Chapter 4: Electric Fields in Matter

Contents:

Electric Fields in Matter: An electric field is a region around a charged particle where a force would be exerted on other charged particles. It is represented by the symbol E and is measured in volts per meter (***V/m***).

Electric fields in matter refer to the study of how electric fields interact with materials, which can affect the behavior of the field and the material itself. When an electric field is applied to matter, the material responds in ways that depend on its electrical properties, such as whether it is a conductor, insulator (dielectric), or something in between.

They play a critical role in materials science, electronics, and many technologies we rely on today. Understanding how materials respond to electric fields helps design better devices and materials for various applications.

Concepts of Electric Fields in Matter:

(i) Polarization in Matter
(ii) Effect of Matter on the Electric Field
(iii) Electric Fields in Conductors
(iv) Gauss's Law in Matter
(v) Polarization Charge
(vi) Boundary Conditions at Interfaces
(vii) Applications of Electric Fields in Matter

✓ Follow this link for better understanding: *https://www.youtube.com/watch?v=qe3guUUXx90*

Dielectric:

Dielectrics are generally characterized as materials with very low conductivity of electric current. Essentially insulators, they lack free electrons. When subjected to an electric field, dielectrics can be easily polarized, causing their response to an electric field to be fundamentally different from that of conductors.

Applications of Dielectrics:

- **Capacitors:** Dielectrics play a crucial role in capacitors by increasing capacitance and enabling the storage of more energy to be stored.
- **Insulation:** Dielectrics are used as insulators in electrical systems to prevent current flow and protect against electrical shocks.
- **Energy Storage:** High-permittivity dielectrics are essential in energy storage devices, allowing for compact, high-capacity systems.
- **Electronics:** Dielectric materials are used in various electronic components, such as transistors and integrated circuits, to improve efficiency and stability.

Types of Dielectrics: There are various types of dielectrics, classified based on their structure and behavior:

(i). Polar Dielectrics
(ii). Non-polar Dielectrics
(iii). Solid Dielectrics
(iv). Liquid Dielectrics
(v). Gaseous Dielectrics

Explanation of these classifications:

(i). **Polar dielectrics:** Polar dielectrics are materials with molecules that have permanent electric dipole moments, even without an external electric field. This inherent charge separation allows the dipoles to align when an external field is applied, leading to material polarization and influencing their electrical properties.
Ex: Water (H_2O), Polyvinyl Chloride (PVC), Acetic Acid (CH_3COOH), etc.

(ii). **Non-polar dielectrics:** Non-polar dielectrics are materials without permanent electric dipole moments. In the absence of an external electric field, their charge distribution is balanced. However, when an electric field is applied, they can become polarized, allowing interaction with the field through temporary dipoles.
Ex: Polyethylene (PE), Polypropylene (PP), Polystyrene (PS), etc.

(iii). **Solid Dielectrics:** Solid dielectrics are non-conductive materials that can become polarized in an electric field. They are vital for applications like energy storage (capacitors), insulation, and high-voltage systems. These materials can be either polar or non-polar based on their molecular structure and response to electric fields.
Ex: Ceramics, Plastics, Glass, etc.

(iv). **Liquid Dielectrics:** Liquid dielectrics are insulating liquids that do not conduct electricity but can be polarized by an electric field. They are commonly used in transformers, high-voltage capacitors, and cable insulation, providing flexibility for applications requiring both cooling and insulation.
Ex: Mineral Oil, Silicone Oil, Synthetic Dielectric Fluids, etc.

(v). **Gaseous Dielectrics:** Gaseous dielectrics are non-conductive gases that can become polarized to provide insulation in electric fields. They are used in high-voltage applications like gas-insulated switchgear (GIS), circuit breakers, and transmission lines.
Gases offer advantages such as lower cost, easier circulation, and self-healing properties, but their dielectric strength is generally lower than that of solid or liquid dielectrics.
Ex: Air, Sulfur Hexafluoride (SF_6), Nitrogen (N_2), etc.

Characteristics of dielectrics:

- Electrical Insulation
- Dielectric Constant (Relative Permittivity)
- Dielectric Strength
- Polarization
- Loss Tangent (Dielectric Loss)
- Permittivity
- Temperature Dependence
- Frequency Dependence
- Thermal Conductivity
- Aging and Durability

- Chemical Stability
- Mechanical Strength

<u>Dielectric Matter:</u>
Dielectric materials are insulators that can be polarized by an applied electric field. While they do not conduct electricity, they can store electrical energy. When exposed to an electric field, their positive and negative charges shift slightly, creating dipoles. This property allows them to reduce the effective electric field within the material.
Though poor conductors of electricity, dielectrics efficiently support electrostatic fields, store electrical charges, and exhibit high specific resistance along with a negative temperature coefficient of resistance.

