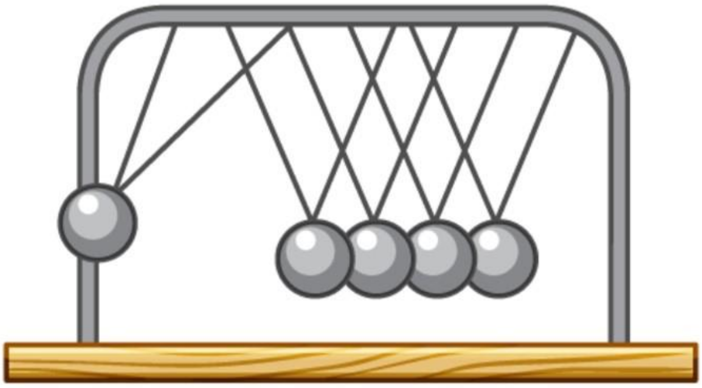
Chapter 1: Mechanics

Contents:

Mechanics:

Mechanics is a part of physics that studies how objects move and how forces affect them. It helps explain why a car speeds up when you press the gas pedal or how the Earth orbits the Sun. Mechanics is important because it helps us understand the world around us and is used in many areas like engineering, building machines, studying planets, and even understanding how our bodies move.



Key Concepts in Mechanics:

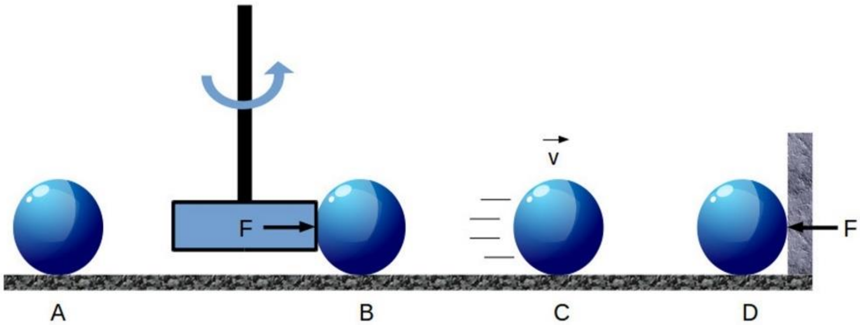
- Force
- Energy There are different kinds of energy:
    - Kinetic energy
    - Potential energy
- Work
- Momentum
- Torque

Everyday Examples of Mechanics:

- Driving a car
- Riding a bicycle
- Throwing a ball
- Building a bridge
- Mechanics is broadly divided into two main branches

1. **<u>Classical Mechanics</u>**

Classical mechanics describes the motion of macroscopic objects—from projectiles to machinery, planets, and even parts of biological systems. The key principles of classical mechanics were established by Isaac Newton in the 17th century, particularly with his formulation of the Three Laws of Motion and the concept of universal gravitation.

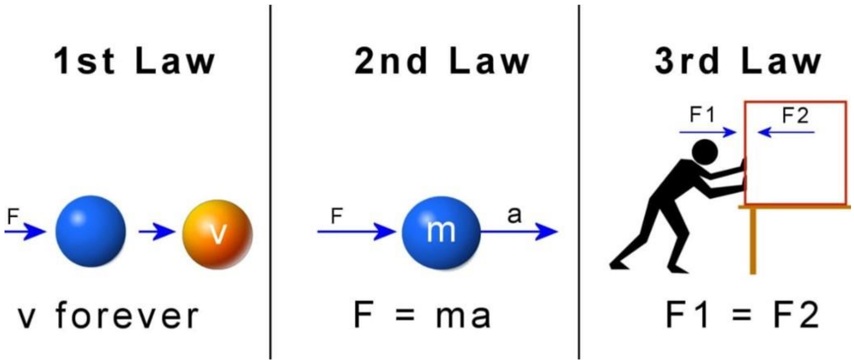


There are several subfields within classical mechanics:

- Kinematics
- Dynamics
- Statics
- Fluid Mechanics
- Rigid Body Mechanics

The fundamental equations in classical mechanics include **Newton's Laws of Motion**:

- **First Law:** A body at rest or in uniform motion will stay that way unless acted upon by an external force.
- **Second Law:** $\boldsymbol{F = ma}$ (Force equals mass times acceleration).
- **Third Law:** For every action, there is an equal and opposite reaction.

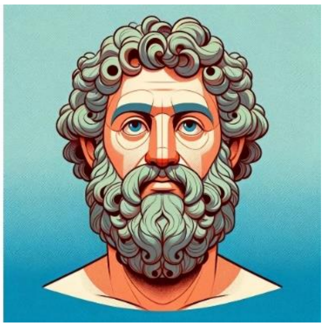


<u>History of Mechanics:</u>

The history of **mechanics** spans centuries and marks the evolution of our understanding of motion and forces:

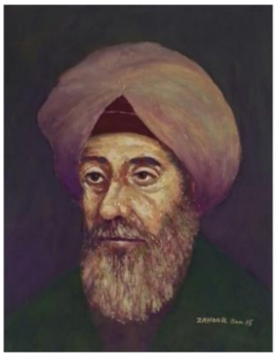
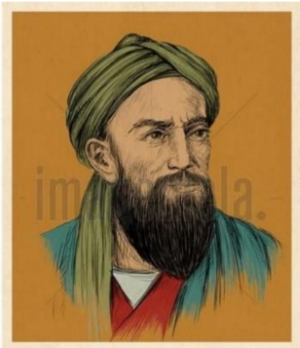
(i) **Ancient Era (Before 300 AD)**:

- Early civilizations like the Egyptians and Greeks used basic mechanical principles for building structures and machines.
- **Aristotle** theorized about motion, though many of his ideas were later proven incorrect.
- **Archimedes** laid the foundation for statistics with his work on levers and buoyancy.

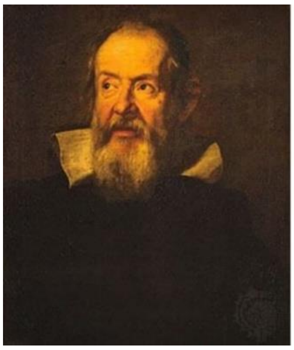
(ii) **Medieval Period (300–1500 AD)**:

- Islamic scholars like **Ibn Al-Haytham** and **Al-Biruni** advanced mechanics during Europe's intellectual decline.
- European thinkers began questioning Aristotle's theories, leading to ideas like **impetus** and momentum.

(iii)**Renaissance (1500–1700)**:

- **Galileo Galilei** revolutionized mechanics by proving that objects fall at the same rate regardless of mass and developed the concept of **inertia**.
- **Johannes Kepler** used mechanics to describe planetary motion.

(iv)**Classical Mechanics (1600–1800)**:

- **Isaac Newton** formalized mechanics with his **Three Laws of Motion** and the **Law of Universal Gravitation** in his work *Principia* (1687). This became the foundation for classical mechanics.

