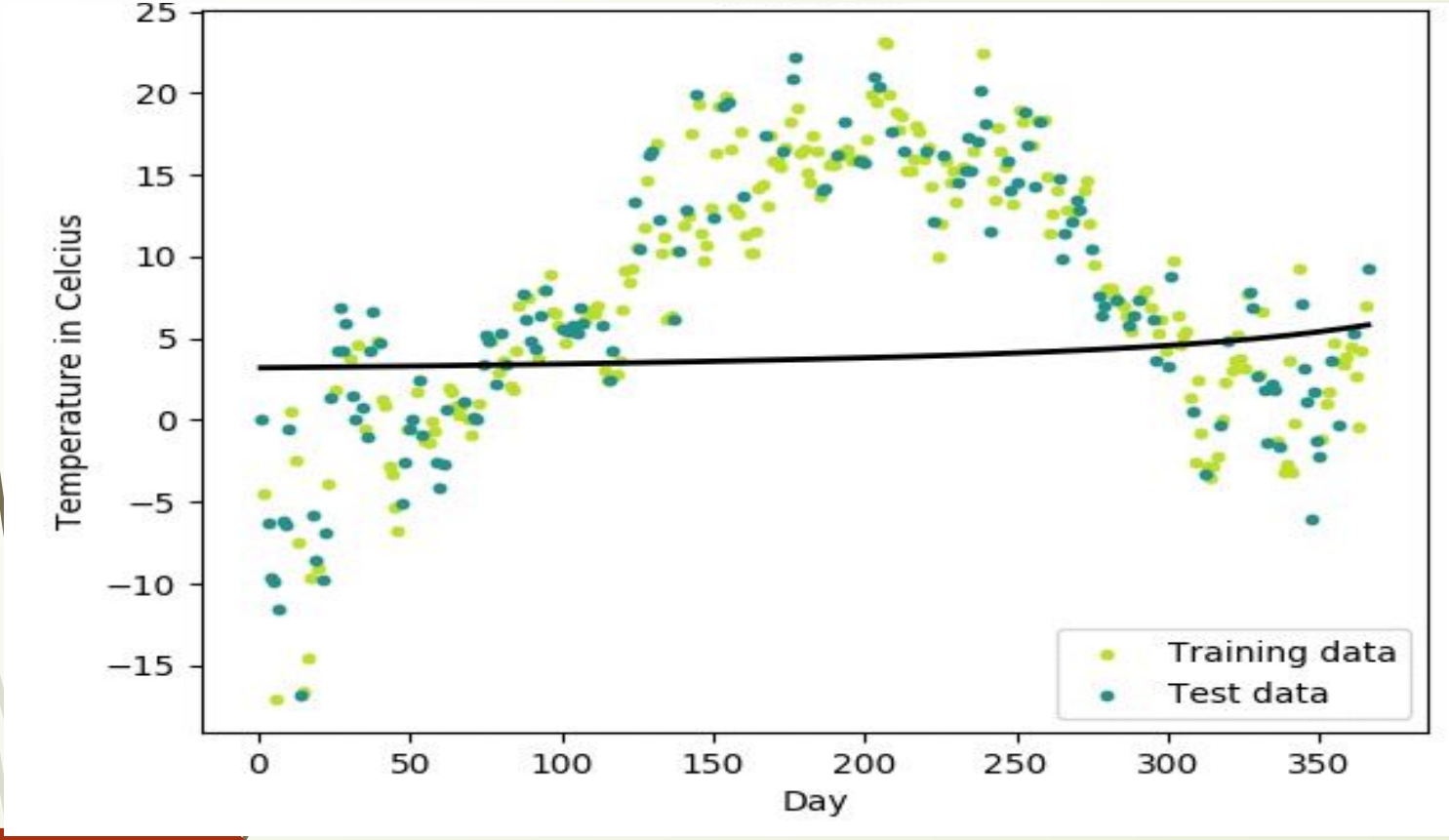
