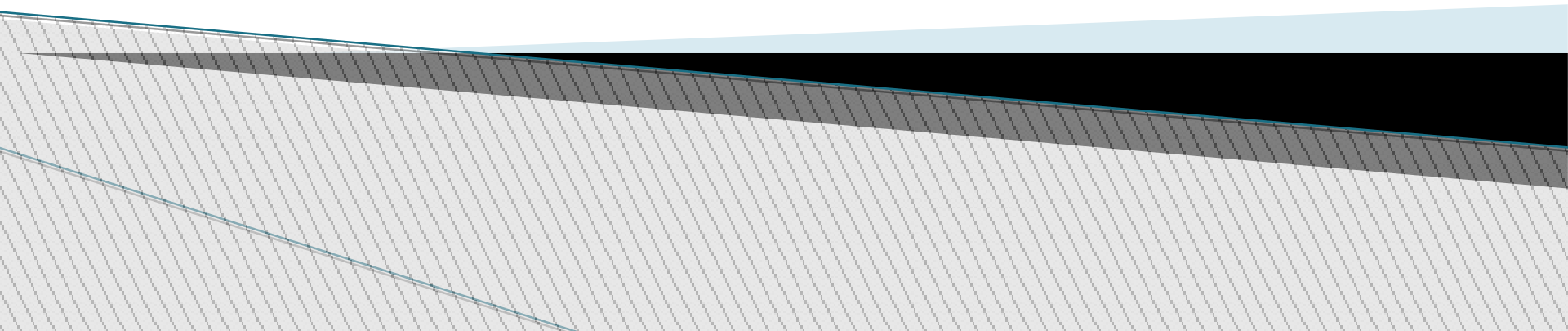
Number System conversions



Number Systems

- There are four systems of arithmetic which are often used in digital electronics.
  - Decimal Number System
  - Binary Number System
  - Octal Number System
  - Hexa Decimal System

Decimal Number system

- Decimal number system contains 10 digits: 0,1,2,3,4,5,6,7,8,9; and that is why its base or radix is 10.
- Here radix means total number of digits used in any system.

Decimal Number System

- The decimal number system is a positional number system.

- Example:

$5\ 6\ 2\ 1$

$10^3\ 10^2\ 10^1\ 10^0$

$1 \times 10^0 = 1$

$2 \times 10^1 = 20$

$6 \times 10^2 = 600$

$5 \times 10^3 = 5000$

Binary Number System

- The binary number system is also a positional numbering system.

- Instead of using ten digits, 0 - 9, the binary system uses only two digits, 0 and 1.

Binary Number System

- The binary number system is also known as **base 2**. The values of the positions are calculated by taking 2 to some power.

- Why is the base 2 for binary numbers?
  - Because we use 2 digits, the digits 0 and 1.

Examples: Binary number system:

11001010

11111111

Octal Number System

- Also known as the Base 8 System
- Uses digits 0 - 7
- Readily converts to binary
- Groups of three (binary) digits can be used to represent each octal digit