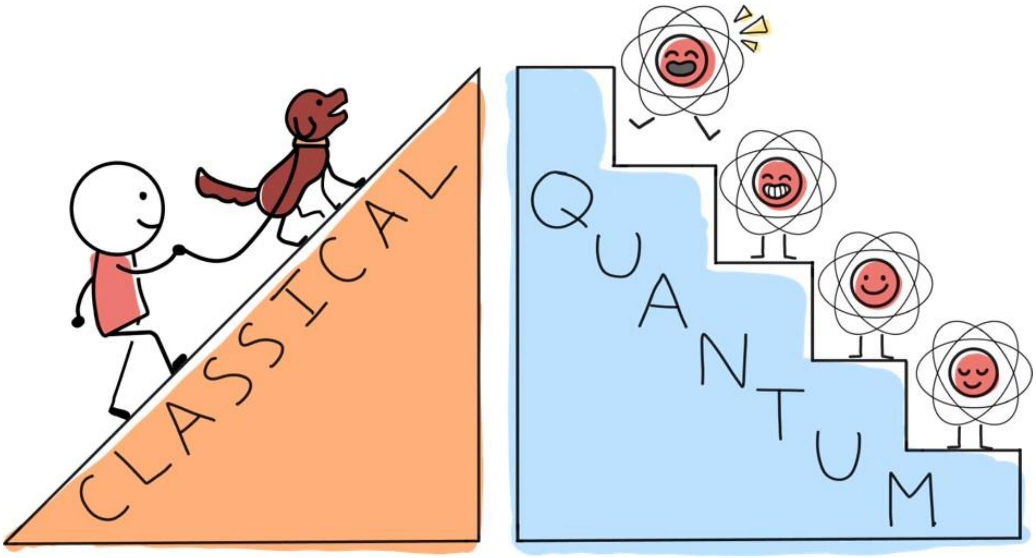
(v) **Modern Developments (1700–1900)**:

- **Joseph-Louis Lagrange** and **Pierre-Simon Laplace** refined Newton's work into **Lagrangian mechanics** and developed tools for solving complex mechanical problems.
- Mechanics became more analytical and mathematically structured, paving the way for engineering and technological advances.

From ancient philosophies to Newtonian physics and beyond, mechanics evolved into a precise science explaining how forces shape the movement of everything from everyday objects to celestial bodies.

2. <u>Quantum Mechanics</u>:

Quantum Mechanics is a fundamental theory in physics that describes the behavior of matter and energy at the smallest scales, such as atoms and subatomic particles. It provides a mathematical framework to understand the physical properties of particles, forces, and their interactions. Unlike classical mechanics, which deals with predictable and continuous motions of large objects, quantum mechanics operates on the principles of uncertainty, probability, and wave-particle duality.



Here's an overview of its core concepts:

(i) Wave-Particle Duality
(ii) Quantization of Energy
(iii) Heisenberg's Uncertainty Principle

***** <u>Difference between Classical and Quantum Mechanics:</u>**

Classical Mechanics (C.M.) and Quantum Mechanics (Q.M.) are two major branches of physics, but they apply to different scales and behaviors of objects. Here's a breakdown of their key differences:

Feature	Classical Mechanics (C.M.)	Quantum Mechanics (Q.M.)
Scope of Application	Deals with the motion of macroscopic objects (like cars, planets, or everyday	Focuses on the behavior of microscopic particles (like

	objects) and works well for objects moving at low speeds relative to the speed of light.	electrons, atoms, and photons) and applies to extremely small scales (the atomic and subatomic level).
Nature of Objects	Treats objects as solid and continuous. It assumes that objects have definite positions, velocities, and paths at any given time.	Treats particles as having both particle-like and wave-like properties (known as wave-particle duality). Particles are described by wave functions and do not have definite positions until measured.
Determinism vs. Probability	It is deterministic, meaning that if you know the initial conditions (position, velocity, etc.) of a system, you can predict its future behavior exactly.	It is probabilistic. You cannot precisely predict the outcome of an event; instead, you calculate the probability of finding a particle in a certain state or location using the Schrödinger equation.
Uncertainty Principle	You can measure both position and velocity (momentum) of an object with any desired accuracy at the same time.	According to the Heisenberg Uncertainty Principle, you cannot measure both the position and momentum of a particle with absolute precision at the same time. The more accurately you measure one, the less accurately you can measure the other.
Energy Quantization	Energy is continuous, meaning objects can have any amount of energy.	Energy is quantized, meaning particles can only have specific discrete energy levels. For example, electrons in atoms can only exist at certain energy levels, not in between.
Mathematical Framework	Described by Newton's Laws of Motion and equations like $\boldsymbol{F = ma}$ (force equals mass times acceleration). It often uses ordinary calculus for solving problems.	Described by complex mathematics, particularly the Schrödinger equation and operators acting on wave functions. It involves more advanced concepts like linear algebra and complex numbers.
Key Phenomena Explained	Explains everyday phenomena like the motion of vehicles, the trajectory of projectiles, planetary orbits, and mechanical systems.	Explains phenomena like the behavior of electrons in atoms, quantum tunneling, superposition, and entanglement, which have no classical equivalents.
Superposition and Entanglement	An object is in one state at a time (definite position, velocity, etc.).	Particles can exist in superposition, meaning they can be in multiple states at once until measured. Quantum entanglement allows particles to be instantaneously connected, regardless of distance.
Classical Limit	Accurately describes the macroscopic world.	Reduces to classical mechanics in the classical limit (i.e., when dealing with large-scale objects or large quantum numbers), as described by the correspondence principle.

*** **Newton's Laws of Motion**:

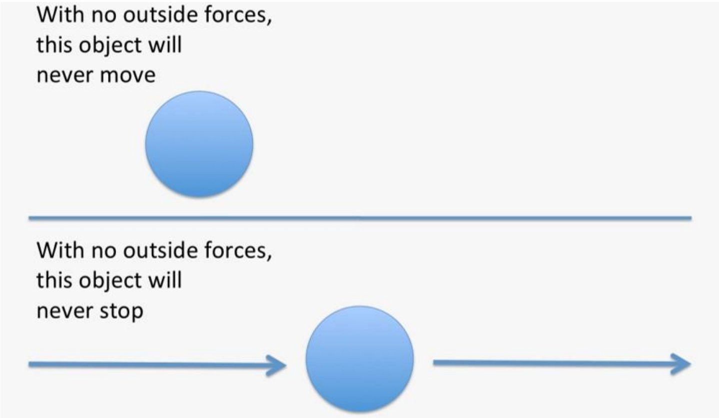
Newton's Laws of Motion are three fundamental principles that form the foundation of classical mechanics. These laws describe the relationship between the motion of an object and the forces acting on it. They were formulated by Sir Isaac Newton in his work ***Philosophiæ Naturalis*** Principia Mathematica (1687) and apply to most everyday objects, providing a framework for understanding the motion of objects under the influence of forces.

1. Newton's First Law of Motion (Law of Inertia):

This law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

Formal Statement:

"***An object at rest will remain at rest, and an object in motion will continue in motion with constant velocity unless acted upon by a net external force***."