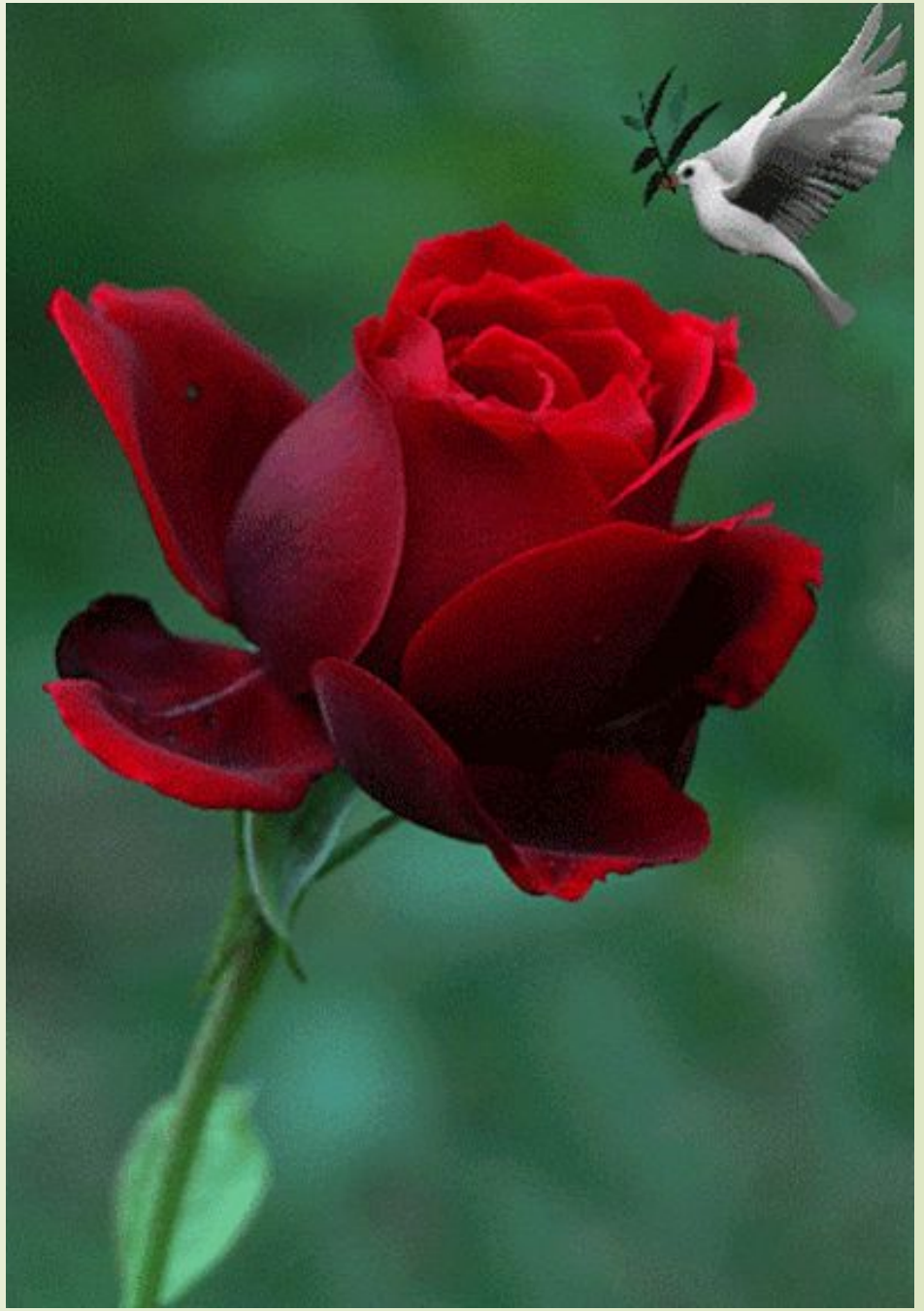
