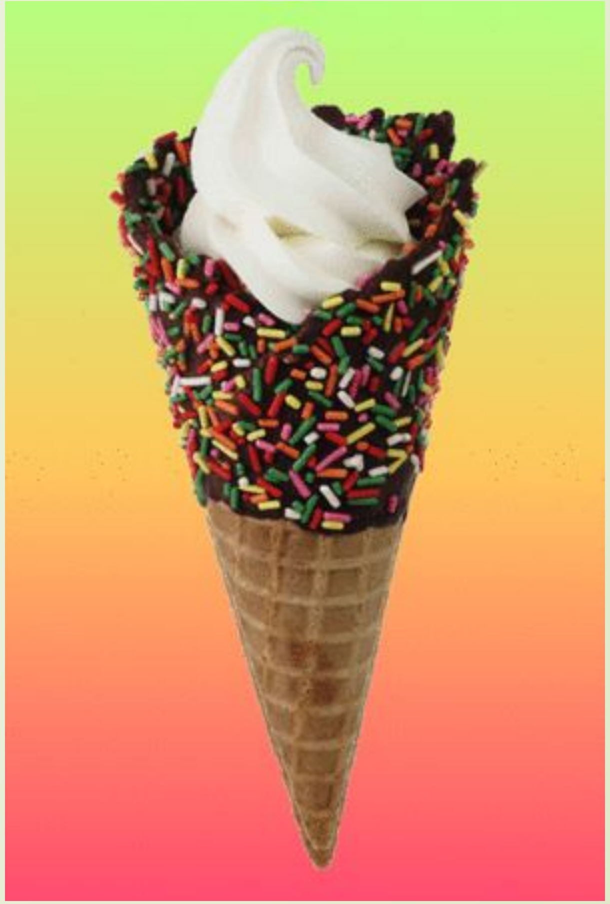
Interpolation

Divided Difference Formula



Newton's Divided Difference Table
(for unequal Intervals)

i						
0			$\frac{y_1 - y_0}{x_1 - x_0} = \Delta y_0$	$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} = \Delta^2 y_0$	$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} = \Delta^3 y_0$	$\frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0} = \Delta^4 y$
1			$\frac{y_2 - y_1}{x_2 - x_1} = \Delta y_1$	$\frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} = \Delta^2 y_1$	$\frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1} = \Delta^3 y_1$	
2			$\frac{y_3 - y_2}{x_3 - x_2} = \Delta y_2$	$\frac{\Delta y_3 - \Delta y_2}{x_4 - x_2} = \Delta^2 y_2$		
3			$\frac{y_4 - y_3}{x_4 - x_3} = \Delta y_3$			
4						

i						
0			$\frac{y_1 - y_0}{x_1 - x_0} = \Delta y_0$			
1			$\frac{y_2 - y_1}{x_2 - x_1} = \Delta y_1$	$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} = \Delta^2 y_0$	$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} = \Delta^3 y_0$	
2			$\frac{y_3 - y_2}{x_3 - x_2} = \Delta y_2$	$\frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} = \Delta^2 y_1$	$\frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1} = \Delta^3 y_1$	$\frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0} = \Delta^4 y_0$
3			$\frac{y_4 - y_3}{x_4 - x_3} = \Delta y_3$	$\frac{\Delta y_3 - \Delta y_2}{x_4 - x_2} = \Delta^2 y_2$		
4						

Example:

i						
0			$\frac{-6-3}{0+1}=-9$	$\frac{15+9}{3+0}=6$	$\frac{41-6}{6+1}=5$	$\frac{13-5}{7+1}=1$
1			$\frac{39+6}{3-0}=15$	$\frac{261-15}{6-0}=41$	$\frac{132-41}{7-0}=13$	
2			$\frac{822-39}{6-3}=261$	$\frac{789-261}{7-3}=132$		
3			$\frac{1611-822}{7-6}=789$			
4						

Newton's Divided Difference Interpolation Formula

$$
\begin{aligned}
y(x) &= y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 \\
&\quad +(x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0 \\
&\quad +(x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0 \\
&\quad + \ldots\ldots\ldots\ldots\ldots\ldots (1)
\end{aligned}
$$

Problem

Problem 01. The values of x and f(x) are given below:

x :	4	5	7	10	11
f(x):	48	100	294	900	1210

with the help of given data construct a divided difference table and

hence find the values of f(8) and f(15).

Solution:

i						
0			$\frac{100-48}{5-4}=52$	$\frac{97-52}{7-4}=15$	$\frac{21-15}{10-4}=1$	$\frac{1-1}{11-4}=0$
1			$\frac{294-100}{7-5}=97$	$\frac{202-97}{10-5}=21$	$\frac{27-21}{11-5}=1$	
2			$\frac{900-294}{10-7}=202$	$\frac{310-202}{11-7}=27$		
3			$\frac{1210-900}{11-10}=310$			
4						

Solution

We have to find the values of f(8) and f(15).

