Probability

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Sample space

It refers to the **set** of all **possible outcomes** or events that could occur in a particular experiment or random process.

It is denoted by the symbol "**S**"

Each element within the sample space is referred to as a **sample point** or an **elementary event**.

Examples:

1. **Coin Toss:** When you toss a fair coin, the sample space is {Heads, Tails}.
2. **Rolling a Die:** When you roll a fair six-sided die, the sample space is {1, 2, 3, 4, 5, 6}.
3. **Drawing Cards:** If you draw a card from a standard deck of 52 playing cards, the sample space is composed of all 52 cards in the deck

Probability

It quantifies the **likelihood or chance** of a particular event or outcome occurring.

It provides a way to express **uncertainty** and **make predictions** in situations involving randomness or uncertainty.

The probability of an event is typically denoted by a number between 0 and 1, where:

- 0 means that the event is impossible and will not occur.
- 1 means that the event is certain and will definitely occur.

Probability theory is a fundamental tool for

- decision-making,
- risk assessment, and
- understanding uncertainty in various fields.

Mathematically, the probability of an event E is often represented as P(E)

$$P(A) = \frac{\text{Number of favorable outcomes to A}}{\text{Total number of outcomes}}$$

Example 1: A coin is thrown 3 times .what is the probability that at least one head is obtained?

Sol: Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]

Total number of ways = 2 × 2 × 2 = 8.

Fav. Cases = 7 P (A) = 7/8

Example 2: Find the probability of getting a numbered card when a card is drawn from the pack of 52 cards.

Sol: Total Cards = 52.

Numbered Cards = (2, 3, 4, 5, 6, 7, 8, 9, 10) 9 from each suit 4 × 9 = 36

P (E) = 36/52 = 9/13

Example 3: There are 5 green 7 red balls. Two balls are selected one by one without replacement. Find the probability that first is green and second is red.

Sol: P (G) × P (R) = (5/12) x (7/11) = 35/132

Example 4: What is the probability of getting a sum of 7 when two dice are thrown?

Sol: Probability math - Total number of ways = 6 × 6 = 36 ways.

Favorable cases = (1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3) --- 6 ways.

P (A) = 6/36 = 1/6

Example 5: 1 card is drawn at random from the pack of 52 cards.

a) Find the Probability that it is an honor card.

b) It is a face card.

Sol: a) honor cards = (A, J, Q, K) 4 cards from each suits = 4 × 4 = 16

P (honor card) = 16/52 = 4/13

b) face cards = (J,Q,K) 3 cards from each suit = 3 × 4 = 12 Cards.

