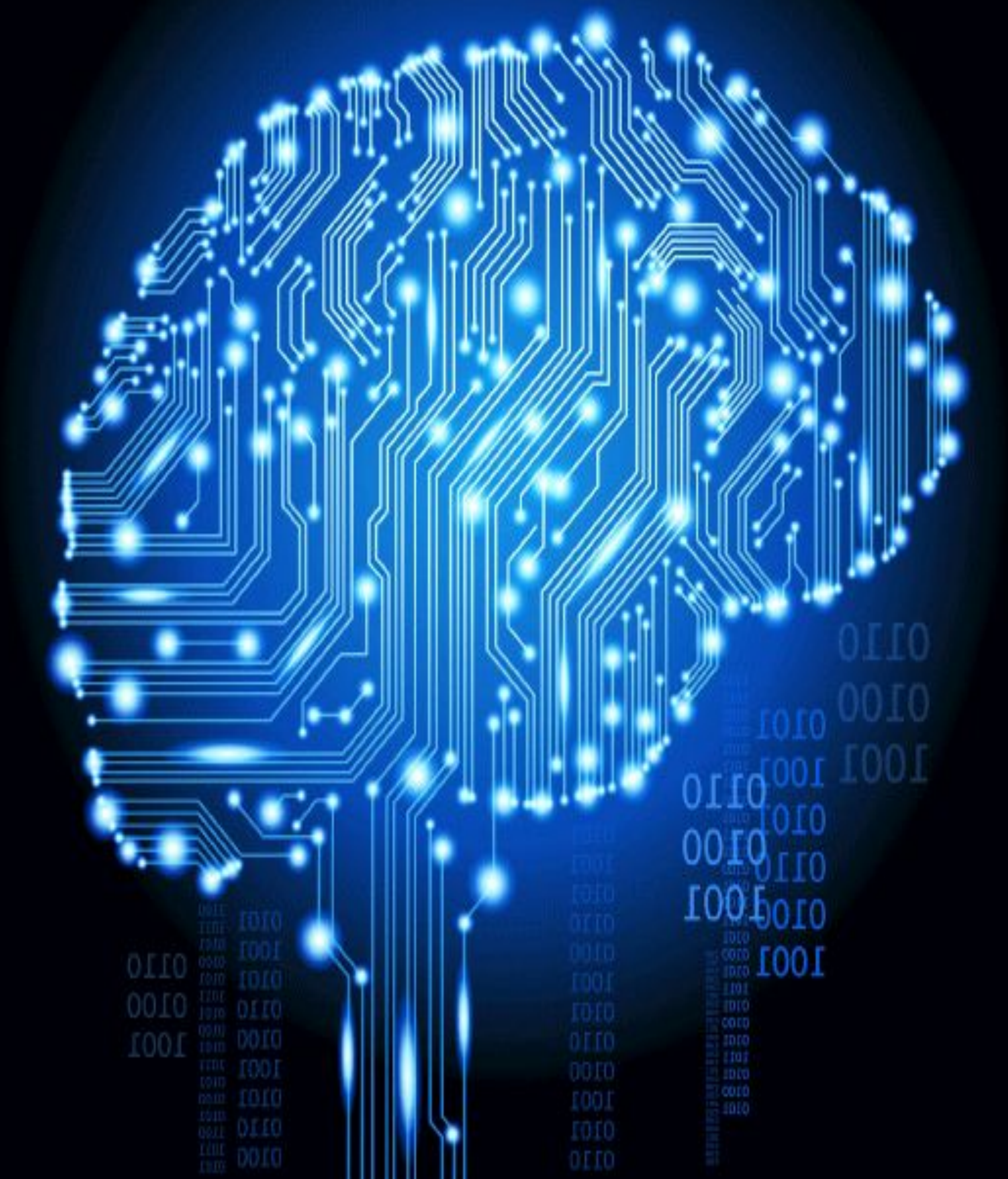

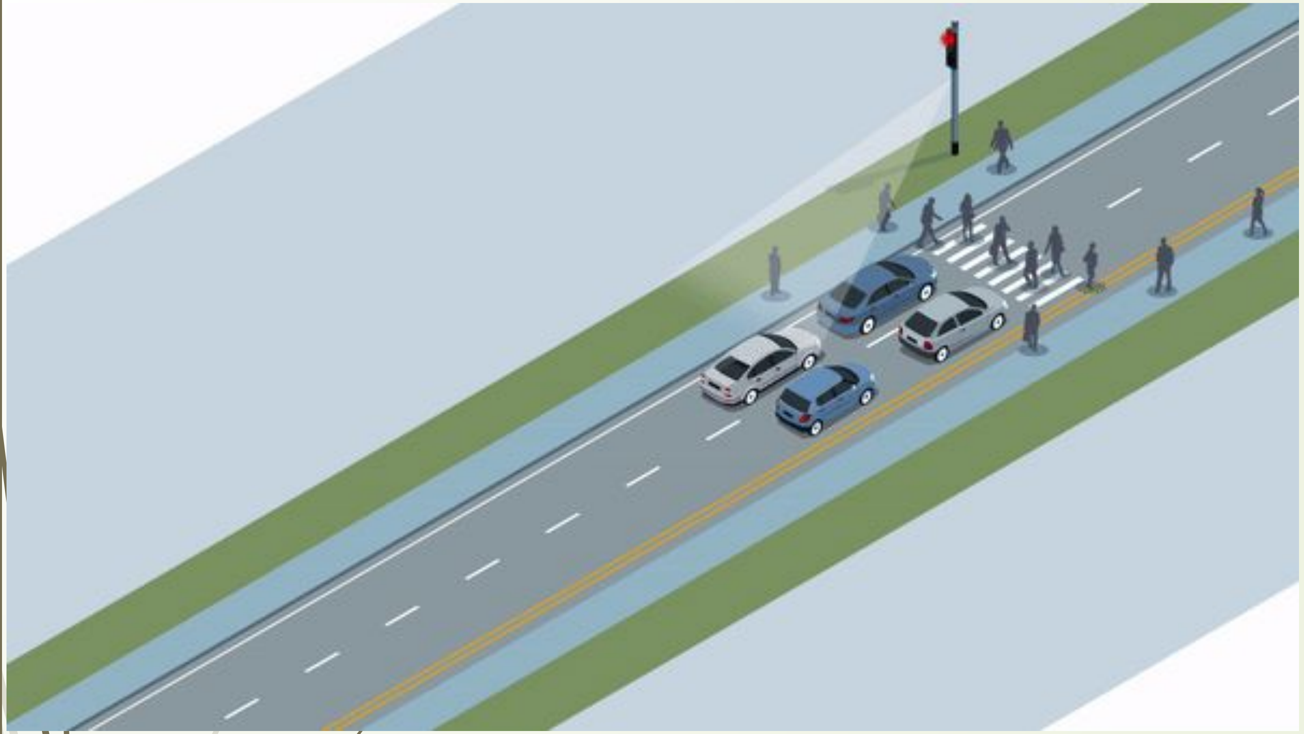

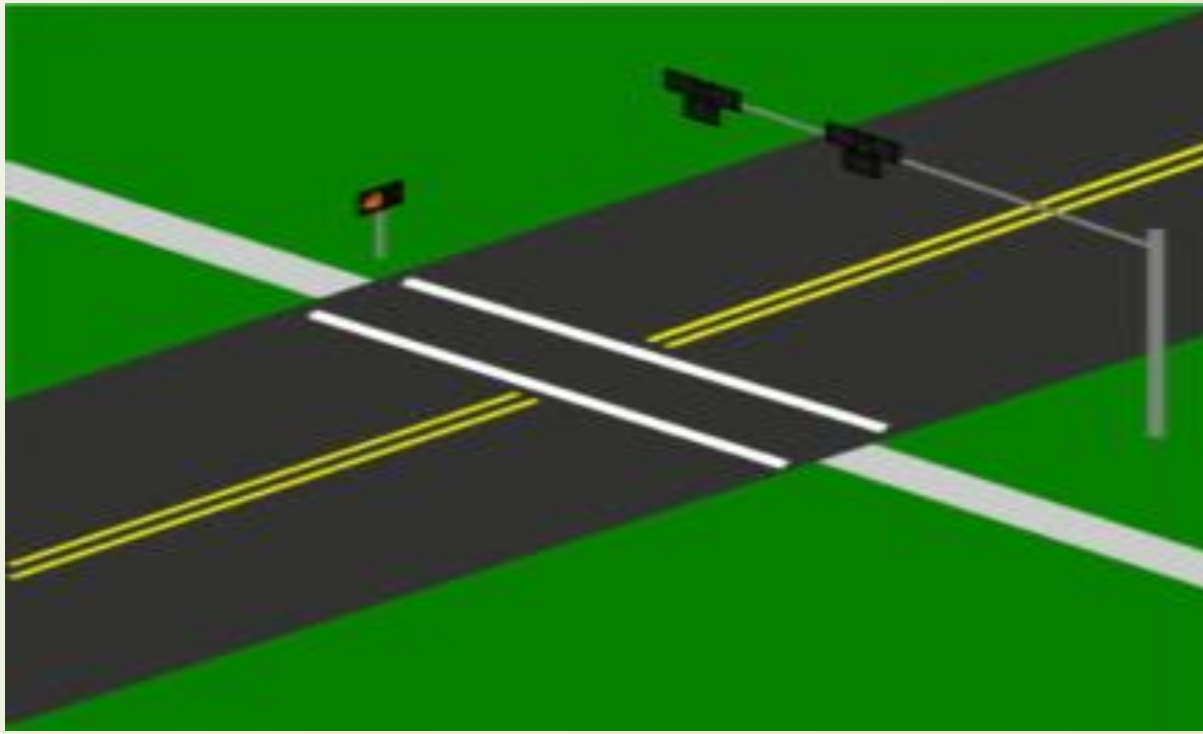

# The solution of system of Linear Equation

## Chapter 05

# System of Linear Equations

A system of linear equations is a collection of equations involving the same set of variables. That is the equations of the form

$$a_{11}x_1 + a_{12}x_2 + \cdots\cdots\cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots\cdots\cdots + a_{2n}x_n = b_2$$

$$a_{31}x_1 + a_{32}x_2 + \cdots\cdots\cdots + a_{3n}x_n = b_3$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots\cdots + a_{mn}x_n = b_m$$

Linear Equation:
$ax + by + c = 0$

# Stricly Diagonally Dominent

**Definition of strictly diagonally dominant matrix:**

A matrix $A = (a_{ij})_{n\times n}$ is called strictly diagonally dominant if $\sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| < |a_{ii}| \quad ,i = 1\,,2,\ldots\ldots,n.$

# Example:

**Problem:**

Check the system whether it is strictly diagonally dominant.