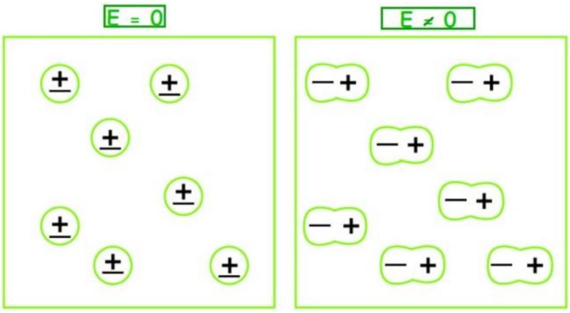
Applications of Dielectric Materials:

- Capacitors
- Insulators
- Electronics
- Energy Storage

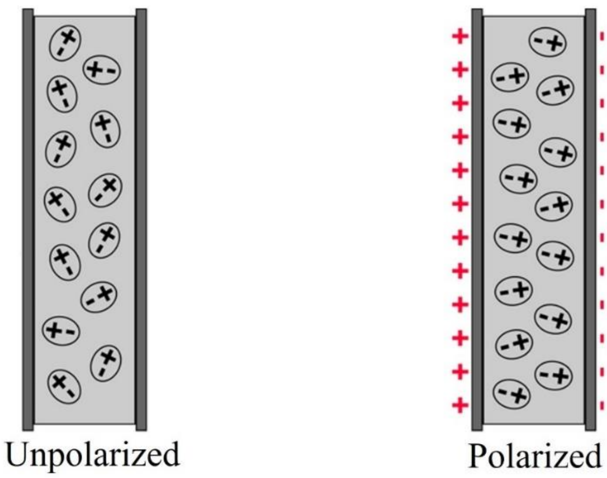
Properties of dielectric materials: These are the most important properties of dielectric materials:

- Electric susceptibility
- Dielectric polarization
- Electric dipole moment
- Electronic polarization
- Relaxation time
- Dielectric breakdown
- Dielectric dispersion

***** <u>Polarization/ Electric Polarization</u>:**



Polarization is the separation of electric charges in a material due to an external electric field, resulting in dipole formation. This phenomenon is key to understanding how dielectric materials behave in electric fields and is important in various electronic applications.

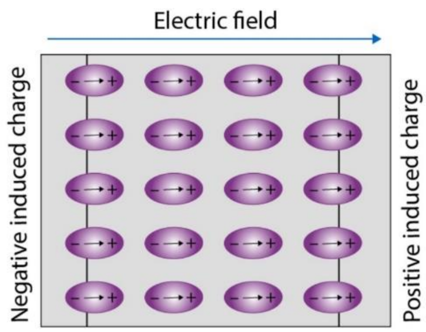


Electronic polarization occurs in dielectric materials when an external electric field is applied, causing the electron cloud to shift relative to the atomic nucleus and forming induced dipoles. This phenomenon is present in both polar and non-polar dielectric materials and is effective across a range of frequencies, impacting both direct current (DC) and alternating current (AC) applications.

Electronic polarization also contributes to the dielectric constant ($\boldsymbol{\epsilon_r}$) of a material, enhancing its capacity to store electrical energy when exposed to an electric field.

*** **Mechanism of Electronic Polarization:**

- **Electron Displacement:** When an external electric field is applied to a dielectric material, the negatively charged electron cloud around the nucleus is displaced slightly in the opposite direction of the field, while the positively charged nucleus stays in place.
- **Formation of Induced Dipoles:** Displacement creates an electric dipole moment in atoms or molecules by separating the positive charge (nucleus) from the negative charge (displaced electron cloud), resulting in induced dipoles throughout the material.
- **Uniform and Instantaneous Response:** Electronic polarization occurs almost instantly when an electric field is applied due to the rapid movement of mobile electrons compared to heavier nuclei.



Electronic polarization in dielectric materials occurs when electron clouds shift in response to an electric field, creating induced dipoles. This phenomenon enhances dielectric properties, improving energy storage in capacitors, insulation, and high-frequency applications. Understanding electronic polarization is essential for optimizing electronic and electrical systems.