Hexadecimal Number System

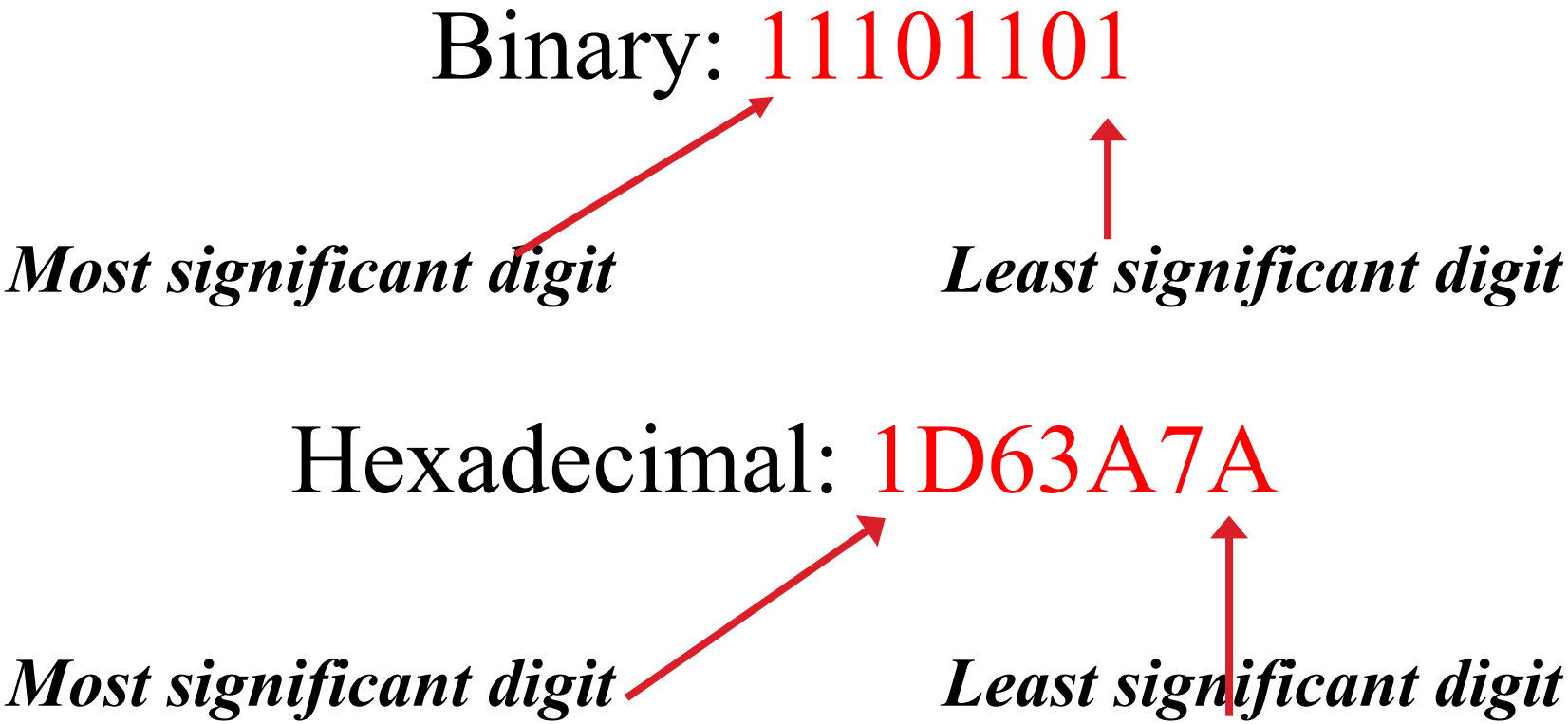
- Base 16 system
- Uses digits 0-9 & letters A,B,C,D,E,F
- Groups of four bits represent each base 16 digit

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Binary Numbering Scale

Base 2 Number	Base 10 Equivalent	Power	Positional Value
000	0	2^0	1
001	1	2^1	2
010	2	2^2	4
011	3	2^3	8
100	4	2^4	16
101	5	2^5	32
110	6	2^6	64
111	7	2^7	128

Significant Digits



Converting From Decimal to Binary

- Make a list of the binary place values up to the number being converted.
- Perform successive divisions by 2, placing the remainder of 0 or 1 in each of the positions from right to left.
- Continue until the quotient is zero.
- Example: 42_{10}

2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1
1	0	1	0	1	0

DECIMAL TO BINARY CONVERSION

- There are two methods to convert it:-

i. Revese of Binary-To-Digital Method

- ✓ Decimal number write as the sum of square
- ✓ 0 & 1 is writen on the byte

Example 1: Convert decimal 45 to the binary value

Solve $= 45_{10} = 32 + 8 + 4 + 1$

$= 2^5 \quad 0 \quad 2^3 \quad 2^2 \quad 0 \quad 2^0$

$= 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1_2$

ii. Repeat division method

- ✓ The numbers is divide by 2.
- ✓ Balance for the question is written until the last answer.