Explanation:

If no net force is applied to an object, its velocity will not change. This means that an object in motion will not stop or change direction on its own, and an object at rest will not start moving unless a force acts on it. The concept of **inertia** means that objects resist changes to their state of motion. The greater the mass of an object, the more inertia it has, and the harder it is to change its motion.

Example:

A book lying on a table will remain at rest unless someone pushes it. A car moving on a straight road at a constant speed will continue moving unless the brakes are applied or the road creates friction.

2. Newton's Second Law of Motion (Law of Force and Acceleration):

This law explains how the velocity of an object changes when it is subjected to an external force.

Formal Statement:

"The acceleration of an object is directly proportional to the net force acting on the object and inversely proportional to its mass. The direction of the acceleration is in the direction of the net force."

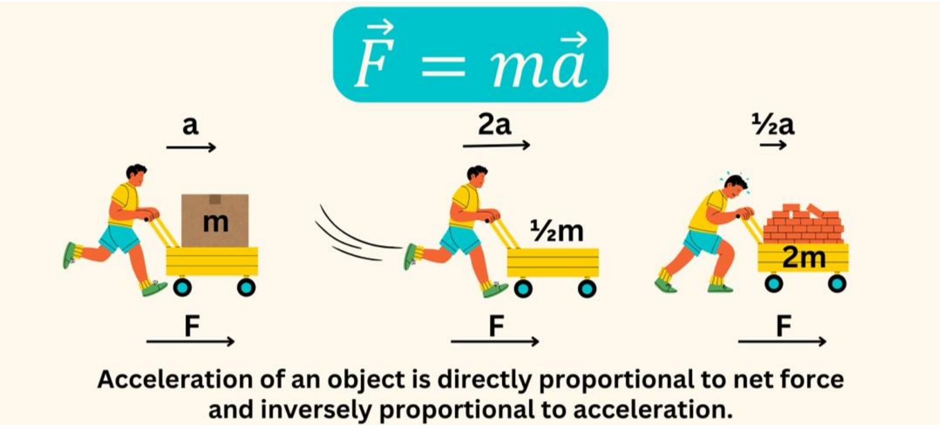
Mathematical Expression:

$$\boldsymbol{F = ma}$$

Where:

- ***F*** is the **net force** acting on the object (measured in newtons, ***N***)
- ***m*** is the **mass** of the object (measured in kilograms, ***kg***)
- ***a*** is the **acceleration** of the object (measured in meters per second squared, m/s^2).

Explanation:
- The greater the force applied to an object, the greater its acceleration.
- For a given force, objects with more mass will experience less acceleration, and objects with less mass will experience more acceleration.

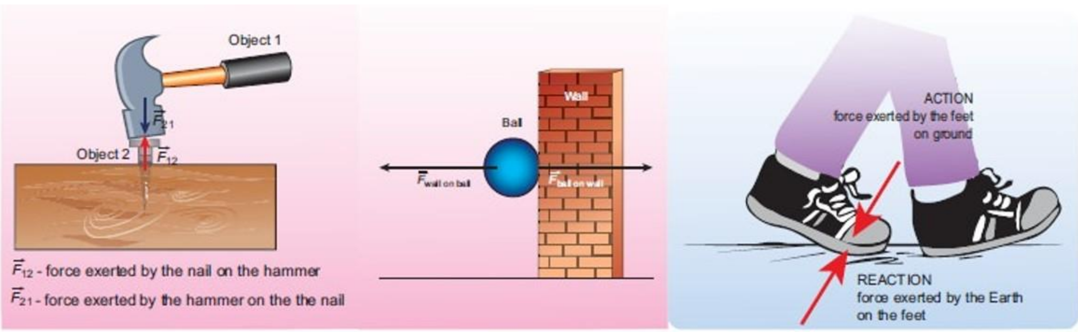


Example:
If you push a light object (like a shopping cart), it accelerates more quickly than a heavy object (like a car), given the same amount of force.

3. Newton's Third Law of Motion (Action and Reaction):
This law states that for every action, there is an equal and opposite reaction.

Formal Statement:

"For every action, there is an equal and opposite reaction."



(a) (b) (c)

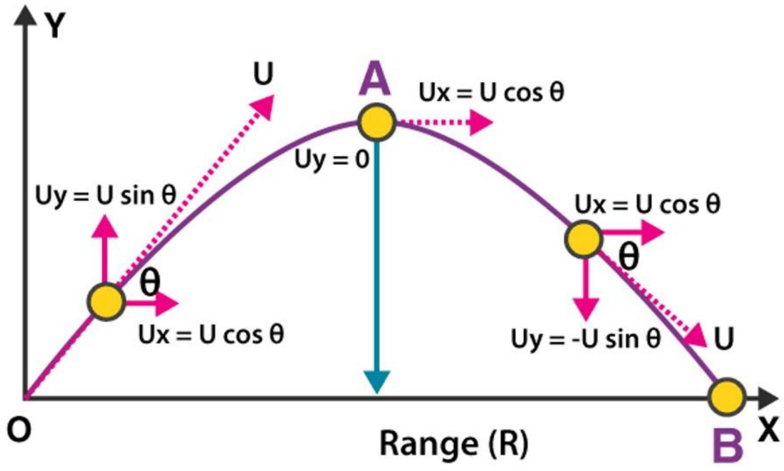
Explanation:
Whenever an object exerts a force on another object, the second object exerts an equal and opposite force back on the first object. These action-reaction force pairs are always equal in magnitude but act in opposite directions. However, they do not cancel out because they act on different objects.

Example:
When you push against a wall, the wall pushes back on you with an equal and opposite force, even though neither you nor the wall moves.

When a rocket launches, the engines push the exhaust gases down, and in return, the gases push the rocket up, allowing it to ascend.

Projectile motion:

When a particle is thrown obliquely near the earth's surface, it takes a curved, constant-acceleration path to the center of the earth (we assume that the particle remains close to the surface of the earth). The path of such a particle is referred to as a projectile, and its motion is referred to as projectile motion.



Characteristics of Projectile Motion:

- **Two-dimensional motion**: Projectile motion occurs in a plane, meaning it has both horizontal (x-axis) and vertical (y-axis) components.
- **Path of the Projectile (Parabolic trajectory)**: The shape of the path followed by the projectile is a **parabola**. This curved path results from the combination of uniform motion along the horizontal direction and accelerated motion along the vertical direction due to gravity.
- **No horizontal acceleration**: Once projected, if we neglect air resistance, the only force acting on the projectile is gravity, which acts vertically downward. Therefore, the horizontal velocity of the projectile remains **constant** throughout its flight.
- **Vertical acceleration due to gravity**: Gravity is the force that accelerates the object downward at a rate of 9.8 m/s^2 on Earth. This causes the vertical velocity to change over time:
  - ➢ When the projectile is moving upward, its vertical velocity decreases.
  - ➢ At the peak of its flight, the vertical velocity becomes zero.
  - ➢ As the projectile moves downward, its vertical velocity increases in the negative direction.
- **Independence of horizontal and vertical motions**: One of the most important features of projectile motion is that the horizontal and vertical motions are independent of each other. The only common factor between them is time. The horizontal displacement is unaffected by the vertical acceleration, and vice versa. This principle allows us to break down and analyze the problem in two parts.