P (face Card) = 12/52 = 3/13

Tree diagram

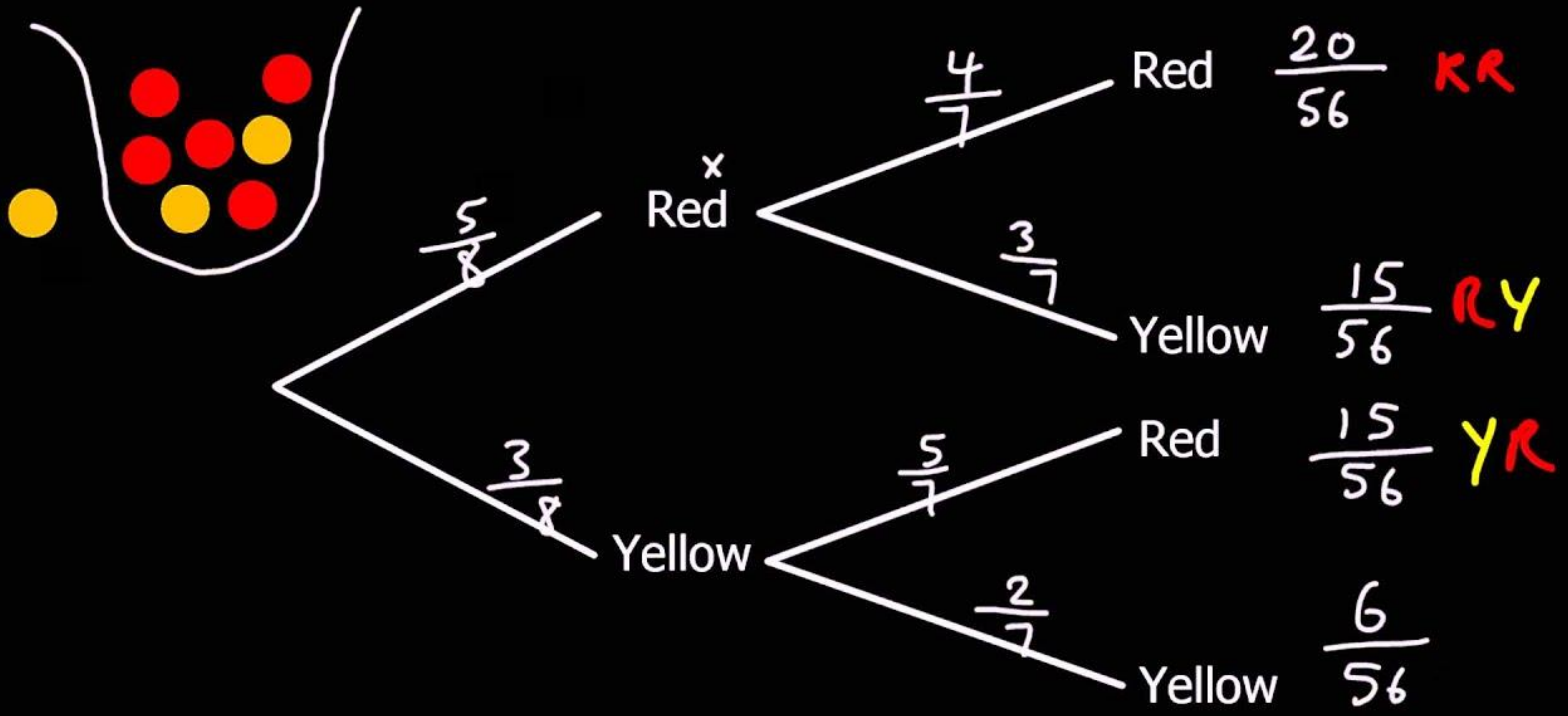
A **graphical representation** commonly used in probability to **visualize** and **calculate** the probabilities of different outcomes in a **sequence of events** or decisions. **Each branch** of the tree represents a **possible outcome**.

Examples:

1. Tina has two bags, Bag A and Bag B. There are 5 red balls and 3 blue balls in bag A. There are 4 red balls and 5 blue balls in bag B. Tina takes at random a counter from each bag. (a) Complete the probability tree diagram. (b) Find the probability that Tina takes two blue balls.
2. Mina is going to play one game of chess and one game of backgammon. The probability she will win the game of chess is 0.6 The probability she will win the game of backgammon is 0.7. (a) Complete the probability tree diagram. (b) Find the probability that Mina will win both games.

