2.Newton's Backward Interpolation

Backward Difference

- If $y_0, y_1, y_2, \ldots., y_n$ denote a set of values of y,
- then $y_1 - y_0, y_2 - y_1, \ldots, y_n - y_{n-1}$ are called the differences of y.
- Denoting the differences by $\nabla y_1, \nabla y_2, \ldots, \nabla y_n$ respectively,

we have ,$\nabla y_1 = y_1 - y_0$,

$\nabla y_2 = y_2 - y_1$,

…………………..

…………………..,

. $\nabla y_n = y_n - y_{n-1}$

Where ∇ is called the backward difference operator
and $\nabla y_1, \nabla y_2, \ldots, \nabla y_n$ are called first backward differences.

Backward Difference

- The differences of the first backward differences are called **second backward differences**
- and **denoted by** $\nabla^2 y_1, \quad \nabla^2 y_2, \dots, \nabla^2 y_n$

We have , $\nabla^2 y_1 = \nabla y_1 - \nabla y_0$

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1,$$

- $$\nabla^2 y_3 = \nabla y_3 - \nabla y_2,$$
- ....,
- ....,
- $$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$$
- Similarly, third, fourth, fifth etc. backward differences can

Third, Fourth backward differences

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$$
$$= y_2 - y_1 - (y_1 - y_0)$$
$$= y_2 - y_1 - y_1 + y_0$$
$$= y_2 - 2y_1 + y_0$$

$$\nabla y_1 = y_1 - y_0$$

$$\nabla y_2 = y_2 - y_1$$

$$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2 = y_3 - 3y_2 + 3y_1 - y_0,$$

Third, Fourth backward differences

$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$

$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$

$\nabla y_4 = y_4 - y_3$

$\nabla y_3 = y_3 - y_2$

$\nabla y_2 = y_2 - y_1$

$$\begin{aligned}
\nabla^3 y_4 &= \nabla^2 y_4 - \nabla^2 y_3 \\
&= \nabla y_4 - \nabla y_3 - (\nabla y_3 - \nabla y_2) \\
&= \nabla y_4 - \nabla y_3 - \nabla y_3 + \nabla y_2 \\
&= \nabla y_4 - 2\nabla y_3 + \nabla y_2 \\
&= y_4 - y_3 - 2(y_3 - y_2) + y_2 - y_1 \\
&= y_4 - y_3 - 2y_3 + 2y_2 + y_2 - y_1 \\
&= y_4 - 3y_3 + 3y_2 - y_1
\end{aligned}$$

Prove That , $\nabla^3 y_4 = y_4 - 3y_3 + 3y_2 - y_1$

Backward Difference Table

x	y						
	y_0						
	y_1	$\nabla y_1 = y_1 - y_0$					
	y_2	$\nabla y_2 = y_2 - y_1$	$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$				
	y_3	$\nabla y_3 = y_3 - y_2$	$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$	$\nabla^3 y_3 = \nabla^2 y_3 \quad \nabla^2 y_2$			
	y_4	$\nabla y_4 = y_4 - y_3$	$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$	$\nabla^3 y_4 = \nabla^2 y_4 \quad \nabla^2 y_3$	$\nabla^4 y_4 = \nabla^2 y_4 - \nabla^2 y_3$		
	y_5	$\nabla y_5 = y_5 - y_4$	$\nabla^2 y_5 = \nabla y_5 - \nabla y_4$	$\nabla^3 y_5 = \nabla^2 y_5 \quad \nabla^2 y_4$	$\nabla^4 y_5 = \nabla^2 y_5 - \nabla^2 y_4$	$\nabla^5 y_5 = \nabla^2 y_5 - \nabla^2 y_4$	
	y_6	$\nabla y_6 = y_6 - y_5$	$\nabla^2 y_6 = \nabla y_6 - \nabla y_5$	$\nabla^3 y_6 = \nabla^2 y_6 \quad \nabla^2 y_5$	$\nabla^4 y_6 = \nabla^2 y_6 - \nabla^2 y_5$	$\nabla^5 y_6 = \nabla^4 y_6 - \nabla^4 y_5$	$\nabla^6 y_6 = \nabla^5 y_6 - \nabla^5 y_5$

Or Backward Difference Table

x	y						
	y_0						
		$\nabla y_1 = y_1 - y_0$					
	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$				
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$			
	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$		
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$		$\nabla^5 y_5 = \nabla^2 y_5 - \nabla^2 y_4$	
	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$		$\nabla^4 y_5 = \nabla^3 y_5 - \nabla^3 y_4$		$\nabla^6 y_6 = \nabla^5 y_6 - \nabla^5 y_5$
		$\nabla y_4 = y_4 - y_3$		$\nabla^3 y_5 = \nabla^2 y_5 - \nabla^2 y_4$		$\nabla^5 y_6 = \nabla^4 y_6 - \nabla^4 y_5$	
	y_4		$\nabla^2 y_5 = \nabla y_5 - \nabla y_4$		$\nabla^4 y_6 = \nabla^2 y_6 - \nabla^2 y_5$		
		$\nabla y_5 = y_5 - y_4$		$\nabla^3 y_6 = \nabla^2 y_6 - \nabla^2 y_5$			
	y_5		$\nabla^2 y_6 = \nabla y_6 - \nabla y_5$				
		$\nabla y_6 = y_6 - y_5$					
	y_6						

Example: Find the Backward Difference table by the following data is :