***** Dipole moment (*p*):**

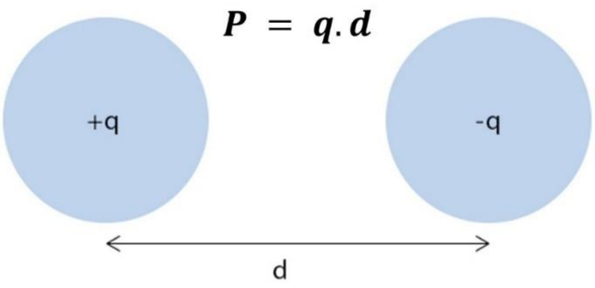
The dipole moment is a vector quantity that signifies the separation of positive and negative charges in a system. It is essential in electromagnetism, chemistry, and materials science for understanding molecular behavior in electric fields.

The dipole moment (p) is defined mathematically as:

$$p = q \cdot d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (i)$$

Where:

- p = Dipole moment **(C.m)**
- q = Magnitude of the charge **(C)**
- d = Displacement vector pointing from the negative charge to the positive charge **(m)**



The dipole moment is a vector with both magnitude and direction, pointing from the negative charge to the positive charge. Its SI unit is coulomb-meter **(C·m)**. In molecular chemistry, it is often expressed in Debye units **(D)**, where 1 D≈3.336×10^{-29} **C·m**.
The magnitude of the dipole moment provides information about the strength of the dipole. Larger dipole moments indicate a greater separation of charge.

<u>*Problem 1:</u>
A diatomic molecule has a bond length of $\mathbf{1.5\,A^{\circ}}$. If the charge on each atom is $\mathbf{+1e}$ and $\mathbf{-1e}$ respectively, calculate the dipole moment of the molecule.

<u>Solution:</u>

Given:

- Bond length $\mathbf{d = 1.5\,A^{\circ} = 1.5\times10^{-10}\,m}$
- Charge $\mathbf{q = 1e = 1.6\times10^{-19}\,C}$

Solution: The dipole moment $\boldsymbol{p}$ is given by:

$$\mathbf{p = q\times d}$$

Substituting the values:

$$\mathbf{p\ =\ (1.6\times10^{-19}\,C)\times(1.5\times10^{-10}\,m)}$$

$$\mathbf{p = 2.4\times10^{-29}\,C.m}$$

Thus, the dipole moment of the molecule is $\mathbf{2.4\times10^{-29}\,C.m}$.

<u>Problem 2:</u>
Two charges, $\mathbf{+5\,\mu C}$ and $\mathbf{-3\,\mu C}$, are placed $\mathbf{4\,cm}$ apart. Calculate the dipole moment of the system.

<u>Solution:</u>

Given:

- Charge $\mathbf{q_1\ = +5\,\mu C = 5\times10^{-6}\,C}$
- Charge $\mathbf{q_1\ = -3\,\mu C = -3\times10^{-6}\,C}$
- Distance $\mathbf{d = 4\,cm = 0.04\,m}$

The dipole moment for an asymmetric charge distribution is calculated using the difference in charges:

$$q_{eff} = \mid q1 - q2 \mid = \mid 5 \times 10^{-6} - (-3 \times 10^{-6}) \mid = 8 \times 10^{-6}\, C$$

We know,

$$p = q_{eff} \times d$$

$$p = (8 \times 10^{-6}\, C) \times (0.04\, m) = 3.2 \times 110^{-7}\, C.m$$

Thus, the dipole moment of the system is $3.2 \times 110^{-7}\, C.m$.

Polarization Density (P):
Polarization density is the dipole moment per unit volume of a dielectric material. As a vector quantity, it indicates both magnitude and direction, measuring the degree of polarization in response to an applied electric field.

$$P = \frac{p}{V} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (ii)$$

Where:

- **P** = Polarization density **(C/m²)**
- ***p*** = Dipole moment **(C·m)**
- ***V*** = Volume of the material **(m³)**

Putting the value of p from **Eq.** (i),

$$P = \frac{q \cdot d}{V} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (iii)$$

The polarization density P is a vector quantity that has both direction and magnitude, direction of P is aligned with the applied electric field. Measured in coulombs per square meter **(C/m²)**, it increases with the strength of the electric field, reflecting how internal dipoles in a material align with the external field.

- ✓ Follow this link for better understanding: https://www.youtube.com/watch?v=_Xwtkw7tA4I

Problem 3:
Consider a dielectric material that contains 5×10^{-22} dipoles per cubic meter, each having a dipole moment of $3 \times 10^{-30} C.m$. Calculate the polarization density P of the material.