Example : convert 25_{10} to binary

Solve $= 25_{10} = ?_2$

$$= \frac{25}{2} \longrightarrow 12 \text{ balance } 1 \longrightarrow \text{LSB}$$

$$= \frac{12}{2} \longrightarrow 6 \text{ balance } 0$$

$$= \frac{6}{2} \longrightarrow 3 \text{ balance } 0$$

$$= \frac{3}{2} \longrightarrow 1 \text{ balance } 1$$

$$= \frac{1}{2} \longrightarrow 0 \text{ balance } 1 \longrightarrow \text{MSB}$$

$\therefore$ Answer $= \underline{11001_2}$

Decimal to Binary Conversion

- The easiest way to convert a decimal number to its binary equivalent is to use the ***Division Algorithm***

- This method repeatedly divides a decimal number by 2 and records the quotient and remainder

  - ***The remainder digits (a sequence of zeros and ones) form the binary equivalent in least significant to most significant digit sequence***

Division Algorithm

Convert 67 to its binary equivalent:

$67_{10} = x_2$

Step 1: 67 / 2 = 33 R 1 — ***Divide 67 by 2. Record quotient in next row***

Step 2: 33 / 2 = 16 R 1 — ***Again divide by 2; record quotient in next row***

Step 3: 16 / 2 = 8 R 0 — ***Repeat again***

Step 4: 8 / 2 = 4 R 0 — ***Repeat again***

Step 5: 4 / 2 = 2 R 0 — ***Repeat again***

Step 6: 2 / 2 = 1 R 0 — ***Repeat again***

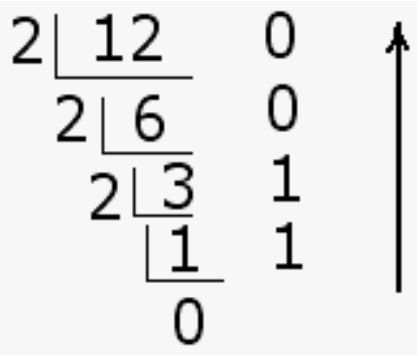
Step 7: 1 / 2 = 0 R 1 — **STOP *when quotient equals 0***

$1\,0\,0\,0\,0\,1\,1_2$

Decimal to binary (fractional)

- Examples:
- Convert the decimal number $(12.0625)_{10}$ into binary number.

Solution:



Fractional part:

0.0625 x 2 = 0.1250 0

0.1250 x 2 = 0.2500 0

0.2500 x 2 = 0.500 0

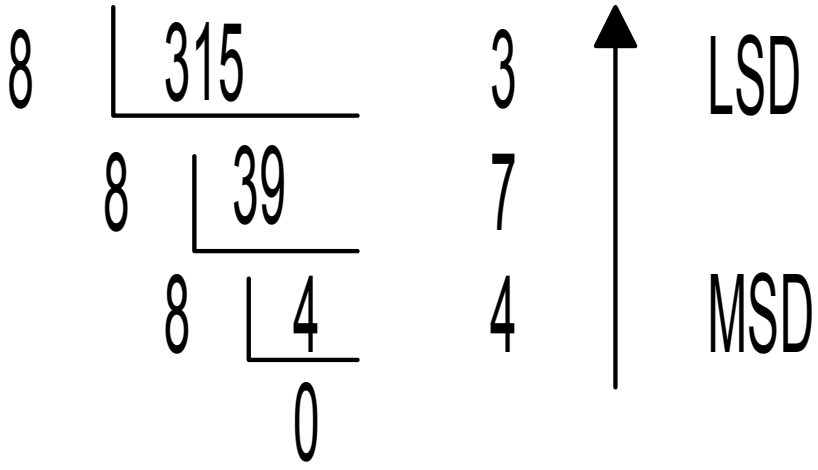
0.500 x 2 = 1.000 1

$(12.0625)_{10} = (1100.0001)_2$

Decimal to Octal Conversion

Examples:

- $(315)_{10} = (473)_8$



DECIMAL TO OCTAL CONVERSION

- Convert from **decimal to octal** by using the repeated division method used for decimal to binary conversion.
- Divide the decimal number by 8
- The first remainder is the LSB and the last is the MSB.