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

$\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

$\nabla y_3 = y_3 - y_2$

$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$

$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$

$\nabla^3 y_3 = \nabla^2 y_3 \quad \nabla^2 y_2$

x	y			
	$y_0 = 0.342$			
	$y_1 = 0.3907$	$\nabla y_1 = 0.0487$		
	$y_2 = 0.4384$	$\nabla y_2 = 0.0477$	$\Delta^2 y_2 = -0.0010$	
	$y_3 = 0.4848$	$\nabla y_3 = 0.0464$	$\Delta^2 y_3 - 0.0013$	$\Delta^3 y_3 = -0.0003$

Example: Find the Backward Difference table by the following data is :

x :	**20**	**23**	**26**	**29**
y :	**0.342**	**0.3907**	**0.4384**	**0.4848**

x	y			
	$y_0 = 0.342$			
		$\nabla y_1 = 0.0487$		
	$y_1 = 0.3907$		$\Delta^2 y_2 = -0.0010$	
		$\nabla y_2 = 0.0477$		$\Delta^3 y_3 = -0.0003$
	$y_2 = 0.4384$		$\Delta^2 y_3 - 0.0013$	
		$\nabla y_3 = 0.0464$		
	$y_3 = 0.4848$			

Newton's backward Interpolation

- If the given data is,

				…	
				…	

- Then the **Newton's Backward interpolation formula** will be,

- $$y(x) = y_n + u\,\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots + \frac{u(u+1)(u+2)\ldots(u+n-1)}{n!}\nabla^{\mathrm{n}} y_n$$

where, $u = \frac{x - x_n}{h}$; h is the difference of x values in the chart.

Problem 1. Find the value of y at x=28 from the following data:

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

$\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

$\nabla y_3 = y_3 - y_2$

$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$

$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$

$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

Solution: we construct the difference table for the given data is as follows:

x	y			
		$\nabla y_1 = 0.0487$		
		0.0477	$\nabla^2 y_2 = -0.0010$	
		0.0464	-0.0013	$\nabla^3 y_3 = -0.0003$

- We have to find y(28). Here ,X = 28 , x_n = 29 , h = 3

From the Given Table,

23-20 = 3

26-23 = 3 h = 3

29-26 = 3

Here $u = \frac{x - x_n}{h}$

$$= \frac{28 - 29}{3}$$

= - 0.3333

Problem 1. Find the value of y at x=28 from the following data:

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

Since x=28 is the last 2nd of the table, so we use Newton's Backward formula.

$y_n = 0.4848$

$u = -0.3333$

$\nabla y_n = 0.0464$

$\nabla^2 y_n = -0.0013$

$\nabla^3 y_n = -0.0003$

Now, by Newton's Backward interpolation formula, we get

$$y(x) = y_n + u\,\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$y(28) = 0.4848 + (-0.3333) \times 0.0464$$

$$+ \frac{(-0.33333)(-0.3333+1)}{2!}(-0.0013)$$

$$+ \frac{(-0.3333\,)(-0.3333+1)\,(-0.3333+2)}{3!}(-0.0003)$$

$= 0.46946$

Problem 1. Find the value of y at x=28 from the following data:

x :	20	23	26	29
y :	0.342	0.3907	0.4384	0.4848

Problem 2. A second degree polynomial passes through (0,1),(1,3),(2,7), and (3,13). Find the polynomial.

Solution: We construct the difference table for the given data is as follows:

x	y			
0	1			
		$\nabla y_1 = 3 - 1 = 2$		
1	3		$\nabla^2 y_2 = 4 - 2 = 2$	
		$\nabla y_2 = 7 - 3 = 4$		$\nabla^3 y_3 = 2 - 2 = 0$
2	7		$\nabla^2 y_3 = 6 - 4 = 2$	
		$\nabla y_2 = 13 - 7 = 6$		
3	13			

By Newton's Backward interpolation formula, we get

$$y(x) = y_n + u\,\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

Where $u = \frac{x - x_n}{h}$

x_n = 3 ; h = 1

1-0=1

2-1=1

3-2=1

$u = \frac{x-3}{1} = x - 3$

Problem 2. A second degree polynomial passes through (0,1),(1,3),(2,7), and (3,13). Find the polynomial.

Here X is not given
So you can Choose Any methods from Newtons Interpolation Formula , Choose either Forward or Backward Interpolation Formula.

$$y(x) = y_n + u\,\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$= 13 + (x-3) \times 6 + \frac{(x-3)\times(x-3+1)}{2} \times 2 + \frac{(x-3)\times(x-3+1)\times(x-3+2)}{6} \times 0$$

$\therefore y(x) = 13 + 6x - 18 + 6\,(x-3) \times (x-2)$

$= -5 + 6x + 6\,(x^2 - 5x + 6)$

$= 6x^2 - 30x + 36 + 6x - 5$

$= 6x^2 - 24x + 31$

Which is the required polynomial.

$y_n = 13$

$\nabla y_n = 6$

$\nabla^2 y_n = 2$

$\nabla^3 y_n = 0$

Practice Work

3. The values of $\sin x$ are given below for different values of x, find the value of $\sin 38^0$.

x	15	20	25	30	35	40
$y = \sin x$	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

4. The population of a town in the last six censuses was as given below. Estimate the population for the year 1946.

Year(x)	1911	1921	1931	1941	1951	1961
Population in thousands(y)	12	15	20	27	39	52

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