$$20x_1 + 2x_2 + 6x_3 = 28$$

$$x_1 + 20x_2 + 9x_3 = -23$$

$$2x_1 - 7x_2 - 20x_3 = -57$$

**Solution:**

Here $|a_{11}| = 20$ , $|a_{22}| = 20$ , $|a_{33}| = 20.$

We have

$$|a_{12}| + |a_{13}| = 2 + 6 = 8 < 20 = |a_{11}|$$

$$|a_{21}| + |a_{23}| = 1 + 9 = 10 < 20 = |a_{22}|$$

$$|a_{31}| + |a_{32}| = 2 + 7 = 9 < 20 = |a_{33}|$$

So $$\sum_{\substack{j \neq i \\ j=1}}^{3} |a_{ij}| < |a_{ii}| \quad , i = 1, 2, 3.$$

Hence the system is strictly diagonally dominant.

# Categories of S.L.E

**The solution of linear system of equations can be accomplished by a numerical method which falls in one of two categories:**

# S.L.E

Iteration Method

Direct Method

Gauss Seidal

Gauss Jacobi

Gauss Jardan

# Gauss Seidal Method

# Procedure of Gauss –Seidal Iteration Method

## Gauss-Seidel iteration method:

Let

We consider the system Ax=b where A= $(a_{ij})$ is non-singular and x= $(x_i)_{n\times 1}; b=(b_i)_{n\times 1}$