Example : convert 359_{10} to Decimal Value

Solve = $359_{10} = ?_8$

$$= \frac{359}{8} \longrightarrow 44 \text{ balance } 7 \longrightarrow \text{LSB}$$

$$= \frac{44}{8} \longrightarrow 5 \quad \text{balance } 4$$

$$= \frac{5}{8} \longrightarrow 0 \quad \text{balance } 5 \longrightarrow \text{MSB}$$

$\therefore$ Answer = $\underline{547_8}$

Decimal to Octal Conversion

Convert 427_{10} to its octal equivalent:

427 / 8 = 53 R-3 Divide by 8; R is LSD

53 / 8 = 6 R-5 Divide Q by 8; R is next digit

6 / 8 = 0 R-6 Repeat until Q = 0

Decimal to Octal (fractional)

- Examples:
- Convert the decimal number $(225.225)_{10}$ into octal number.

Solution:

Fractional part:

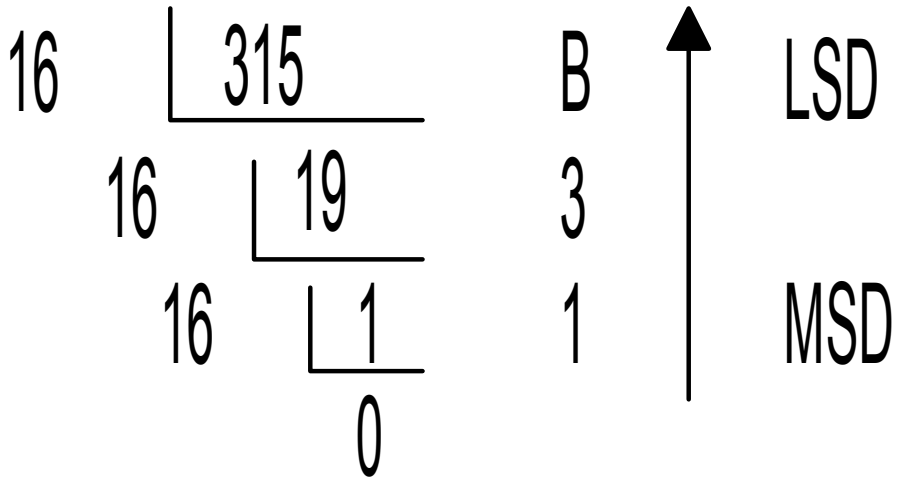
8 | 225		0.225 x 8 = 1.800 1
8 | 28	1	0.800 x 8 = 6.400 6
8 | 3	4	0.400 x 8 = 3.200 3
0	3	0.200 x 8 = 1.600 1
		0.600 x 8 = 4.800 4