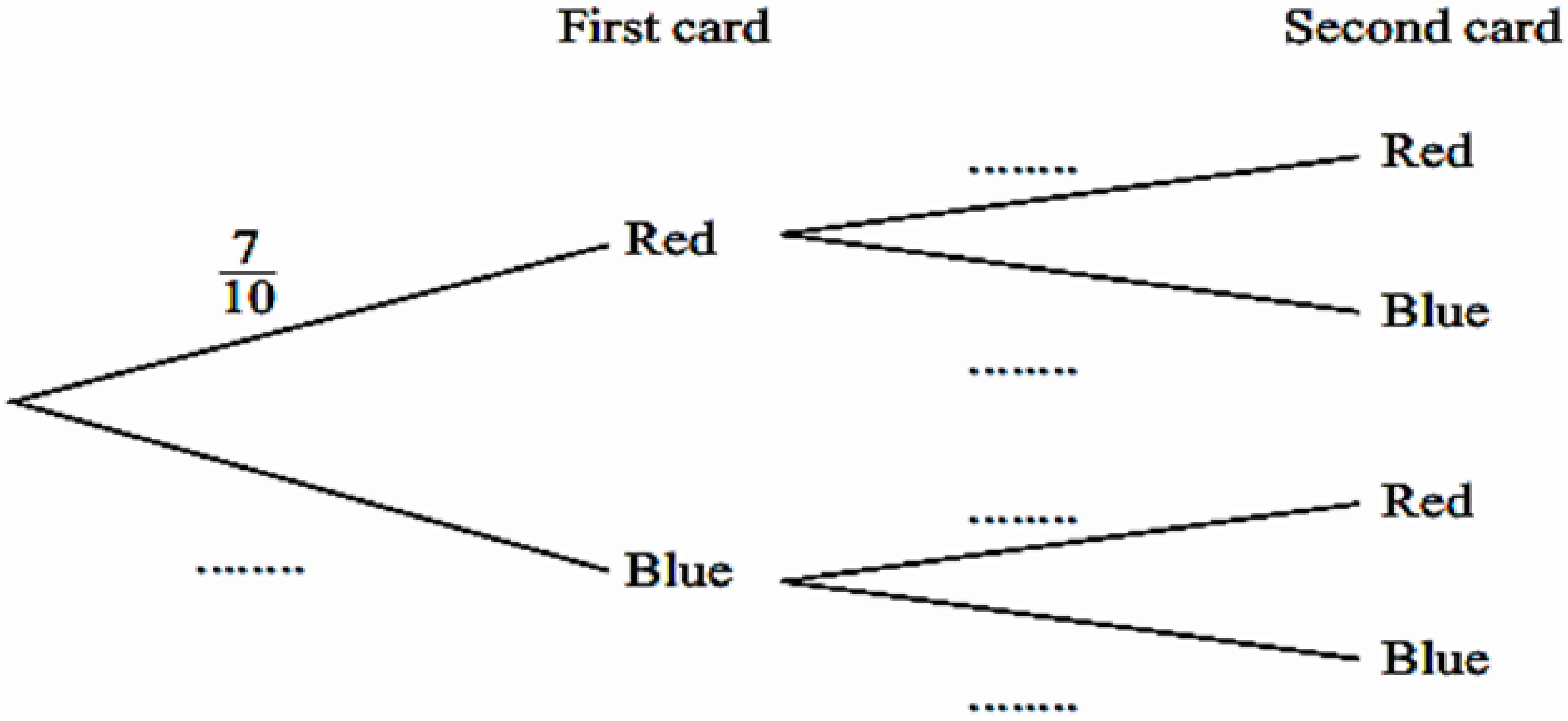
NOT REPLACING

Probability Trees



In a box there are 7 red cards and 3 blue cards.
A card is drawn at random from the box and is not replaced.
A second card is then drawn at random from the box.

(a) Complete this tree diagram.



Joint probability

Probability of **two or more** events or outcomes **occurring simultaneously** or in **combination**.

It is used to describe the **likelihood of multiple events** happening together.

It is denoted as P(A and B), where A and B represent two distinct events or outcomes.

The **formula** depends on whether the events are **independent or dependent**.

- Two or more events are **independent** if one event **doesn't** effect the probability of the others happening. Getting a **head** both times of **2 coin flips** are independent events.
- Two or more events are **dependent** if one event **does** effect the probability of the others happening. **Picking a red** marble at random from a bag, **then picking a green** marble without replacing the red marble are dependent events.

1.Independent Events: we can calculate the joint probability by multiplying the individual probabilities of the events:

P(A and B)=P(A)∗P(B) or

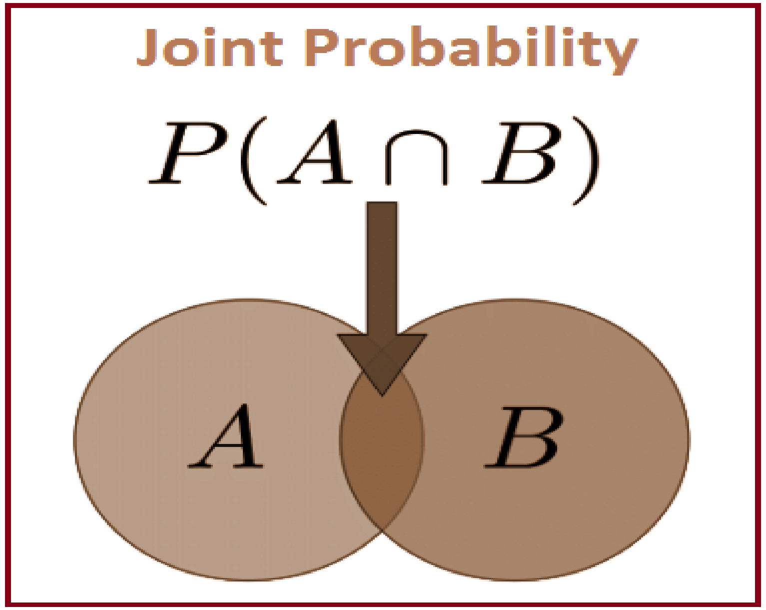
P(A and B and C)=P(A)·P(B)·P(C)

2. Dependent Events: In this case we need to consider conditional probabilities. The joint probability in this case is calculated as:

P(A and B)=P(A)∗P(B|A)

Here, P(B|A) represents the probability of event B occurring given that event A has occurred.

These formulas are called “**The AND Rules**” or **“Multiplication Rule”**



Example: 1. Find the probability that the number three will occur twice when two dice are rolled at the same time. (**Ans**: 1/36)

2. Find the probability of drawing two aces consecutively from a standard deck of 52 cards when we don't replace the cards.

Ans: Initially, the deck contains four aces, so the likelihood of drawing an ace on the first draw is 4/52 or 1/13. If we draw an ace (event A1), only three aces and 51 cards remain in the deck. Consequently, the conditional probability of drawing another ace (event A2) is now 3/51.

the joint probability of drawing two aces in a row:

$P(A1 \cap A2) = P(A1) * P(A2|A1)$

$P(A1) = 4/52 = 1/13$

$P(A2|A1) = 3/51$

$P(A1 \cap A2) = (1/13) * (3/51) = 3/663 = 1/221$

3. A bag contains 4 **red** balls and 5 **blue** balls. Rahim picks 2 balls at random. Calculate the probability that he selects the same coloured ball each time, given that after each time a ball is selected, **it is replaced**. (**Ans**: 41/81)

4. A bag contains 4 **red** balls and 5 **blue** balls. Rahim picks 2 balls at random. Calculate the probability that he selects the same coloured ball each time, given that after each time a ball is selected, **it is not replaced**. (**Ans**: 32/72)

5. Anna and Rob take their driving tests on the same day. The probability of Anna passing her driving test is 0.7. The probability of both Anna and Rob passing is 0.35

(a) Work out the probability of Rob passing his driving test. (**Ans**: 1/2)

(b) Work out the probability of both Anna and Rob failing their driving tests. (**Ans**: 0.15)