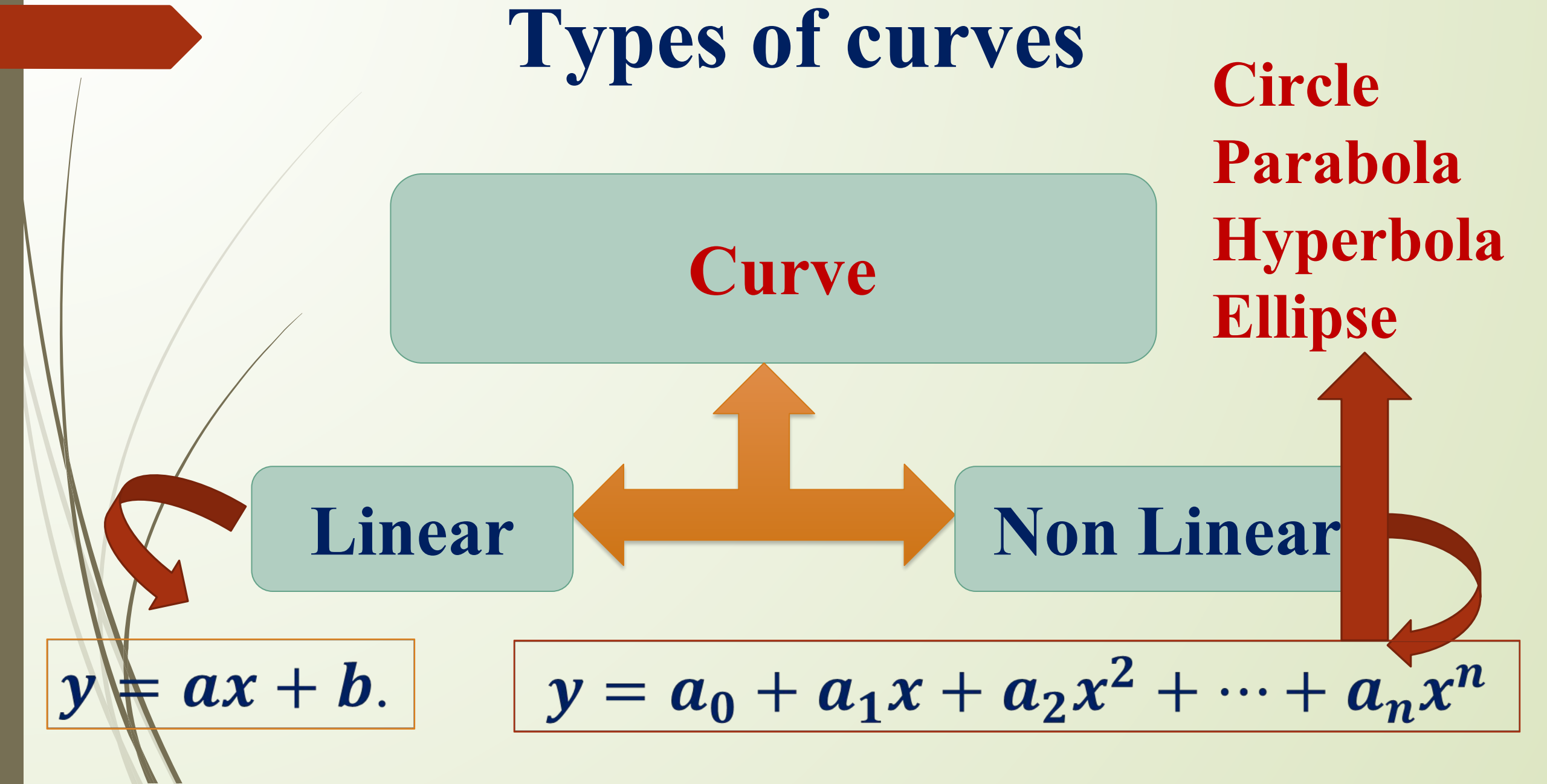