*** Equations Governing Projectile Motion:

To describe projectile motion, we use the following equations derived from the basic kinematic equations.

(i) **Horizontal Motion** (Uniform motion):

Horizontal motion is defined as a projectile motion in a horizontal plane depending upon the force acting on it. For a short distance, the vertical and horizontal components of a projectile are perpendicular and independent of each other.

To calculate the distance covered by a projectile, the following equation is used;

$$\boldsymbol{Distance = speed \times Time}$$

$$\boldsymbol{x = v_x \cdot t}$$

Where:

- $\boldsymbol{x}$ is the horizontal displacement
- $\boldsymbol{v_x}$ is the horizontal velocity
- $\boldsymbol{t}$ is the time of flight

(ii) **Vertical Motion** (Accelerated motion):

Vertical motion refers to the movement of an object in the vertical (up or down) direction. This type of motion is typically studied in physics, especially under the influence of gravity.

Using kinematic equations:

$$\boldsymbol{y = v_{y0} \cdot t - \frac{1}{2} g \cdot t^2}$$

Where:

- $\boldsymbol{y}$ is the vertical displacement
- $\boldsymbol{v_{y0}}$ is the initial vertical velocity
- $\boldsymbol{g}$ is the acceleration due to gravity (approximately 9.8 m/s^2)
- $\boldsymbol{t}$ is the time of flight

(iii)**Time of flight**:

The total time the projectile spends in the air depends on the initial vertical velocity and the height from which it is projected.

For a projectile launched from ground level with an initial velocity $\boldsymbol{v_0}$ and angle $\boldsymbol{\theta}$, the time of flight is:

$$\boldsymbol{t_{flight} = \frac{2v_0 sin\theta}{g}}$$

(iv)**Range of the projectile**:

The horizontal distance traveled by the projectile, or the range, is given by:

$$\boldsymbol{R = \frac{{v_0}^2 sin(2\theta)}{g}}$$

Where $\boldsymbol{v_0}$ is the initial velocity and $\boldsymbol{\theta}$ is the launch angle. The range depends on both the speed of the launch and the angle.

(v) **Maximum height**:

The maximum height reached by the projectile can be found using:

$$\boldsymbol{H = \frac{{v_0}^2 sin^2\theta}{2g}}$$

This height depends on the vertical component of the initial velocity.

*** **Example problems:**

Imagine you throw a ball at an initial speed of $v_0 = 20\, m/s$ at an angle of $45°$ with the horizontal. The ball follows a parabolic path, reaching a certain maximum height before descending back to the ground.

- **Initial velocity components**:

Horizontal velocity:

$$v_x = v_0\, cos\theta = 20 \times cos(45°) = 14.14 m/s$$

Vertical velocity:

$$v_y = v_0\, sin\,\theta = 20 \times sin\,(45°) = 14.14 m/s$$

- **Time of flight**:

Using the equation for the time of flight:

$$t_{flight} = \frac{2v_0 sin\theta}{g} = \frac{2 \times 20 \times sin(45^{\circ})}{9.82} = 2.88\ s$$

So, the ball stays in the air for 2.88 seconds.

- **Range**:

Using the equation for range:

$$R = \frac{{v_0}^2 sin(2\theta)}{g} = \frac{(20)^2 sin(2 \times 45^{\circ})}{9.8} = 40.82\text{m}$$

Thus, the ball covers a horizontal distance of about 40.82 meters before hitting the ground.

- **Maximum height**:

Using the maximum height equation:

$$H = \frac{{v_0}^2 sin^2\theta}{2g} = \frac{(20)^2 sin^2(45^{\circ})}{2 \times 9.8} = 10.2\ m$$

The ball reaches a peak height of 10.2 meters.

Factors Affecting Projectile Motion:

1. **Initial speed**: Higher initial speed increases both the range and height of the projectile.
2. **Launch angle**: The launch angle determines the balance between horizontal and vertical components. The optimal angle for maximum range on flat ground (without air resistance) is $45°$.
3. **Height of release**: If the object is released from a height (like a cliff), this adds extra time for the projectile to be in the air, increasing its range.
4. **Air resistance** (ignored in basic models): In real-world scenarios, air resistance slows down the projectile, reducing its range and altering its trajectory.

Force:

Force is a fundamental concept in physics that describes any interaction that, when unopposed, changes the motion of an object. In simpler terms, a force can cause an object to accelerate, slow down, change direction, or remain in place. Force is a vector quantity, meaning it has both magnitude and direction.

The unit of force in the International System of Units (SI) is the **Newton (N)**, where 1 newton is the force required to accelerate a 1 kg mass by 1 m/s^2.

Types of Forces

1. **Gravitational Force:** The attractive force between two masses due to gravity. On Earth, the gravitational force is the weight of an object:

$$F_g = m \cdot g$$

   Where g is the acceleration due to gravity ($9.8\ m.s^2$).

2. **Frictional Force:** The force that opposes the relative motion of two surfaces in contact. It acts parallel to the surfaces and opposes the direction of motion.
3. **Normal Force:** The support force exerted by a surface, perpendicular to the surface. For example, when an object rests on a table, the table exerts an upward normal force equal to the object's weight.
4. **Tension Force:** The force transmitted through a string, rope, or cable when it is pulled tight by forces acting from opposite ends.
5. **Applied Force:** Any force that is applied to an object by a person or another object.
6. **Air Resistance:** A type of frictional force that acts on objects as they move through air, opposing their motion.

Example Problem: Force and Acceleration

A 5 kg box is pushed across a frictionless surface with a force of 20 N. What is the acceleration of the box?

Solution:

Using Newton's second law, $F = m \cdot a$:

$$a = \frac{F}{m} = \frac{20\,\mathrm{N}}{5\,\mathrm{kg}} = 4\,\mathrm{m/s}^2$$

Answer:

The box accelerates at $4\,\mathrm{m/s}^2$.

*** **Friction:**

Friction is a force that opposes the relative motion or the tendency of such motion between two surfaces in contact. It acts parallel to the surfaces in contact and always resists motion. Friction plays a crucial role in everyday activities, from walking to driving a car, as it allows us to grip surfaces and control movements.

Types of Friction:

(i) **Static Friction:**

This is the frictional force that prevents two surfaces from sliding past each other when a force is applied. It acts when there is no relative motion between the surfaces.

The maximum static friction is given by: $f_s = \mu_s \cdot N$

Where:

- f_s is the maximum static friction,
- μ_s is the coefficient of static friction (a dimensionless number depending on the surfaces),
- N is the normal force.

(ii) **Kinetic (Sliding) Friction:**

Once an object is in motion, kinetic friction takes over. It is generally less than static friction.

The force of kinetic friction is given by: $f_k = \mu_k \cdot N$

Where:

- f_k is the kinetic friction
- μ_k is the coefficient of kinetic friction
- N is the normal force

(iii)**Rolling Friction:**

- ➢ Rolling friction occurs when an object rolls over a surface, such as a wheel or a ball. It is generally much smaller than both static and kinetic friction.
- ➢ The force of rolling friction depends on factors like the nature of the surfaces in contact and the deformation of the rolling object.