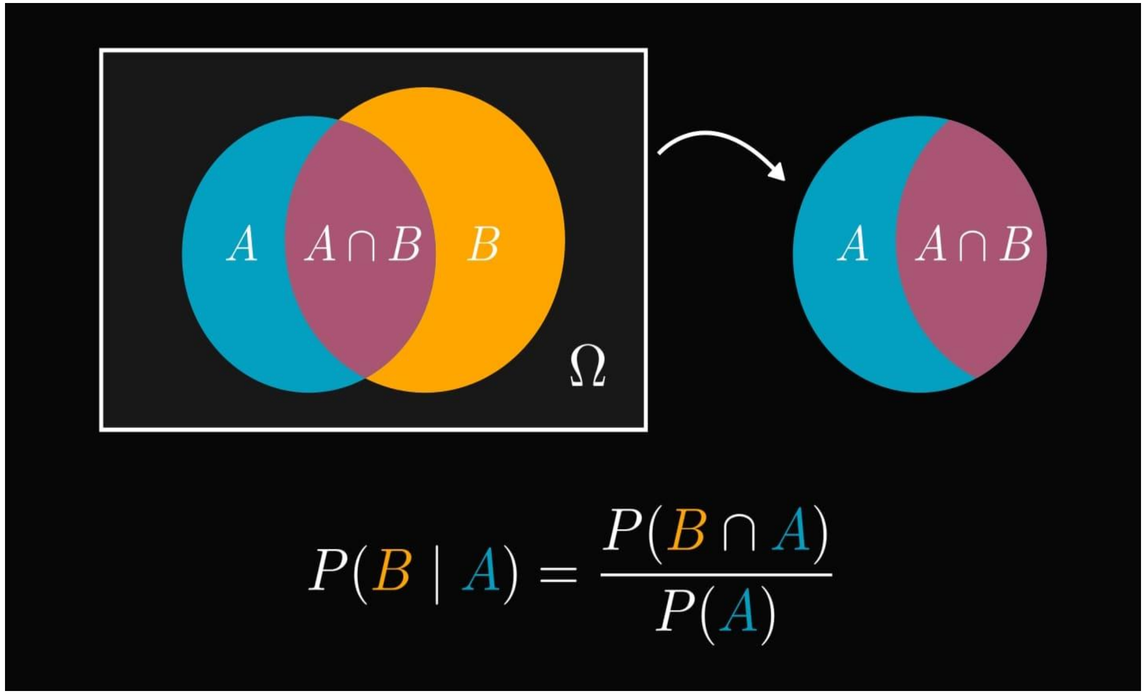
<u>Conditional probability</u>

the probability of **one event** occurring given that **another** event has **already occurred**.

It represents the **likelihood** of an **event** under a **specific condition** or context. Conditional probability is denoted as P(A|B), where A is the event of interest, and B is the given condition or context.

The formula for conditional probability is as follows:

P(A|B)=P(B)/P(A and B)



Example:
you draw two cards from the deck without replacement. Find the conditional probability of drawing a red card on the second draw (event A) given that you drew a red card on the first draw (event B).

1. The probability of drawing a red card on the first draw (P(B)) is 26/52 because there are 26 red cards in a standard deck of 52 cards.
2. The probability of drawing a red card on the second draw (P(A)) is 25/51 because there are now 25 red cards left out of 51 cards after the first draw.
3. The joint probability of drawing a red card on both draws (P(A and B)) is (26/52) * (25/51)

You can then find the conditional probability:

$$P(A|B) = P(A \text{ and } B)/P(B)$$

$$= (26/52) * (25/51)/(26/52)$$

$$= 25/51$$

Example 1: A bag contains 5 red balls and 7 blue balls. Two balls are drawn without replacement. What is the probability that the second ball drawn is red, given that the first ball drawn was red?

Solution: 4/11

2: A box contains 5 green balls and 3 yellow balls. Two balls are drawn without replacement. What is the probability that both balls are green?

Solution: 5/14

3: In a bag, there are 8 red marbles, 4 blue marbles, and 3 green marbles. If one marble is randomly drawn, what is the probability that it is not blue?

Solution: 11/15

4: In a survey among a group of students, 70% play football, 60% play basketball, and 40% play both sports. If a student is chosen at random and it is known that the student plays basketball, what is the probability that the student also plays football?

