Lesson 06

Boolean Algebra and Logic Gate

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Review from Previous Lesson

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Today’s Class Agenda

- Axioms
- Theorems
- Operation
- Logic gates
- Boolean function
- Truth table

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Boolean axioms

- Boolean algebra: an algebraic system of logic introduced by George Boole in 1854.
- The axioms (postulates) of an algebra are the basic assumptions from which all theorems of the algebra can be proved.
- The axioms are independent; none can be proved from the others.
- Boolean algebra is calculated using only addition and multiplication by following these rules:
  - 0+0=0, 0+1=1, 1+0=1, 1+1=1
  - 0.0=0, 0.1=0, 1.0=0, 1.1=1

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Theorems

- Theorem 1 (Idempotency)
  - (a) x + x = x; (b) x . x = x
- Theorem 2
  - (a) x + 1 = 1; (b) x . 0 = 0
- Theorem 3 (Absorption)
  - (a) yx + x = x; (b) (y + x)x = x
- Theorem 4 (Involution)
  - $\bar{\bar{x}} = x$
  - $x\bar{x} = 0$
- Theorem 5 (Associativity)
  - (a) x + (y + z) = (x + y) + z; (b) x(yz) = (xy)z
- Theorem 6 (De Morgan)
  - (a) $\overline{x + y} = \bar{x} . \bar{y}$ (b) $\overline{x . y} = \bar{x} + \bar{y}$
- Theorem 7
  - $E(x_1, x_2, \ldots, x_n) = E_d(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$
- Theorem 8 (De Morgan (generalized))
  - $\overline{(x_1 + x_2 + \ldots + x_n)} = \bar{x}_1\bar{x}_2 \ldots \bar{x}_n$
  - $\overline{(x_1 . x_2 . \ldots . x_n)} = \bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_n$
- Other theorems:
  - $x + \bar{x} = 1$
  - x + yz = (x + y) (x + z)

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Boolean Algebra Operation

- $A\,B + \bar{A}\,B + A\,\bar{B}$

$= B\,(A + \bar{A}) + A\,\bar{B}$

$= B.\,1 + A\,\bar{B}$

$= B + A\,\bar{B}$

$= (B + A)\,(B + \bar{B})$

$= (B + A)\,.\,1$

$= A + B$

- $A\,\bar{B} + \overline{(\bar{A} + \bar{B})} + C\,\bar{C}$

$= A\,\bar{B} + \overline{(\bar{A} + \bar{B})} + 0$

$= A\,\bar{B} + (\bar{\bar{A}}\,.\,\bar{\bar{B}})$

$= A\,\bar{B} + A\,B$

$= A\,(\bar{B} + B)$

$= A\,.\,1$

$= A$

- $X = \overline{(A + \bar{C})\,(\bar{B} + D)}$

$= \overline{(A + \bar{C})} + \overline{(\bar{B} + D)}$

$= \bar{A}\,\bar{\bar{C}} + \bar{\bar{B}}\,\bar{D}$

$= \bar{A}\,C + B\,\bar{D}$

- $XY + XZ + \bar{X}Z + ZX$

$= XY + XZ + \bar{X}Z$

$= XY + Z\,(X + \bar{X})$

$= XY + Z.1$

$= XY + Z$

- $\bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$

$= (\bar{X}YZ + XYZ) + (X\bar{Y}Z + XYZ) + (XY\bar{Z} + XYZ)$

$= YZ\,(\bar{X} + X) + XZ\,(\bar{Y} + Y) + XY\,(\bar{Z} + Z)$

$= YZ\,.\,1 + XZ\,.\,1 + XY\,.\,1$

$= YZ + XZ + XY$

Boolean Algebra Operation…

- Prove the following equations:
- $XYZ + X\bar{Y}Z + XYZ + \bar{X}Z = C$

L.H.S $= XYZ + X\bar{Y}Z + XYZ + \bar{X}Z$

$= X\bar{Y}Z + XYZ + \bar{X}Z$

$= XZ\,(\bar{Y} + Y) + \bar{X}Z$

$= XZ\,.\,1 + \bar{X}Z$

$= XZ + \bar{X}Z$

$= Z\,(X + \bar{X})$

$= C\,.\,1$

$= C =$ R.H.S.