Now we shall write the system in detail:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots\dots\dots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots\dots\dots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots\dots\dots + a_{3n}x_n &= b_3 \qquad \dots\dots\dots\dots\dots\dots\dots\dots (1) \\
\dots \quad \dots \quad \dots \quad \dots \quad \dots & \quad \dots \\
a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots\dots\dots + a_{nn}x_n &= b_n
\end{aligned}
$$

# Example of S.L.E

$$20x_1 + 2x_2 + 6x_3 = 28$$
$$x_1 + 20x_2 + 9x_3 = -23$$
$$2x_1 - 7x_2 - 20x_3 = -57$$

## Step - 1

# Check :is it strictly diagonally dominent

If necessary, we re-arrange the given system (1) making strictly diagonally dominant, such that

$$|a_{ii}| > \sum_{\substack{j=1 \\ j=i}}^{n} |a_{ij}|, (i = 1,2,3,..........., n).$$

**Problem:**

Check the system whether it is strictly diagonally dominant.

$$20x_1 + 2x_2 + 6x_3 = 28$$
$$x_1 + 20x_2 + 9x_3 = -23$$
$$2x_1 - 7x_2 - 20x_3 = -57$$

**Solution:**

Here $|a_{11}| = 20$, $|a_{22}| = 20$, $|a_{33}| = 20$.