Curve Fitting

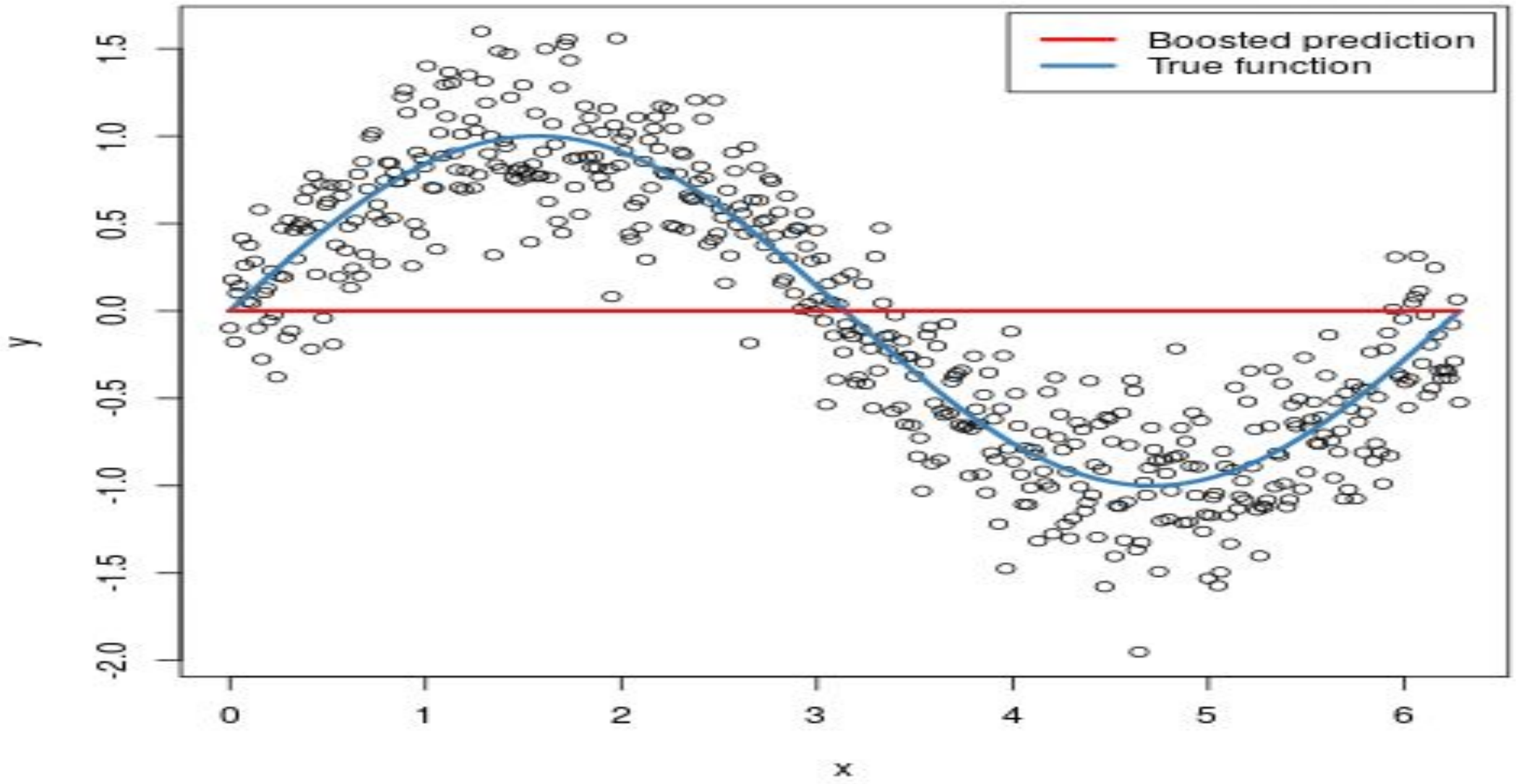
Chapter 4 : Non Linear



Types of curves



Non Linear Curve



Non Linear Curve Fitting

There are two useful methods for finding a straight line.