L.H.S. = R.H.S. (Proved)

- $\overline{(M + \bar{N})\,(\bar{M} + N)} = \bar{M}N + M\bar{N}$
- L.H.S. $= \overline{(M + \bar{N})\,(\bar{M} + N)}$

$= \overline{(M + \bar{N})} + \overline{(\bar{M} + N)}$

$= \bar{M}\,\bar{\bar{N}} + \bar{\bar{M}}\,\bar{N}$

$= \bar{M}N + M\bar{N} =$ R.H.S.

- L.H.S. = R.H.S. (Proved)

$(X + Y)\,(\bar{X} + Z)\,(Y + Z) = (X + Y)\,(\bar{X} + Z)$

- L.H.S. $= (X + Y)\,(\bar{X} + Z)\,(Y + Z)$

$= (X.\bar{X} + XZ + Y\bar{X} + YZ)\,(Y + Z)$

$= (0 + XZ + Y\bar{X} + YZ)\,(Y + Z)$

$= (XZ + Y\bar{X} + YZ)\,(Y + Z)$

$= XYZ + \bar{X}YY + YYZ + XZZ + \bar{X}YZ + YZZ$

$= XYZ + Y\bar{X} + YZ + XZ + \bar{X}YZ + YZ$

$= XYZ + Y\bar{X} + YZ + XZ + \bar{X}YZ$

$= YZ\,(1 + \bar{X} + X) + \bar{X}Y + XZ$

$= YZ + \bar{X}Y + XZ$

- R.H.S. $= (X + Y)\,(\bar{X} + Z)$

$= X\,\bar{X} + XZ + \bar{X}\,Y + YZ$

$= 0 + XZ + \bar{X}\,Y + YZ$

$= YZ + \bar{X}\,Y + XZ$

= R.H.S.

L.H.S. = R.H.S. (Proved)

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Exercise

1. $X = \overline{(A + B + C)\,\bar{B}\,C}$
2. $(A + \bar{A}B) + (A + B)$
3. $\overline{X\,(X + Y)}\,.\,\overline{Y\,(Y + X)}$
4. $(B\bar{C} + \bar{A}D)\,(A\bar{B} + C\bar{D})$
5. $\overline{(A + B + C)}\,\bar{B}\,C$

Prove the following equations:

1. $A + \bar{A}\,B + \bar{A}\,\bar{B} = 1$
2. $\overline{(A + \bar{B} + C\bar{D})} = \bar{A}\,B\,(\bar{C} + D)$
3. $\overline{(A + B)}\;\overline{(\bar{A} + \bar{B})} = 0$
4. $(X + Y + Z)\,\bar{Y}\,Z = X\,\bar{Y}\,Z$
5. $XYZ + X\,\bar{Y}\,Z + X\,Y\,\bar{Z} = X(Y + C)$
6. $\overline{(A + C)} + \overline{(B + \bar{D})} = A\,\bar{C} + \bar{B}\,D$

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Answer key

1. $\overline{A} + B + \overline{C}$
2. $A + B$
3. $\overline{X} . \overline{Y}$
4. 0
5. 0

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Logic gate

Logic gates are the basic building blocks of any digital system. It is an **electronic circuit** having one or more than one input and only one output. The relationship between the input and the output is based on a certain **logic**. Based on this, logic gates are named as **AND** gate, **OR** gate, **NOT** gate etc.

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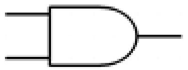
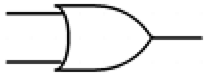
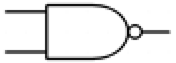
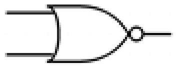
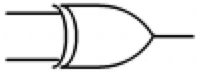
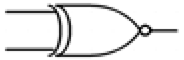
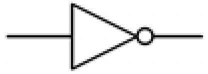
Logic gate…