$$(225.225)_{10} = (341.16314)_8$$

Decimal to Hexadecimal Conversion

- ***Examples***

  - $(315)_{10} = (13B)_{16}$



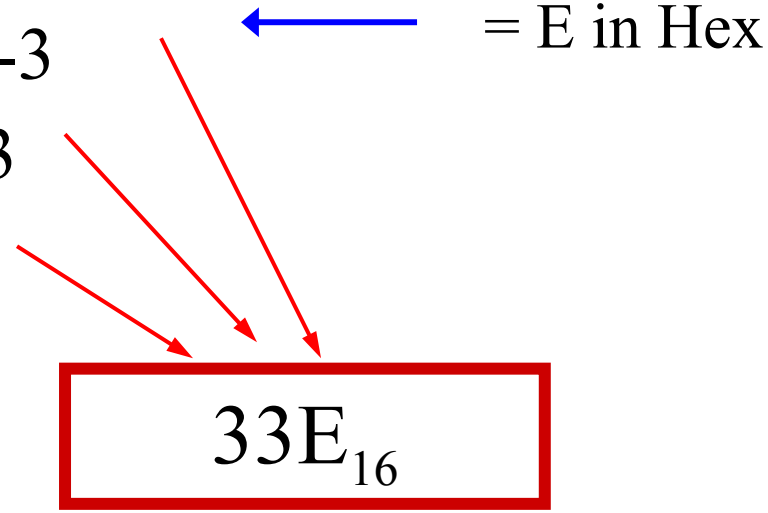
Decimal to Hexadecimal Conversion

Convert 830_{10} to its hexadecimal equivalent:

830 / 16 = 51 R-14

51 / 16 = 3 R-3

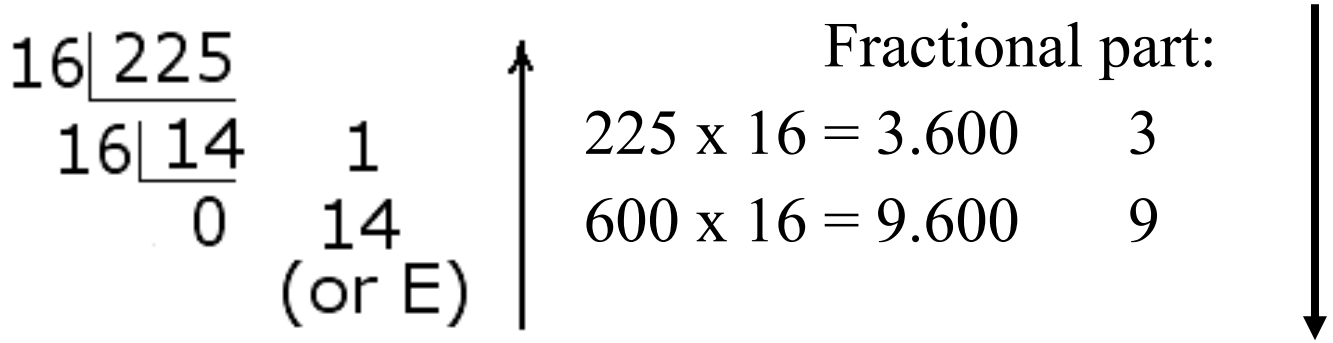
3 / 16 = 0 R-3



Decimal to Hexadecimal (fractional)

- Examples:
- Convert the decimal number $(225.225)_{10}$ into hexadecimal number.

Solution:



$$(225.225)_{10} = (E1.39)_{16}$$

Converting from Binary to Decimal

- Example of a binary number and the values of the positions:

1	0	0	1	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Converting from Binary to Decimal

$\underline{1}\ \underline{0}\ \underline{0}\ \underline{1}\ \underline{1}\ \underline{0}\ \underline{1}$

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$2^0 = 1$ $2^4 = 16$

$2^1 = 2$ $2^5 = 32$

$2^2 = 4$ $2^6 = 64$

$2^3 = 8$

$1 \text{ X } 2^0 = 1$

$0 \text{ X } 2^1 = 0$

$1 \text{ X } 2^2 = 4$

$1 \text{ X } 2^3 = 8$

$0 \text{ X } 2^4 = 0$

$0 \text{ X } 2^5 = 0$

$1 \text{ X } 2^6 = \underline{64}$

Answer: 77_{10}

Binary to Decimal Conversion

- The easiest method for converting a binary number to its decimal equivalent is to use the ***Multiplication Algorithm***
- Multiply the binary digits by increasing powers of two, starting from the right
- Then, to find the decimal number equivalent, sum those products

Multiplication Algorithm

Convert $(10101101)_2$ to its decimal equivalent:

Binary → 1 0 1 0 1 1 0 1

X X X X X X X X

Positional Values → 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

Products → 128 + 32 + 8 + 4 + 1

173_{10}

Converting from Binary to Decimal

Practice conversions:

Binary	Decimal
11101	
1010101	
100111	

Converting From Decimal to Binary

Practice conversions:

Decimal	Binary
59	
82	
175	

binary to hexadecimal system

- Group the digits of the binary number by **four** starting from the right.
- Replace each group of **four** digits by an equivalent hexadecimal digit.