- The Least square method
- The graphical method

Non Linear Curve Fitting

The Least Square method for finding any non Linear equations.

The Least square method

Non Linear Curve Fitting

- **Least Square Formula for fitting the Non linear Curve :**

Polynomial of nth degree,

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

to be fitted to the data points (x_i, y_i), $i = 1, \ldots, m$

For Example :

- $y = ax^2 + bx + c$

- **Least Square Formula for fitting the Non linear Curve :**

$$y = ax^2 + bx + c \ldots\ldots(1)$$

The Normal equations:

$$\sum y = a\sum x^2 + b\sum x + cn \ldots\ldots(2)$$

$$\sum xy = a\sum x^3 + b\sum x^2 + c\sum x \ldots\ldots(3)$$

$$\sum x^2y = a\sum x^4 + b\sum x^3 + c\sum x^2 \ldots\ldots(4)$$

Problem

Problem 01: Use the method of least squares to fit a polynomial of a second degree to the following data:

x	0	1	2
y	1	6	17

Estimate the value of y when x = 25.

Solution:

Let, **a polynomial of a second degree**

$$y = ax^2 + bx + c \ldots\ldots(1)$$

The Normal equations:

$$\sum y = a\sum x^2 + b\sum x + cn \ldots\ldots(2)$$

$$\sum xy = a\sum x^3 + b\sum x^2 + c\sum x \ldots\ldots(3)$$

$$\sum x^2y = a\sum x^4 + b\sum x^3 + c\sum x^2 \ldots\ldots(4)$$

Now we construct a table for finding the values of

$$\sum x, \sum y, \sum xy, \sum x^2,$$

$$\sum x^3, \sum x^4, \sum x^2 y,$$

x	y				xy	
0	1	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68
$\sum x=3$	$\sum y=24$	$\sum x^2=5$	$\sum x^3=9$	$\sum x^4=17$	$\sum xy=40$	$\sum x^2y=74$

Now putting these values in the above equations (2) and (3) and (4) we get

$$5a + 3b + 3c = 24 \ldots\ldots (5)$$

$$9a + 5b + 3c = 40 \ \ldots\ldots (6)$$

$$17a + 9b + 5c = 74 \ \ldots\ldots (7)$$

From (6)- (5)

$$9a + 5b + 3c - 5a - 3b - 3c = 40 - 24$$

$$\therefore 4a + 2b = 16$$

$$\therefore 2a + b = 8 \ldots\ldots(8)$$

From (7× 3)- (6 ×5)

$$51a + 27b + 15c - 45a - 25b - 15c = 222 - 200$$

$$\therefore 6a + 2b = 22$$

$$\therefore 3a + b = 11 \ldots\ldots(9)$$

$\therefore 2a + b = 8 \ldots\ldots(8)$

$\therefore 3a + b = 11 \ldots\ldots(9)$

From (9)- (8)

$$\therefore a = 3$$

From (8)

$$\therefore b = 2$$

putting these values in the above equations (5)

$$5 \times 3 + 3 \times 2 + 3c = 24$$

$$15 + 6 + 3c = 24$$

$$21 + 3c = 24$$

$$3c = 24 - 21$$

$$3c = 3$$

$$\therefore c = 1$$

Putting the values of a , b , c in the above equations (1)

$$y = 3x^2 + 2x + 1$$

When , x = 25

$$y = 3(25)^2 + 2(25) + 1$$

$$\therefore y = 3(625) + 51 = 1926$$

3. Fit a second-degree parabola to the following data:

x	0	1	2	3	4
y	1	5	10	22	38

3. Fit a curve of the form $y = ax^2 + bx + c$ to the data:

x	87.5	84	77.8	63.7	46.7
y	292	283	270	235	197