225

3. The values of $\sin x$ are given below for different values of x, find the value of $\sin 18^0$.

x	15	20	25	30	35	40
$y = \sin x$	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

For Stay With