Solution:

Given:
Number of dipoles per unit volume $V = 5 \times 10^{22}\, m^3$
Dipole moment $p = 3 \times 10^{-30} C.m$

The polarization density P is given by the product of the dipole moment and the number of dipoles per unit volume:

$$P = \frac{p}{V}$$

Substituting the values:

$$P = \frac{3 \times 10^{-30}\, C.m}{5 \times 10^{22}\, m^3} = 6 \times 10^{-33} C/\, m^2$$

Relation to Electric Field (E):

Polarization density ($\boldsymbol{P}$) is a fundamental concept in electromagnetism that quantifies the electric dipole moment per unit volume in a dielectric material. It indicates how much a material polarizes in response to an external electric field, reflecting the alignment of its internal dipoles. This concept is crucial for understanding the behavior of dielectric materials in electrical and electronic applications.

The polarization density is related to the electric field $\boldsymbol{E}$ through the electric susceptibility (χ) of the material as,

$$\boldsymbol{P} = \epsilon_0 \chi_e \boldsymbol{E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{iv})$$

Where:

- $\boldsymbol{\epsilon_0}$ = Vacuum permittivity (approximately 8.854×10^{-12} **F/m**)
- $\boldsymbol{\chi_e}$ = Electric susceptibility (dimensionless)
- **P** = Polarization density (**C/m²**)

Electric susceptibiliy χ [$\boldsymbol{\chi_e = \epsilon_r - 1}$] is a key property of dielectric materials that measures how easily they polarize in response to an external electric field. It indicates the degree of polarization relative to the strength of the applied field.Putting the value of $\boldsymbol{P}$ from Eq. (iii) into Eq. (iv), we get,

$$\frac{\boldsymbol{q \cdot d}}{\boldsymbol{V}} = \epsilon_0 \chi_e \boldsymbol{E}$$

Or, $\boldsymbol{\epsilon_0 \chi_e E} = \frac{q \cdot d}{V}$

Or, $\boldsymbol{E} = \frac{q \cdot d}{V \times \epsilon_0 \chi_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{v})$

Vacuum permittivity $\boldsymbol{\epsilon_0}$ is the permittivity of free space, meaning it describes how much electric field is generated per unit charge in a vacuum. It relates electric field $\boldsymbol{E}$ and electric displacement field $\boldsymbol{D}$ in free space.

Thus, the Electric Field ($\boldsymbol{E}$) for the polarization of dielectric matter can be calculated by using Eq. **(v).**

***Problem 4:**

A dielectric material is placed in an external electric field of $\boldsymbol{2 \times 10^5 V.m^{-1}}$. If the dielectric constant of the material is $\boldsymbol{\epsilon_r = 5}$, and the permittivity of free space is $\boldsymbol{\epsilon_0 = 8.85 \times 10^{-12}\ C^2.N.m^{-2}}$, calculate the polarization density $\boldsymbol{P}$ in the dielectric.

Solution:

Given:

Electric field $\boldsymbol{E = 2 \times 10^5\ V.m^{-1}}$
Dielectric constant $\boldsymbol{\epsilon_r = 5}$
The permittivity of free space $\boldsymbol{\epsilon_0 = 8.85 \times 10^{-12}\ C^2.N.m^{-2}}$

The polarization density $\boldsymbol{P}$ is related to the electric susceptibility $\boldsymbol{\chi_e}$ by the following relation:

$$\boldsymbol{P = \epsilon_0 \chi_e E}$$

The electric susceptibility $\boldsymbol{\chi_e}$ is related to the dielectric constant by:

$$\boldsymbol{\chi_e = \epsilon_r - 1}$$

$$\boldsymbol{\chi_e = 5 - 1 = 4}$$

Now, substitute the values to find $\boldsymbol{P}$:

$$P = \epsilon_0 \chi_e E = \{(8.85 \times 10^{-12}) \times 4 \times (2 \times 10^5)\}\, C.m^2 = 7.08 \times 10^{-6}\, C.m^2$$

**** Dielectric Constant (ε_r):**

The dielectric constant, or relative permittivity ($\boldsymbol{\varepsilon_r}$), is a measures a material's ability to store electrical energy in an electric field compared to a vacuum. It indicates how well the material can polarize, reducing the field's strength inside it.