(iv)**Fluid Friction (Air Resistance/Drag):**

- ➢ Fluid friction occurs when an object moves through a fluid (liquid or gas). For example, air resistance is a type of fluid friction that acts against objects moving through the air.

A 10 kg box is pushed along a horizontal surface with a coefficient of kinetic friction $\mu_k = 0.4$. What is the force of kinetic friction acting on the box?

Solution:

First, calculate the normal force:

$$N = m \cdot g = 10\,\text{kg} \cdot 9.8\,\text{m/s}^2 = 98\,\text{N}$$

Now, use the formula for kinetic friction:

$$f_k = \mu_k \cdot N = 0.4 \cdot 98\,\text{N} = 39.2\,\text{N}$$

Answer:

The kinetic friction acting on the box is $39.2\,\text{N}$.

Importance of Friction:

Friction is necessary for many daily activities:

- **Walking:** Static friction between your shoes and the ground prevents slipping.
- **Driving:** Tires rely on friction with the road to grip and move forward.
- **Braking:** Friction between the brake pads and wheels slows down vehicles.

Momentum:

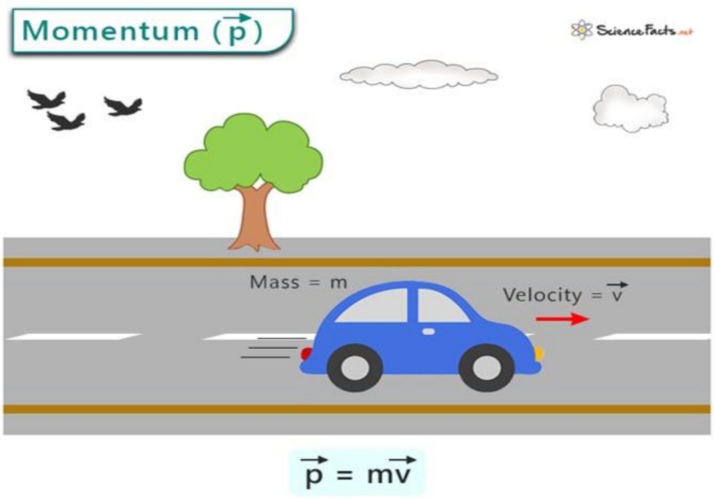
Momentum refers to the quantity of motion an object has and is a key concept in physics. It's typically defined as the product of an object's mass and velocity, represented by the equation:

$$\boldsymbol{p} = \boldsymbol{mv}$$

Where:

- $\boldsymbol{p}$ is momentum,
- $\boldsymbol{m}$ is mass,
- $\boldsymbol{v}$ is velocity

Momentum is a vector, meaning it has both magnitude and direction. The law of conservation of momentum states that the total momentum before and after an event (like a collision) remains constant in a closed system, without external forces. This principle is vital in analyzing collisions, explosions, and other physical systems.



A car with a mass of 1,200 kg is moving at a speed of 15 m/s.

Calculate the momentum of the car.

To calculate the momentum of the car, we can use the formula:

$$p = m \times v$$

where:

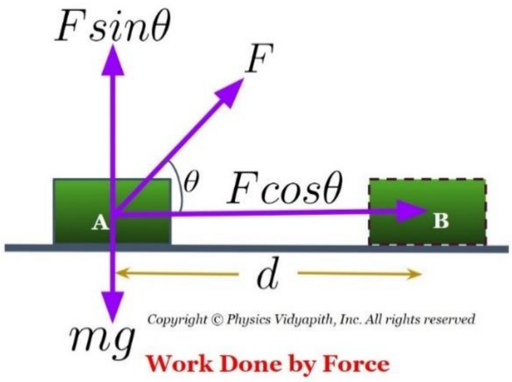
- p is momentum,
- m is mass (1,200 kg), and
- v is velocity (15 m/s).

*** **Work:**

Work in physics refers to the process of transferring energy by applying a force over a distance. It is a scalar quantity, meaning it only has magnitude and no direction. Work is done when a force causes an object to move in the direction of the force.

Formula for Work:

$$W = F \cdot d \cdot cos(\theta)$$

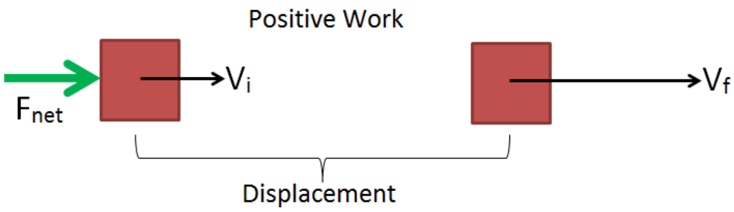


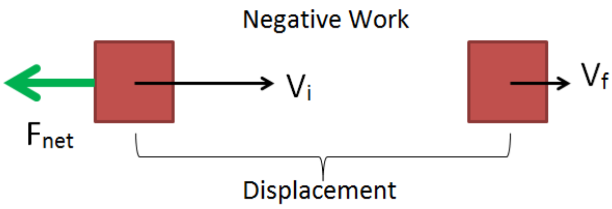
Where:

- W is the **work** done (measured in joules, J),
- F is the **force** applied (measured in newtons, N),
- d is the **distance** the object moves (measured in meters, m),
- θ is the angle between the force and the direction of motion

Work is only done if the object moves in the direction of the applied force'

- **Positive Work**: When the force applied is in the direction of the object's motion (e.g., pushing a box forward).
- **Negative Work**: When the force applied is opposite to the direction of motion (e.g., friction acting on a moving object).





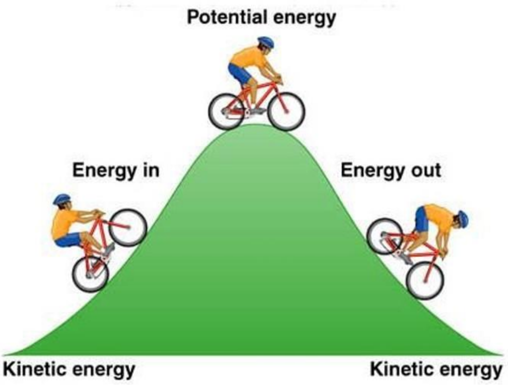
Example:
If you push a box with a force of 10 N over a distance of 5 meters in the same direction as the force, the work done is:

$$W = 10\,N \times 5\,m = 50\,J$$

This means 50 joules of energy is transferred to the box.

*** **Energy:**

Energy is the capacity to do work. It is a conserved quantity, meaning it can neither be created nor destroyed, only transformed from one form to another. There are many forms of energy, including mechanical energy, thermal energy, chemical energy, electrical energy, and more.



In mechanics, we primarily deal with **kinetic energy** and **potential energy**.

Types of energy:

1. Kinetic Energy (KE):

Kinetic energy is the energy an object possesses due to its motion.