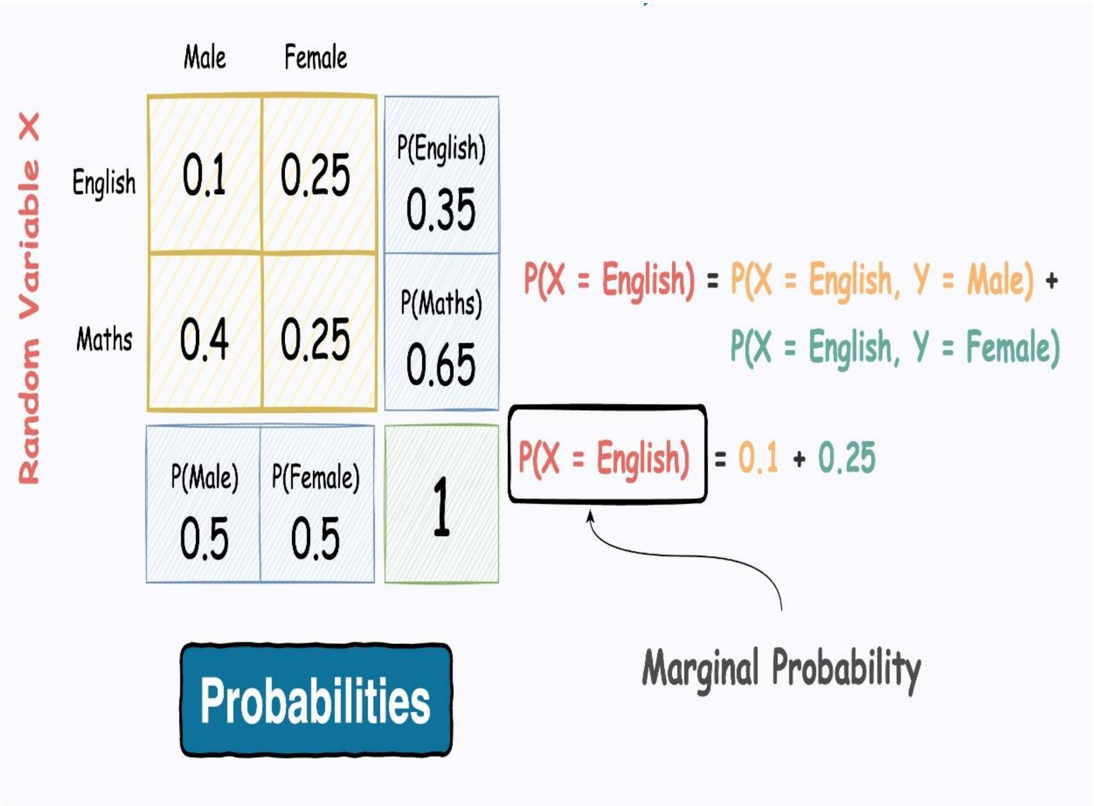
Solution: 2/3

5: In a deck of 52 playing cards, 4 cards are drawn without replacement. What is the probability that all 4 cards are aces, given that the first card drawn is an ace?

Solution: 1/270725

Marginal probability

the probability of a **single** event or outcome **without** considering **other** events or outcomes.
It is derived from a joint probability distribution that involves multiple variables or events.
In the context of a probability distribution with multiple variables, the marginal probability of a specific variable is calculated by **summing** or integrating over all other variables, effectively "**marginalizing**" the distribution to **focus** on a **single** variable.



Marginal probabilities

	Pass	Fail	Total	
Males	46	56	102	P(male) = 0.51
Females	68	30	98	P(female) = 0.49
Total	114	86	200	
	P(passed) = 0.57	P(failed = 0.43)		

Calculating Marginal Probabilities

Subjects are asked if they would vote for a qualified woman for President. Results are broken down by gender.

- Task: Fill in the marginal probabilities.

Vote for Female?

Gender		Yes	No	Total
	Male	.41	.07	
	Female	.47	.05	
	Total			

Conditional Probability Vs Joint Probability Vs Marginal Probability

Parameter	Conditional Probability	Joint Probability	Marginal Probability
Definition	The probability of an event occurring given that another event has already occurred.	The probability of two or more events occurring simultaneously.	The probability of an event occurring without considering any other events.
Calculation	P(A | B)	P(A ∩ B)	P(A)
Variables involved	Two or more events	Two or more events	Single event

Addition rule or OR Rule

a way to calculate the probability of the union of two or more events. There are two variations of the addition rule:

1. **Addition Rule for Mutually Exclusive Events**:

$$P(A \text{ or } B)=P(A)+P(B)$$

Example: Find the probability of rolling a 3 (event A) or rolling a 4 (event B) on a fair six-sided die.

Ans: 1/6+1/6=1/3.

2. Addition Rule for Non-Mutually Exclusive Events:

$$P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$$

For more

$$P(A \text{ or } B \text{ or } C)=P(A)+P(B)+P(C)-P(A \text{ and } B)-P(A \text{ and } C)-P(B \text{ and } C)+P(A \text{ and } B \text{ and } C)$$

Example: Find the probability of drawing a red card (event A) or drawing a face card (event B) from a standard deck of cards.