The dielectric constant is defined as the ratio of the permittivity of the material ($\boldsymbol{\varepsilon}$) to the permittivity of free space ($\boldsymbol{\varepsilon_0}$):

$$\boldsymbol{\varepsilon_r} = \frac{\varepsilon}{\varepsilon_0} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (\boldsymbol{vi})$$

Where:

- $\boldsymbol{\varepsilon}$ is the permittivity of the material.
- $\boldsymbol{\varepsilon_0}$ is the permittivity of free space, approximately 8.854×10^{-12} **F/m**.

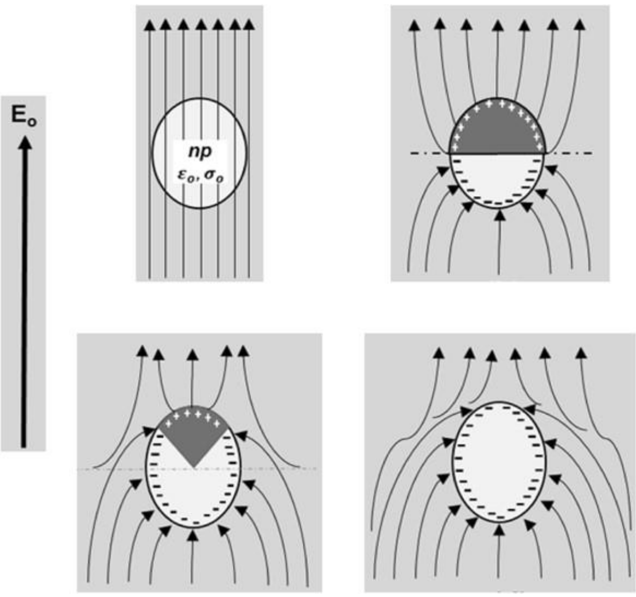
The dielectric constant is a dimensionless ratio; it is defined as 1 for a vacuum and is greater than 1 for all other materials. This value indicates a material's ability to reduce the internal electric field.

When an external electric field is applied, charges within the material shift slightly, creating electric dipoles. This process, known as polarization, generates an opposing electric field that diminishes the original field's strength. A higher dielectric constant means the material can polarize more effectively, enhancing its capacity to store electrical energy.

This property is crucial for designing capacitors, insulators, and various electronic components, as it allows for improved energy storage and control of electrical signals, making dielectric materials vital in many technological applications.

*** Electric Displacement Field ($\boldsymbol{D}$):**

The electric displacement field, or electric flux density (symbolized as ($\boldsymbol{D}$), is a vector field that describes how electric fields behave in materials, especially dielectrics and insulators where charges are bound.



The electric displacement field relates the electric field $\boldsymbol{E}$, the free charge in a system, and the polarization of the material. It is defined by the equation:

$$\boldsymbol{D} = \boldsymbol{\varepsilon_0 E} + \boldsymbol{P} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (\boldsymbol{vii})$$

Where:

- $\boldsymbol{D}$ is the electric displacement field (measured in **C/m²**)
- $\boldsymbol{\varepsilon_0}$ is the permittivity of free space (8.854×10−12 **F/m**
- $\boldsymbol{E}$ is the electric field (measured in **V/m**)
- $\boldsymbol{P}$ is the polarization of the material, which describes how much the material is polarized in response to the applied electric field

***** Relationship Between E, D, and P:**
The relationship between electric field $\boldsymbol{E}$, electric displacement field $\boldsymbol{D}$, and polarization $\boldsymbol{P}$ in the context of electromagnetism can be described by the following equations.
The electric field $\boldsymbol{E}$ represents the force per unit charge exerted on a charged particle in space. It is measured in volts per meter (**V/m**).
The electric displacement field relates the electric field $\boldsymbol{E}$, the free charge in a system, and the polarization of the material.

It is defined by the equation:

$$\boldsymbol{D = \varepsilon_0 E + P} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (\boldsymbol{vii})$$

Polarization $\boldsymbol{P}$ represents the electric dipole moment per unit volume of the material. It arises due to the alignment of dipoles within a dielectric material when exposed to an electric field.
For a linear, homogeneous dielectric material, the polarization $\boldsymbol{P}$ is typically proportional to the electric field $\boldsymbol{E}$:

$$\boldsymbol{P = \epsilon_0 \chi_e E} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (\boldsymbol{viii})$$

where $\boldsymbol{\epsilon_0}$ is the electric susceptibility of the material, which quantifies how easily the material becomes polarized in response to $\boldsymbol{E}$.