Convert 10110101_2 into a hexadecimal number.

$$\underbrace{1011}_{B}\ \underbrace{0101}_{5}{}_2 = B5_{16}$$

Exercise

- Convert 101000_2 into a hexadecimal number.

- Convert 11101111_2 into a hexadecimal number.

Octal to Decimal Conversion

Convert 653_8 to its decimal equivalent:

Octal Digits →	6	5	3
	x	x	x
Positional Values →	8^2	8^1	8^0
Products →	384	+ 40	+ 3

427_{10}

OCTAL TO BINARY CONVERSION

- Convert from **octal to binary** by converting each octal digit to a three bit binary equivalent

Octal digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

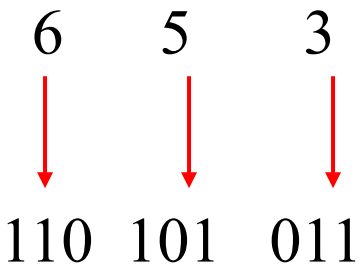
- Convert from **binary to octal** by grouping bits in threes starting with the LSB.
- Each group is then converted to the octal equivalent
- Leading zeros can be added to the left of the MSB to fill out the last group.

Octal to Binary Conversion

Each octal number converts to 3 binary digits

Code
0 - 000
1 - 001
2 - 010
3 - 011
4 - 100
5 - 101
6 - 110
7 - 111

To convert 653_8 to binary, just substitute code:



Hexadecimal to Decimal Conversion

Convert 3B4F (Hexadecimal) to its decimal equivalent:

Hex Digits	→	3	B	4	F
		x	x	x	x
Positional Values	→	16^3	16^2	16^1	16^0
Products	→	12288	+2816	+64	+15

$15{,}183_{10}$

HEXADECIMAL TO binary

- To convert a hexadecimal to binary number, convert each hexadecimal digit to its 4 bit equivalent using the hexa number.

- Example: $(23.AB)_{16} = ()_2$

Solution: $(23.AB)_{16}$ = 2 3 . A B

0010 0011 1010 1011

$(23.AB)_{16} = (00100011.10101011)_2$

Hexadecimal Number System

Binary	Decimal	Hexadecimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	8	8
1001	9	9

Binary	Decimal	Hex
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F

Exercise

- Practice conversions:

Binary	Decimal	Octal	Hex
01111101			
1110101			
1101010111			

- Practice conversions:

Decimal	Binary	Octal	Hex
72			
92			
185			

- Convert 11011111_2 into a hexadecimal number.

Answer Exercise

- Practice conversions:

Binary	Decimal	Octal	Hex
01111101	$125_{(10)}$	$175_{(8)}$	$7D_{(16)}$
1110101	$117_{(10)}$	$165_{(8)}$	$75_{(16)}$
1101010111	$855_{(10)}$	$1527_{(8)}$	$357_{(16)}$

- Practice conversions:

Decimal	Binary	Octal	Hex
72	$1001000_{(2)}$	$110_{(8)}$	$48_{(16)}$
92	$1011100_{(2)}$	$134_{(8)}$	$5C_{(16)}$
185	$10111001_{(2)}$	$271_{(8)}$	$B9_{(16)}$

- Convert 11011111_2 into a hexadecimal number.

$$11011111_{(2)} = DF_{(16)}$$

Any Questions ????????

End of Session

End of Session