Formula for Kinetic Energy:

$$K_E = \frac{1}{2}mv^2$$

Where:

- K_E is the **kinetic energy** (measured in joules, J)
- m is the **mass** of the object (measured in kilograms, kg)
- v is the **velocity** of the object (measured in meters per second, m/s).

Example:

A car with a mass of 1000 kg moving at a speed of 20 m/s has kinetic energy:

$$K_E = \frac{1}{2}mv^2 = \frac{1}{2} \times 1000 \times (20)^2 = 200,000\, J$$

2. Potential Energy (PE):

Potential energy is the energy stored in an object due to its position, shape, or configuration. The most common form is **gravitational potential energy**, which is energy due to an object's height above the ground.

The formula for Gravitational Potential Energy:

$$P_E = mgh$$

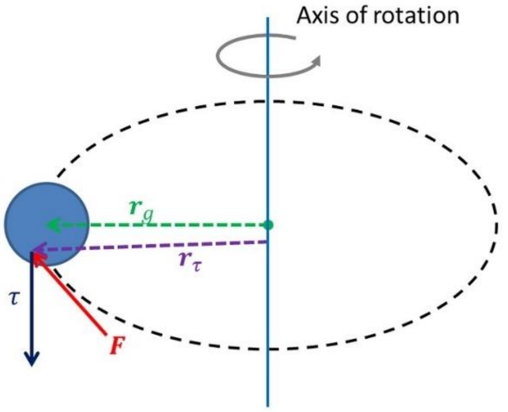
Where:

- P_E is the **potential energy** (measured in joules, J)
- m is the **mass** of the object (measured in kilograms, kg),
- g is the **acceleration due to gravity** (approximately $9.8\ m/s^2$ on Earth),
- h is the **height** of the object above a reference level (measured in meters, m).

Example:
A rock with a mass of 5 kg at a height of 10 meters above the ground has potential energy:

$$P_E = mgh = (5 \times 9.8 \times 10)J = 490\,J$$

Rotational body:
A rotational body refers to an object that rotates around an axis. Studying such bodies involves understanding the motion and forces that result from rotation, typically in the context of classical mechanics.



Here are a few examples:

- Earth
- Bicycle Wheel
- Spinning Top
- Fan Blades
- Planets and Satellites
- Wind Turbine
- Gyroscope
- Rotating Gears
- Potter's Wheel
- Hard Drives or CD/DVD Discs
- Car Tires
- Ferris Wheel
- Merry-Go-Round

Below are key ideas about rotational bodies, touching on their physical principles, real-world applications, and broader implications:

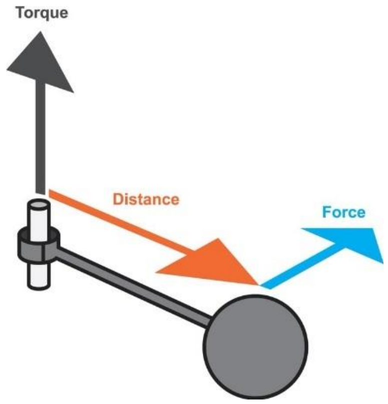
Axis of Rotation:
The **axis of rotation** is a central concept when discussing rotational bodies. This is an imaginary line around which the object spins. A body can rotate around an **internal axis** (like the Earth spinning on its own axis) or an **external axis** (like the Earth revolving around the Sun).

*** **Torque:**

Torque is the measure of the tendency of a force to cause an object to rotate about an axis. It's a fundamental concept in physics and engineering, especially in the context of rotational motion and mechanics. Torque is what causes objects to spin or turn and is essential for understanding how things like engines, gears, and even human muscles work.

"Torque is the rotational equivalent of linear force, determining how force causes an object to rotate around an axis."

Just as a force pushes or pulls an object in a straight line, torque twists or rotates an object around a pivot point or axis. The SI unit of torque is the Newton-meter (Nm).



Mathematically, torque $\boldsymbol{\tau}$ (tau) is expressed as:

$$\boldsymbol{Torque(\tau) = Force(F) \times Lever\ Arm(r)}$$

$$\boldsymbol{\tau = F \times r}$$

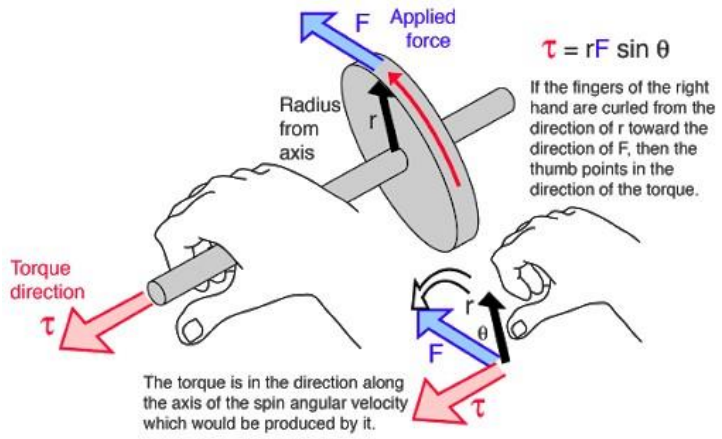
Where:

- $\boldsymbol{\tau}$ is the torque,
- $\boldsymbol{r}$ is the **lever arm** or **moment arm**—the distance from the axis of rotation to the point where the force is applied,
- $\boldsymbol{F}$ is the force applied, and
- $\times$ represents the cross product, indicating that torque is a vector quantity, which means it has both **magnitude** and **direction**.

Direction of Torque (Right-Hand Rule):

The direction of the torque vector can be determined using the **right-hand rule**. Here's how it works:

- Point your right hand's fingers in the direction of the force.
- Curl your fingers toward the axis of rotation.
- Your thumb, when extended, will point in the direction of the torque vector.



If the torque causes a counterclockwise rotation, the torque is considered **positive**. If it causes a clockwise rotation, the torque is considered **negative**.

Torque depends on three main factors:

- **Magnitude of the applied force**: The larger the force, the greater the torque, assuming the lever arm stays constant.
- **Distance from the pivot point (lever arm)**: The farther the force is applied from the axis of rotation, the larger the torque.
- **Angle of application**: The angle between the force vector and the lever arm also affects torque. If the force is applied perpendicular to the lever arm, torque is maximized. Mathematically, this relationship is:

$$\boldsymbol{\tau} = \boldsymbol{F} \times \boldsymbol{r}\, \mathbf{sin}(\boldsymbol{\theta})$$

Where:

- $\boldsymbol{\theta}$ is the angle between the force vector and the lever arm.

For maximum torque, $\boldsymbol{\theta}$ should be $\mathbf{90^\circ}$, since $\boldsymbol{sin}(\mathbf{90^\circ}) = \mathbf{1}$. When the force is applied at an angle less than 90°, the torque decreases.

Application of Torque:

Torque is essential in many real-world applications:

- Automotive Industry.
- Mechanical Tools
- Industrial Machinery
- Bicycles and Motorcycles
- Wind Turbines
- Electric Motors
- Aircraft and Spacecraft
- Sports and Biomechanics
- Clocks and Watches
- Robotics
- Construction Equipment
- Marine Applications

*** **Problem**

You apply a force of 50 N perpendicular to a door at a distance of 0.8 meters from the hinges. What is the torque exerted on the door?