Substituting $\boldsymbol{P}$ into the equation for $\boldsymbol{D}$, we get:

$$\boldsymbol{D = \varepsilon_0 E + \epsilon_0 \chi_e E}$$

$$\boldsymbol{D = \varepsilon_0 (1 + \chi_e) E} \dots\dots\dots\dots\dots\dots\dots\dots (\boldsymbol{ix})$$

This can be simplified to:

$$\boldsymbol{D = \epsilon E} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (\boldsymbol{x})$$

where $\boldsymbol{\epsilon = \epsilon_0 (1 + \chi_e)}$ is the permittivity of the material.

*** **Problem 5:**
A parallel plate capacitor has an electric displacement field $\boldsymbol{D = 3 \times 10^{-5}\ C.m^{-2}}$, in a dielectric material with a dielectric constant $\boldsymbol{\epsilon_r = 4}$. Calculate the polarization density $\boldsymbol{P}$ in the material.

Solution:

Given:
Electric displacement field $\boldsymbol{D = 3 \times 10^{-5}\ C.m^{-2}}$
Dielectric constant $\boldsymbol{\epsilon_r = 4}$
The permittivity of free space $\boldsymbol{\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C}^2.\mathrm{N}.m^{-2}}$

The electric displacement field $\boldsymbol{D}$ is related to the polarization density $\boldsymbol{P}$ and the electric field $\boldsymbol{E}$ by the equation:

$$\boldsymbol{D = \varepsilon_0 E + P} \dots\dots\dots\dots\dots\dots (\boldsymbol{i})$$

We can also express $\boldsymbol{D}$ in terms of $\boldsymbol{\epsilon_r}$ and $\boldsymbol{E}$ as:

$$\boldsymbol{D = \varepsilon_r \varepsilon_0 E} \dots \dots \dots \dots \dots \dots (\boldsymbol{ii})$$

Now, solving for $\boldsymbol{P}$ from $\boldsymbol{Eq.}(\boldsymbol{i})$:

$$\boldsymbol{P = D - \varepsilon_0 E} \dots \dots \dots \dots \dots \dots (\boldsymbol{iii})$$

First, calculate the electric field $\boldsymbol{E}$ from $\boldsymbol{Eq.}(\boldsymbol{ii})$:

$$\boldsymbol{E = \frac{D}{\varepsilon_r \varepsilon_0}}$$

$$\boldsymbol{E = \frac{3 \times 10^{-5}}{4 \times 8.85 \times 10^{-12}} \approx 8.47 \times 10^{5}\, V.m^{-1}}$$

Now calculate the polarization density by putting the value of $\boldsymbol{Eq.}(\boldsymbol{ii})$ into $\boldsymbol{Eq.}(\boldsymbol{iii})$ we get:

$$\boldsymbol{P = \varepsilon_r \varepsilon_0 E - \varepsilon_0 E}$$

$$\boldsymbol{= \varepsilon_0 E(\varepsilon_r - 1)}$$

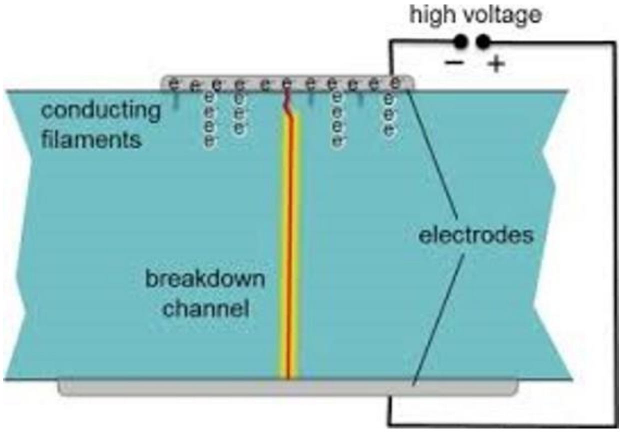
$$= \left\{\boldsymbol{8.85 \times 10^{-12} \times 8.47 \times 10^{5} \times (4 - 1)}\right\} \mathbf{C.}\boldsymbol{m^{-2}}$$

$$= \boldsymbol{2.25 \times 10^{-5}}\, \mathbf{C.}\boldsymbol{m^{-2}}$$

Thus, the polarization density $\boldsymbol{P}$ is $\boldsymbol{2.25 \times 10^{-5}}\, \mathbf{C.}\boldsymbol{m^{-2}}$

Dielectric Breakdown:

Dielectric breakdown is a phenomenon that occurs in insulating materials (dielectrics) when they are subjected to an electric field strong enough to cause a sudden increase in electrical conductivity, effectively transforming the material from an insulator to a conductor. This leads to the failure of the dielectric to block electric current, which can cause damage to electronic components and other systems.