From the table we get,

$x = 8$

$x_0 = 4$

$x_1 = 5$

$x_2 = 7$

$x_3 = 10$

$x_4 = 11$

$y_0 = 48$

$\Delta y_0 = 52$

$\Delta^2 y_0 = 15$

$\Delta^3 y_0 = 1$

$\Delta^4 y_0 = 0$

From Newton's Divided Difference Interpolation Formula

$$y(x) = y_0 + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0 + (x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0$$

..............................(1)

Solution

We have to find the values of f(8) and f(15).

Therefore,

$$y(8) = 48 + (8-4)\times 52 + (8-4)(8-5)\times 15 + (8-4)(8-5)(8-7)\times 1 = 448$$

& $$y(15) = 48 + (15-4)\times 52 + (15-4)(15-5)\times 15 + (15-4)(15-5)(15-7)\times 1 = 3150$$

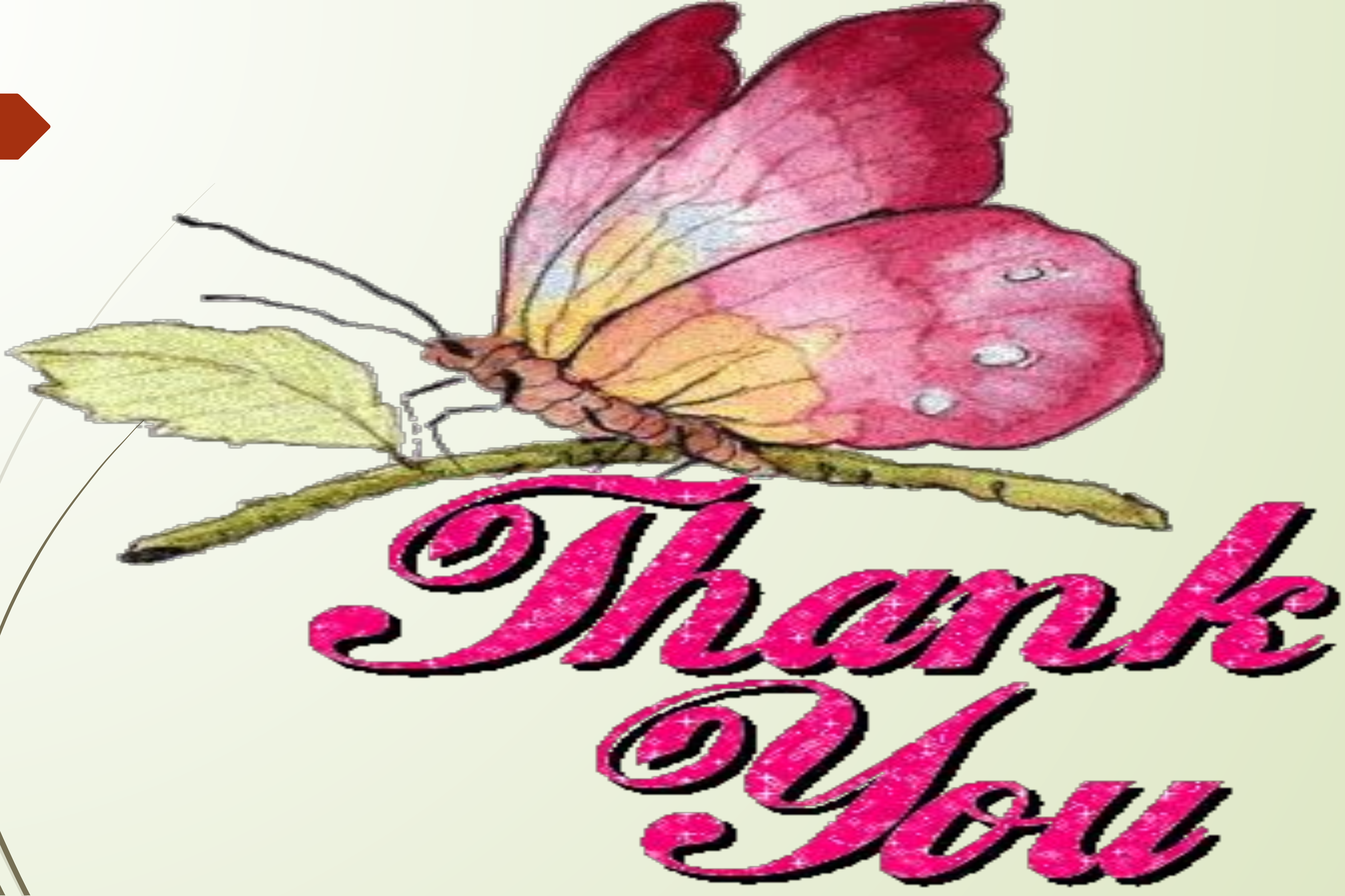
Problem 01. Find the cubic polynomial which takes the following values f(0) =1, f(1) =3, f(3) = 31, and f(10) =1011. Hence or otherwise obtain f(2.5).

Problem 02. Using Newton's Divided difference formula estimate f(6) from the following table

x	5	7	11	13	21
F(x)	**150**	**392**	**1452**	**2366**	**9702**

Problem 3: Apply Lagrange's formula to estimate sin 39^{o} using the following values:

x	0	10	20	30	40
f(x)= sin(x)	0	1.1736	0.3420	0.5	0.6428


Thank You