Solution:

Given:

- Force $\boldsymbol{F = 50\,N}$
- Distance from the hinge (lever arm) $\boldsymbol{r = 0.8\,m}$
- Angle between the force and lever arm $\boldsymbol{\theta = 90^\circ}$ $\{\boldsymbol{sin(90^\circ) = 1}\}$.

The torque is given by:

$$\boldsymbol{\tau = F \times r \times \sin(\theta)}$$

$$\boldsymbol{\tau = 50\,N \times 0.8\,m \times \sin(90^\circ) = 50\,N \times 0.8\,m \times 1 = 40\,Nm}$$

The torque on the door is **40 Nm**.

*** **Moment of inertia:**

The moment of inertia is a measure of an object's resistance to rotational motion around a specific axis. It depends on the object's mass and the distribution of that mass relative to the axis of rotation.

The moment of inertia depends on:

- **Mass** of the object.
- **Distribution of the mass** relative to the axis of rotation mass farther from the axis increases the moment of inertia.

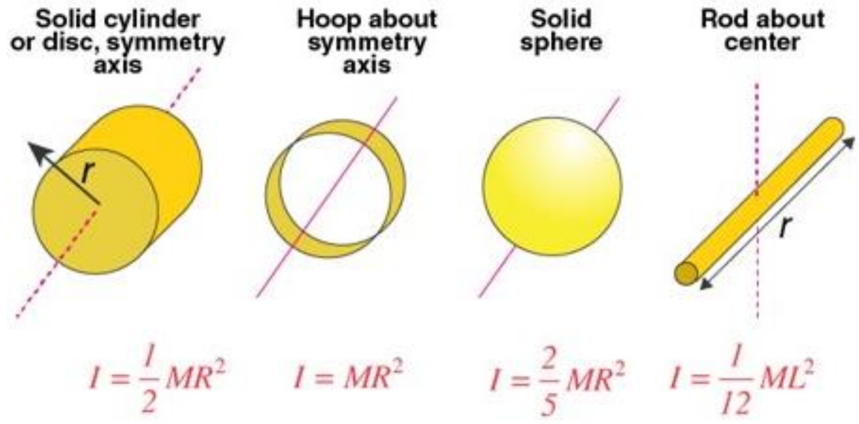
The moment of inertia $\boldsymbol{I}$ for a discrete system of point masses is mathematically defined as:

$$\boldsymbol{I = \sum m_i\, r_i^{\,2}}$$

Where:

- $\boldsymbol{m_i}$ is the mass of each particle or element
- $\boldsymbol{r_i}$ is the distance of each mass from the axis of rotation
- The summation $\sum$ indicates that you sum this value for every particle in the system.

In the International System of Units (SI), the moment of inertia is measured in **kilogram meter squared** $(\boldsymbol{kg \cdot m^2})$. This unit comes from the fact that the moment of inertia involves mass $(\boldsymbol{kg})$ and the square of distance $(\boldsymbol{m^2})$.



Moment of Inertia for Common Shapes:
The moment of inertia varies depending on the geometry of the object and the axis of rotation. Below are the moments of inertia for some common shapes with respect to their principal axes:

- **Solid Cylinder (Axis through the center)**:

$$I = \frac{1}{2}MR^2$$

Where: M is the mass and R is the radius.

- **Solid Sphere (Axis through the center)**:

$$I = \frac{2}{5}MR^2$$

- **Thin Rod (Axis through the center)**:

$$I = \frac{1}{12}ML^2$$

Where: L is the length of the rod.

- **Thin Rod (Axis through one end)**:

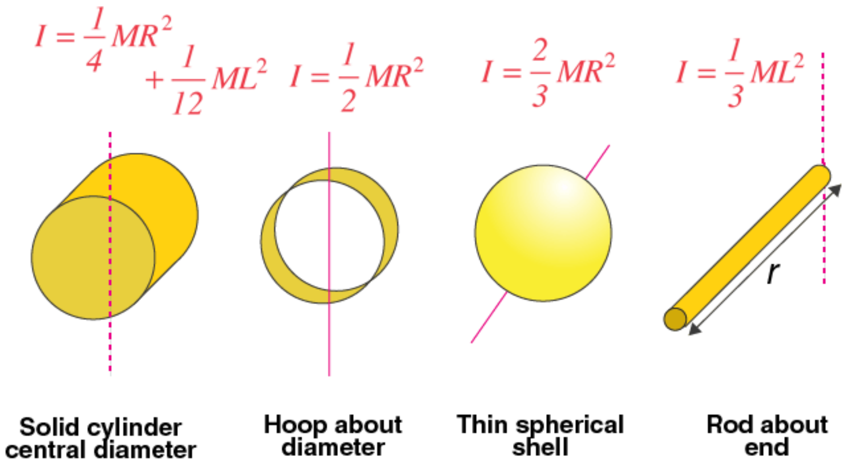
$$I = \frac{1}{3}ML^2$$

- **Hollow Cylinder (Axis through the center)**:

$$I = MR^2$$

- **Rectangular Plate (Axis along one edge)**:

$$I = \frac{1}{v}ML^2$$



***** Problem:**
Find the moment of inertia of a solid disk with a mass of 5 kg and a radius of 0.4 meters.

Solution:
Given:

- Mass $\boldsymbol{M = 5\,kg}$
- Radius $\boldsymbol{R = 0.4\,m}$

Using the formula:

$$\boldsymbol{I = \frac{1}{2}MR^2}$$

Substitute the values:

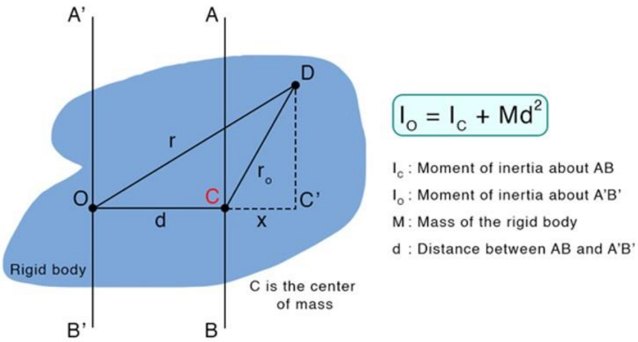
$$\boldsymbol{I = \frac{1}{2}MR^2 = \frac{1}{2} \times 5 \times (0.4)^2 = 0.4\,kg \cdot m^2}$$

The moment of inertia of the disk is $\boldsymbol{0.4\,kg \cdot m^2}$.

** **<u>Theorem of Moment of Inertia:</u>**

(i) Parallel Axis Theorem:

The **parallel axis theorem** allows us to calculate the moment of inertia about any axis parallel to an axis that passes through the object's center of mass. If the moment of inertia about the center of mass (axis $\boldsymbol{I_{cm}}$) is known, then the moment of inertia about any parallel axis located a distance ddd from the center of mass is:



$$I = I_{cm} + Md^2$$

Where:

- I is the moment of inertia about the new axis
- I_{cm} is the moment of inertia about the center of mass
- M is the mass of the object, and
- d is the distance between the center of mass and the new axis.