<u>Mechanism of Dielectric Breakdown:</u>

In normal conditions, a dielectric material is an insulator because its electrons are tightly bound to atoms, preventing the free movement of charges. However, when an extremely high electric field is applied to the dielectric, the following occurs:

i. **Electron Acceleration**: The strong electric field accelerates free electrons within the material. Even though dielectrics have few free electrons under normal conditions, thermal excitation or impurities may release a small number of them.

ii. **Collisional Ionization**: As these accelerated electrons move through the dielectric, they gain enough energy from the electric field to knock other bound electrons out of their atomic orbits, leading to ionization. The newly freed electrons, in turn, are also accelerated, creating a chain reaction known as an avalanche breakdown.

iii. **Conductive Path**: The avalanche of free electrons results in a surge of electrical conductivity, creating a conductive path through the material. At this point, the dielectric no longer behaves as an insulator but as a conductor, allowing current to flow through it.

iv. **Thermal Effects**: The sudden flow of current can also cause localized heating, leading to physical damage such as melting, carbonization, or even catastrophic failure of the material.

Polarization Energy in Dielectrics:

Polarization energy in dielectrics refers to the potential energy stored in a dielectric material when it becomes polarized by an external electric field.

When a dielectric material is subjected to an external electric field $\boldsymbol{E}$, the electric dipoles inside the material tend to align with the field. This alignment causes the formation of bound charges within the dielectric, which, in turn, generates the polarization vector $\boldsymbol{P}$. The induced polarization leads to the storage of electrostatic energy in the dielectric medium.

In a dielectric material, the total electrostatic energy can be divided into two parts:

i. The energy due to the external electric field.

ii. The energy stored in the dielectric because of its polarization.

The **polarization energy density** (energy per unit volume) stored in the dielectric can be expressed using the relationship between the electric displacement field $\boldsymbol{D}$, the electric field $\boldsymbol{E}$, and the polarization $\boldsymbol{P}$.

For a linear dielectric, where polarization $\boldsymbol{P}$ is proportional to the electric field $\boldsymbol{E}$, the total electrostatic energy density u stored in the material is given by:

$$\boldsymbol{u} = \frac{1}{2}\boldsymbol{E} \cdot \boldsymbol{D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{xi})$$

Since $\boldsymbol{D} = \boldsymbol{\varepsilon_0 E} + \boldsymbol{P}$, this can also be written as:

$$\boldsymbol{u} = \frac{1}{2}\boldsymbol{E} \cdot (\boldsymbol{\varepsilon_0 E} + \boldsymbol{P}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{xii})$$

$$\boldsymbol{u} = \frac{1}{2}\boldsymbol{\varepsilon_0 E^2} + \frac{1}{2}\boldsymbol{E}.\boldsymbol{P} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{xiii})$$

where:

- $\boldsymbol{\varepsilon_0}$ is the permittivity of free space
- $\boldsymbol{E}$ is the external electric field
- $\boldsymbol{P}$ is the polarization vector

The first term, $\frac{1}{2}\boldsymbol{\varepsilon_0 E^2}$, is the energy stored in the electric field in the absence of the dielectric, while the second term, $\frac{1}{2}\boldsymbol{E}.\boldsymbol{P}$, represents the additional energy due to the polarization of the dielectric.

*** **Problem 6:**

A dielectric material is placed in a uniform electric field of strength $\boldsymbol{E} = \mathbf{1.5} \times \mathbf{10^5}\ \boldsymbol{V.m^{-1}}$. The relative permittivity (dielectric constant) of the material is $\boldsymbol{\epsilon_r} = \mathbf{3}$, and the permittivity of free space is $\boldsymbol{\epsilon_0} = \mathbf{8.85} \times \mathbf{10^{-12}}\ \mathbf{C^2.N.}\boldsymbol{m^{-2}}$. Calculate the energy density stored in the dielectric material.