We have

$$|a_{12}| + |a_{13}| = 2 + 6 = 8 < 20 = |a_{11}|$$
$$|a_{21}| + |a_{23}| = 1 + 9 = 10 < 20 = |a_{22}|$$
$$|a_{31}| + |a_{32}| = 2 + 7 = 9 < 20 = |a_{33}|$$

So $$\sum_{\substack{j=1 \\ j \neq i}}^{3} |a_{ij}| < |a_{ii}| \quad, i = 1, 2, 3.$$

Hence the system is strictly diagonally dominant.

Step - 2

# To make the above those equations (1)by the following equations

Suppose that the system (1) is (strictly) diagonally dominant. Now we re-write the system as follows:

$$x_1 = \frac{1}{a_{11}}\left[b_1 - \left(a_{12}x_2 + a_{13}x_3 + \text{.........} + a_{1n}x_n\right)\right]$$

$$x_2 = \frac{1}{a_{22}}\left[b_2 - \left(a_{21}x_1 + a_{23}x_3 + \text{.........} + a_{2n}x_n\right)\right]$$

$$x_3 = \frac{1}{a_{33}}\left[b_3 - \left(a_{31}x_1 + a_{32}x_2 + \text{.........} + a_{3n}x_n\right)\right] \qquad \text{.....................} \ (2)$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$x_n = \frac{1}{a_{nn}}\left[b_n - \left(a_{n1}x_1 + a_{n2}x_2 + \text{.........} + a_{n(n-1)}x_{n-1}\right)\right]$$

# Example:

$$28x + 4y - z = 32 \dots\dots\dots\dots\dots (1)$$

$$2x + 17y + 4z = 35 \dots\dots\dots\dots\dots (2)$$

$$x + 3y + 10z = 24 \dots\dots\dots\dots\dots (3)$$

$$\therefore (1) \Rightarrow x = \frac{1}{28}(-4y + z + 32) \dots\dots\dots\dots (4)$$

$$(2) \Rightarrow y = \frac{1}{17}(-2x - 4z + 36) \dots\dots\dots\dots (5)$$

$$(3) \Rightarrow z = \frac{1}{10}(-x - 3y + 24) \dots\dots\dots\dots (6)$$

## Step - 4

Take the Initial values of y = 0 , z = 0 for finding the value of x in the euations ( 2 ).

Let $y^{(0)} = 0 \; and \; z^{(0)} = 0$ be the initial values of y and z respectively.

**First iteration:**

$$x^{(1)} = \frac{1}{28}\left(-4y^{(0)} + z^{(0)} + 32\right) = \frac{1}{28}\left(-4.0 + 0 + 32\right) = 1.1429$$

$$y^{(1)} = \frac{1}{27}\left(-2x^{(1)} - 4z^{(0)} + 35\right) = \frac{1}{27}\left(-2 \times 1.1429 - 4.0 + 35\right) = 1.2116$$

$$z^{(1)} = \frac{1}{10}\left(-x^{(1)} - 3y^{(1)} + 24\right) = \frac{1}{10}\left(-1.1429 - 3 \times 1.2116 + 24\right) = 1.9322$$

# Step - 5

**Now these values of first iteration in the equations (2).**

**Second iteration:**

$$x^{(2)} = \frac{1}{28}(-4\times1.2116+1.9322+32)=1.0388$$

$$y^{(2)} = \frac{1}{17}(-2\times1.0388-4\times1.9322+35)=1.4820$$

$$z^{(2)} = \frac{1}{10}(-1.0388-3\times1.4820+24)=1.8515$$

For Example:the values of $x^1$ , $y^1$ , $z^1$ in equations 4, 5,6

# Step - 6

In this way , this iteration will be continued untill last two iterations are same or equal .

Hence the solution of the given system of equations by Gauss-Seidel method.

