Numerical Differentiation
2^{nd} Part

Chapter 6

Learning results

1. Derive the Numerical Derivative For Newton's Backward Difference Formula

2. How to solve the Problem of Numerical Derivative For Newton's Backward Difference Formula

Newton's backward difference formula gives

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \frac{1}{6}\nabla^6 y_n \text{ ....} + (n \text{ terms})\right]$$

…………(1)

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \frac{137}{180}\nabla^6 y_n \text{ ......} + (n \text{ terms})\right]$$

…………(2)

Problem

Calculate the first and Second derivatives from the following tables of the values of x and y when $x = 2.0$ and $x = 2.2$

Solution

Since $x = 2.0$, towards the last of the table . The difference table is:

x	y	∇y_n	∇^2	∇^3	∇^4	∇^5	∇^6
1.0	2.7183						
		0.6018					
1.2	3.3201		0.1333				
		0.7351		0.0294			
1.4	4.0552		0.1627		0.0067		
		0.8978		0.0361		0.0013	
1.6	4.9530		0.1988		0.0080		0.0001
		1.0966		0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891		0.2964				
		1.6359					
2.2	9.0250						

For X = 2.0 we have , x_n = 2.0 and y_n = 7.3891 and h = 0.2

We know that,

Newton's backward difference formula gives

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \ldots\ldots\ \ldots\ldots\ldots\ldots + (n\ terms)\right] \quad \ldots\ldots\ldots\ldots(1)$$

$$\left(\frac{dy}{dx}\right)_{x=2.0} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \ldots\ldots\ \ldots\ldots\ldots\ldots + (n\ terms)\right]$$

$\nabla y_n = 1.3395$

$\nabla^4 y_n = 0.0080$

$\nabla^2 y_n = 0.2429$

$\nabla^5 y_n = 0.0013$

$\nabla^3 y_n = 0.0441$

$$\left(\frac{dy}{dx}\right)_{x=2.0} = \frac{1}{0.2}\left[\, 1.3395 + \frac{1}{2}0.2429 + \frac{1}{3}\, 0.0441 + \frac{1}{4}\, 0.0080 + \frac{1}{5}0.0013 + \ldots\ldots\ \ldots\ldots\ldots + (n\ terms)\,\right]$$

$$= 7.3896$$

For X = 2.0 we have , $\boldsymbol{x_n}$ = 2.0 and y_n = 7.3891 and h = 0.2

We know that,

Newton's backward difference formula gives

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \right.$$

$$\left. \ldots\ldots + (n\ terms)\ \right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=2.0} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\ \nabla^4 y_n + \frac{5}{6}\ \nabla^5 y_n\right]$$

………………………………(2)

$$\left(\frac{d^2y}{dx^2}\right)_{x=2.0} = \frac{1}{0.4}\left[0.2429 + 0.0441 + \frac{11}{12}\ 0.0080 + \frac{5}{6}\ 0.0013\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=2.0} = \frac{1}{0.4}\left[0.29541666667\right] = 0.73854166667$$

<u>For $x = 2.2$</u>, we have

$$x_n = 2.2,\ y_n = 9.0250\ ,\ \text{and}\ h = 0.2.$$

We know that,

Newton's backward difference formula gives

$$\left[\frac{dy}{dx}\right]_{x=x_n} = \frac{1}{h}\left(\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \right.$$

$$\left. \ldots\ldots + (n\ terms)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

x	y	∇y_n	∇^2	∇^3	∇^4	∇^5	∇^6
1.0	2.7183						
		0.6018					
1.2	3.3201		0.1333				
		0.7351		0.0294			
1.4	4.0552		0.1627		0.0067		
		0.8978		0.0361		0.0013	
1.6	4.9530		0.1988		0.0080		0.0001
		1.0966		0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891		0.2964				
		1.6359					
2.2	9.0250						

For $x = 2.2$, we have

$$x_n = 2.2,\ y_n = 9.0250,\ \text{and } h = 0.2.$$

We know that,

Newton's backward difference formula gives

$$\left[\frac{dy}{dx}\right]_{x=x_n} = \frac{1}{h}\left(\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right)$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{x=2.2} = \frac{1}{0.2}\left[1.6359 + \frac{1}{2}(0.2964) + \frac{1}{3}(0.0535) + \frac{1}{4}(0.0094) + \frac{1}{5}(0.0014) + \frac{1}{6}(0.001)\right]$$

$$= 9.0229$$

Again for second derivative at $x = 2.2$ we obtain,

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_n} = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right).$$

$$\left[\frac{d^2y}{dx^2}\right]_{x=2.2} = \frac{1}{0.04}\left[0.2964 + 0.0535 + \frac{11}{12}(0.0094) + \frac{5}{6}(0.0014) + \frac{137}{180}(0.0001\right]$$

$$= 8.994$$

Practice Work

1. From the following table of values of x and y, Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for

X= 1.8

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250


Thank You