Solution:

Given:

Electric field $\boldsymbol{E} = \mathbf{1.5} \times \mathbf{10^5}\ \boldsymbol{V.m^{-1}}$
Relative permittivity $\boldsymbol{\epsilon_r} = \mathbf{3}$
The permittivity of free space $\boldsymbol{\epsilon_0} = \mathbf{8.85} \times \mathbf{10^{-12}}\ \mathbf{C^2.N.}\boldsymbol{m^{-2}}$

The energy density $\boldsymbol{u}$ stored in a dielectric in the presence of an electric field is given by:

$$\boldsymbol{u} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\varepsilon_r \varepsilon_0 E^2}$$

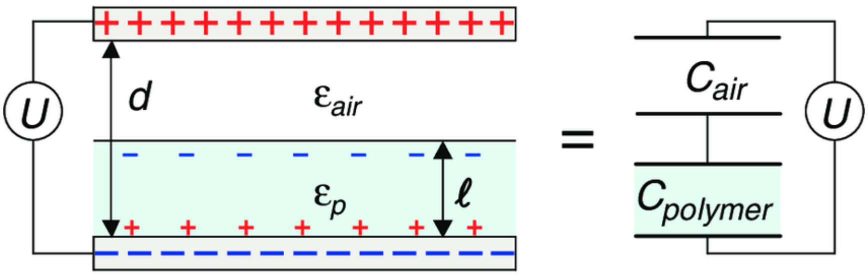
Substitute the given values:

$$\boldsymbol{u} = \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{3} \times (\mathbf{8.85} \times \mathbf{10^{-12}}) \times (\mathbf{1.5} \times \mathbf{10^5})^2\ \mathbf{J.}\boldsymbol{m^{-3}} = \mathbf{2.98} \times \mathbf{10^{-1}}\mathbf{J.}\boldsymbol{m^{-3}}$$

Thus, the energy density stored in the dielectric material is approximately $\mathbf{0.298\ J.}\boldsymbol{m^{-3}}$

Polarization Force on Dielectrics:

The polarization force on a dielectric material occurs when the dielectric is placed in a non-uniform electric field. This force arises due to the interaction between the induced dipoles in the dielectric and the varying strength of the electric field. The force tends to pull the dielectric toward regions of higher electric field intensity, a phenomenon commonly observed in capacitors and other electric field setups involving dielectrics.



In a uniform electric field, a dielectric polarizes as its molecules form dipoles, but the balanced forces result in no net movement. In a non-uniform field, however, varying field strength exerts unequal forces on the dipoles, creating a net force that pulls the dielectric toward the stronger field. This polarization force per unit volume, ($\boldsymbol{f}$), is derived from the gradient of the electric field energy density in the dielectric.

For a dielectric with polarization $\boldsymbol{P}$ in a non-uniform electric field, the force per unit volume $\boldsymbol{f}$ can be expressed as:

$$\boldsymbol{f} = (\boldsymbol{P} \cdot \boldsymbol{\nabla})\boldsymbol{E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{xiv})$$

Here:

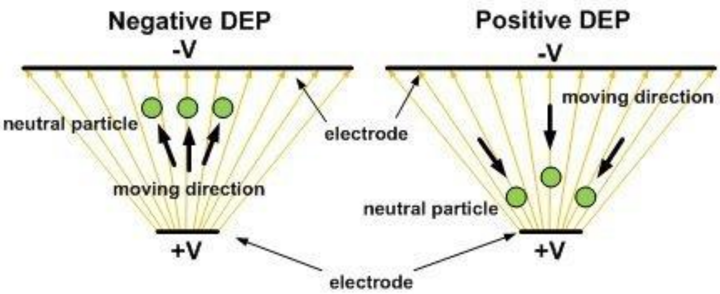
- $\boldsymbol{P}$ is the polarization vector (electric dipole moment per unit volume)
- $\boldsymbol{E}$ is the electric field
- $\boldsymbol{\nabla}$ represents the spatial variation (gradient) of the electric field

This equation implies that the force is proportional to both the polarization ***P*** and the gradient of the electric field. In regions where the electric field changes more rapidly, the force on the dielectric will be stronger.

Examples of Polarization Force Applications

i. **Capacitor Systems**: In capacitors, a dielectric experiences a force pulling it into the stronger electric field between the plates, a principle used in devices like variable capacitors.

ii. **Dielectrophoresis**: When a neutral dielectric particle is placed in a non-uniform electric field, the polarization force moves it toward higher field strength. This dielectrophoresis effect is used in biotechnology to manipulate cells and particles.



iii. **Optical Tweezers**: In light, polarization forces can act on dielectric particles in a non-uniform electromagnetic field (like a focused laser), allowing precise manipulation of microscopic particles.