**Problem:**

Solve the following system of equations by Gauss-Seidel method.

$$28x + 4y - z = 32$$

$$x + 3y + 10z = 24$$

$$2x + 17y + 4z = 35$$

**Solution:** The given systems of equations are

$$\begin{aligned} 28x + 4y - z &= 32 \\ x + 3y + 10z &= 24 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)} \\ 2x + 17y + 4z &= 35 \end{aligned}$$

We observe that the coefficient matrix of (1) is not diagonally dominant because $|3| > |1| + |10|$

Now $A = \begin{pmatrix} 28 & 4 & -1 \\ 1 & 3 & 10 \\ 2 & 17 & 4 \end{pmatrix} R_2 \leftrightarrow R_3 \begin{pmatrix} 28 & 4 & -1 \\ 2 & 17 & 4 \\ 1 & 3 & 10 \end{pmatrix}$

This is diagonally dominant.

So Gauss-Seidel iteration method is applicable.

$\therefore$ The given system (1) becomes

$$28x + 4y - z = 32 \ldots\ldots\ldots\ldots\ldots(1)$$

$$2x + 17y + 4z = 35 \ldots\ldots\ldots\ldots\ldots(2)$$

$$x + 3y + 10z = 24 \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\therefore (1) \Rightarrow x = \frac{1}{28}(-4y + z + 32) \ldots\ldots\ldots\ldots(4)$$

$$(2) \Rightarrow y = \frac{1}{17}(-2x - 4z + 36) \ldots\ldots\ldots\ldots\ldots(5)$$

$$(3) \Rightarrow z = \frac{1}{10}(-x - 3y + 24) \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Let $y^{(0)} = 0 \; and \; z^{(0)} = 0$ be the initial values of y and z respectively.

**First iteration:**

$$x^{(1)} = \frac{1}{28}\left(-4y^{(0)} + z^{(0)} + 32\right) = \frac{1}{28}\left(-4.0 + 0 + 32\right) = 1.1429$$

$$y^{(1)} = \frac{1}{27}\left(-2x^{(1)} - 4z^{(0)} + 35\right) = \frac{1}{27}\left(-2 \times 1.1429 - 4.0 + 35\right) = 1.2116$$

$$z^{(1)} = \frac{1}{10}\left(-x^{(1)} - 3y^{(1)} + 24\right) = \frac{1}{10}\left(-1.1429 - 3 \times 1.2116 + 24\right) = 1.9322$$

Second iteration:

$$x^{(2)} = \frac{1}{28}\left(-4 \times 1.2116 + 1.9322 + 32\right) = 1.0388$$

$$y^{(2)} = \frac{1}{17}\left(-2 \times 1.0388 - 4 \times 1.9322 + 35\right) = 1.4820$$

$$z^{(2)} = \frac{1}{10}\left(-1.0388 - 3 \times 1.4820 + 24\right) = 1.8515$$

**Third iteration:**

$$x^{(3)} = \frac{1}{28}\left(-4\times 1.4820 + 1.8515 + 32\right) = 0.9973$$

$$y^{(3)} = \frac{1}{17}\left(-2\times 0.9973 - 4\times 1.8515 + 35\right) = 1.5059$$

$$z^{(3)} = \frac{1}{10}\left(-0.9973 - 2\times 1.5059 + 24\right) = 1.8485$$

Fourth iteration:

$$x^{(4)} = \frac{1}{28}\left(-4\times 1.5059 + 1.8485 + 32\right) = 0.9938$$

$$y^{(4)} = \frac{1}{17}\left(-2\times 0.8838 - 4\times 4\times 1.8485 + 35\right) = 1.5070$$

$$z^{(4)} = \frac{1}{10}\left(-0.9938 - 3\times 1.5070 + 24\right) = 1.8485$$

**Fifth iteration:**

$$x^{(5)} = \frac{1}{28}\left(-4\times1.5070+1.8485+32\right)=0.9936$$

$$y^{(5)} = \frac{1}{17}\left(-2\times0.9936-4\times1.8485+35\right)=1.5070$$

$$z^{(5)} = \frac{1}{10}\left(-0.9936-3\times1.5070+24\right)=1.8485$$

After five iterations the difference between 4th and 5th iteration is very negligible. Hence the solution of the given system of equations by Gauss-Seidel method is $x=0.994$, $y=1.507$, $z=1.849$ correct up to three decimal places.


.Solve by Gauss-Seidel Method of Iteration the equations

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$2x + 2y + 10z = 14$$