Logic Gate	Symbol	Description	Boolean
AND		Output is at logic 1 when, and only when all its inputs are at logic 1,otherwise the output is at logic 0.	$X = A \cdot B$
OR		Output is at logic 1 when one or more are at logic 1.If all inputs are at logic 0,output is at logic 0.	$X = A + B$
NAND		Output is at logic 0 when,and only when all its inputs are at logic 1,otherwise the output is at logic 1	$X = \overline{A \cdot B}$
NOR		Output is at logic 0 when one or more of its inputs are at logic 1.If all the inputs are at logic 0,the output is at logic 1.	$X = \overline{A + B}$
XOR		Output is at logic 1 when one and Only one of its inputs is at logic 1. Otherwise is it logic 0.	$X = A \oplus B$
XNOR		Output is at logic 0 when one and only one of its inputs is at logic1.Otherwise it is logic 1. Similar to XOR but inverted.	$X = \overline{A \oplus B}$
NOT		Output is at logic 0 when its only input is at logic 1, and at logic 1 when its only input is at logic 0.That's why it is called and INVERTER	$X = \overline{A}$

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Boolean Function using Digital Circuit

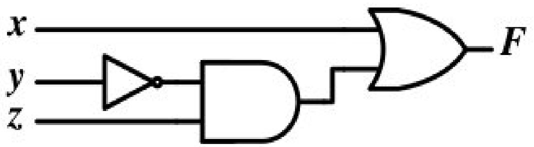
- **Boolean Expression/Function**

  Example: $F(x, y) = x + y'z$

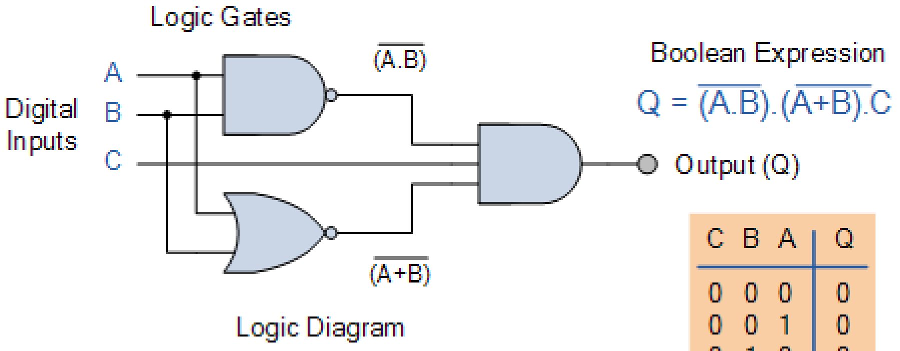
- **Truth Table**

  All possible combinations of input variables

- **Logic Circuit**



x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



Typical Truth Table

C	B	A	Q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

For Exercise:

$Y = \bar{A}B\overline{(A + C)}$

$X = \bar{A}\bar{B} + C$

$X = \bar{A} + \bar{B} + C$

$X = (A + B)C$

$X = (A + \bar{A}B + (A + B)$

$X = A\bar{B} + \bar{C}$

$Y = A + BC$

$F = \overline{AB} . \overline{BC} . \overline{CA}$

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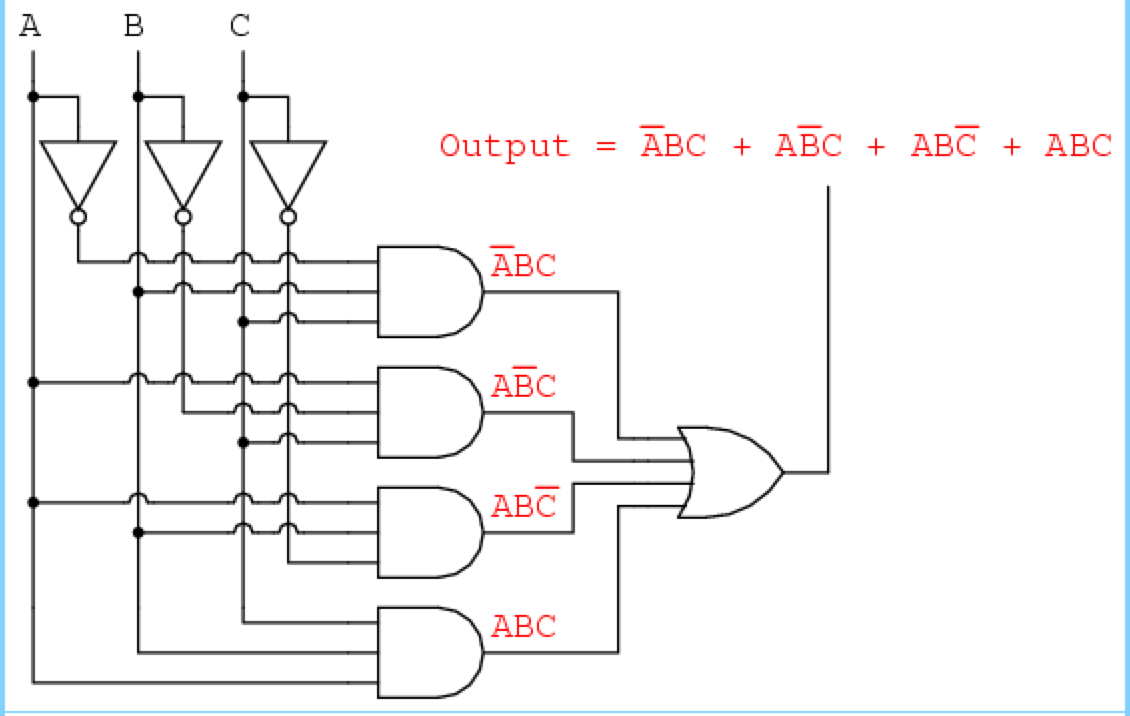
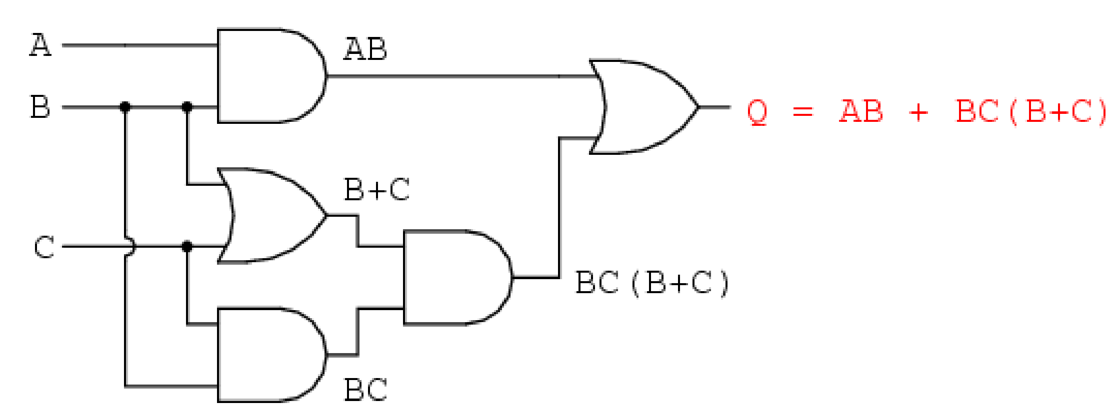
Boolean Function using Digital Circuit…





For Exercise:

YCi + XCi + XY

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Truth table

- **Any boolean function can be represented by a truth table. The number of rows in the table is 2^n, where n is the number of variables in the function.**
- **$X = (A + \overline{B})\,C$**

A	B	C	$\overline{B}$	$A + \overline{B}$	$X = (A + \overline{B})\,C$
0	0	0	1	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	0	1	1

I Do it!

References

- Peter Norton, M. (2004). *Peter Norton's intro to computers 6/E*. McGraw-Hill Education.
- Rahman, M. L., Kaiser, M. S., Rahman, M. A., & Hossain, M. A. (2017). *Computer Fundamentals and ICT* (1st ed.). Daffodil International University Press.
- Different sites found in internet

Next Lesson: Lesson 07 on Types of Storage Devices

- **Based on today's discussion, activity will be assigned to you. You have to make a solution to the problem.**
- **At the end there will be a short assessment test based on Lecture lesson-06**

Discussion Questions and Learning Summary!

The End of Lesson-6

Thanks!