This theorem is particularly useful for finding the moment of inertia of objects where the axis of rotation is not at the center of mass.

*** Problem:**
A solid disk with a mass of 10 kg and a radius of 0.3 m has a moment of inertia about its center of mass given by $I_{cm} = \frac{1}{2}MR^2$. Calculate the moment of inertia of the disk about an axis that is 0.5 meters away from its center (parallel axis).

Solution:
Given:

- Mass M=10 kg
- Radius R=0.3 m
- Distance from center d=0.5 m

- $I_{cm} = \frac{1}{2} \times 10 \times (0.3)^2 = 0.45\ kg \cdot m^2$.

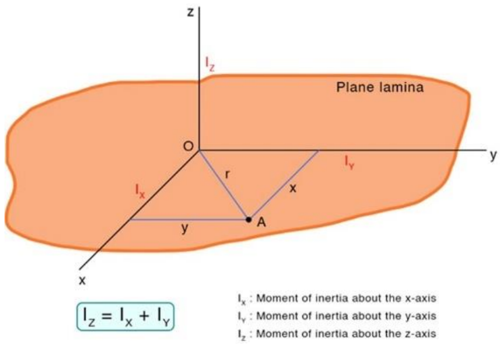
Now, apply the parallel axis theorem:

$$I = I_{cm} + (M \times d^2) = [0.45 + \{10 \times (0.5)^2\}] = 2.95\ kg \cdot m^2$$

Answer: The moment of inertia of the disk about the offset axis is $2.95\ kg \cdot m^2$.

(ii) **Perpendicular Axis Theorem:**

For flat, planar objects (thin, 2D objects like discs or plates), the **perpendicular axis theorem** can be used to calculate the moment of inertia about an axis perpendicular to the plane of the object.



The theorem states:

$$I_z = I_x + I_y$$

Where:

- I_z is the moment of inertia about the perpendicular axis (the axis coming out of the plane),
- I_x and I_y are the moments of inertia about two perpendicular axes lying in the plane of the object.

This theorem is especially useful for calculating moments of inertia of thin objects like discs.

*** **Problem:**
A thin rectangular plate has a mass of 4 kg, with dimensions 2 m along the xxx-axis and 3 m along the y-axis. Calculate the moment of inertia about an axis perpendicular to the plate through the center (I_z).

Solution: For a thin rectangular plate, the moments of inertia about the x-axis and y-axis are given by:

$$I_x = \frac{1}{12} \times M \times {L_y}^2$$

$$I_y = \frac{1}{12} \times M \times {L_x}^2$$

Where: L_x=2 m (length along the x-axis) and L_y=3 m (length along the y-axis).

Given:

- Mass $M = 4\,kg$
- $L_x = 2\,m$
- $L_y = 3\,m$

First, calculate I_x and I_y:

$$I_x = \frac{1}{12} \times M \times {L_y}^2 = \frac{1}{12} \times 4 \times (3)^2 = 3\,kg \cdot m^2$$

$$I_y = \frac{1}{12} \times M \times {L_x}^2 = \frac{1}{12} \times 4 \times (2)^2 = 1.33\,kg \cdot m^2$$

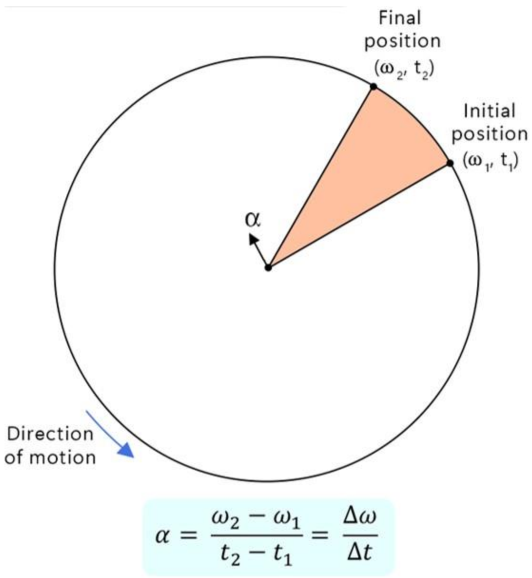
Now, apply the perpendicular axis theorem:

$$I_z = I_x + I_y = 3 + 1.33 = 4.33 = 4.33\ kg \cdot m^2$$

The moment of inertia about the z-axis is $\mathbf{4.33}\ \boldsymbol{kg \cdot m^2}$.

Angular Acceleration:

Angular acceleration is the rate of change of angular velocity over time. It describes how quickly an object's rotational speed is increasing or decreasing. In simpler terms, it measures how fast the rotational speed of an object changes around a particular axis.



Mathematically, angular acceleration α\alphaα is defined as:

$$\boldsymbol{\alpha = \frac{d\omega}{dt}}$$

Where:

- $\boldsymbol{\alpha}$ is the angular acceleration (measured in radians per second squared, rad/s^2)
- $\boldsymbol{\omega}$ is the angular velocity (in radians per second, rad/s)
- $\boldsymbol{t}$ is the time (in seconds).

Angular acceleration is a vector quantity, meaning it has both magnitude and direction. Its direction is determined by the change in angular velocity if an object is speeding up, the angular acceleration is in the same direction as the angular velocity, and if it's slowing down, it is in the opposite direction.

Problem:

A wheel starts from rest and reaches an angular velocity of 20 rad/s in 5 seconds. What is the angular acceleration?

Solution:

Given:

- Initial angular velocity $\boldsymbol{\omega_0 = 0\, rad/s}$
- Final angular velocity $\boldsymbol{\omega = 20\, rad/s}$
- Time $\boldsymbol{t = 5s}$

The angular acceleration is given by:

$$\alpha = \frac{d\omega}{dt} = \frac{\omega - \omega_0}{t} = \frac{(20-0)rad/s}{5s} = 4\,rad/s^{\,2}$$

The angular acceleration is $\boldsymbol{\alpha = 4\,rad/s^{\,2}}$.

*** **Moment of Inertia and Angular Acceleration:**

In rotational dynamics, torque is related to angular acceleration in the same way that force is related to linear acceleration in Newton's second law. The relationship is given by:

$$\tau = I \cdot \alpha$$
$$=> \alpha = \frac{\tau}{I}$$

Where:

- I is the **moment of inertia** (a measure of how much an object resists rotational acceleration)
- α is the **angular acceleration**.

This equation tells us that the greater the moment of inertia, the more torque is needed to achieve a certain angular acceleration.

***** Problem:**

A flywheel with a moment of inertia $I = 10\,kg \cdot m^2$ is spinning with an angular acceleration of $5\,rad/s^{\,2}$. What is the torque required to produce this angular acceleration?

Solution:

Given:

- Moment of inertia $I = 10\,kg \cdot m^2$
- Angular acceleration $\alpha = 5\,rad/s^{\,2}$

The torque is calculated as:

$$\tau = (10 \times 5) = 50\,N.m$$

The required torque is $\mathbf{50\,N.m}$.

Application of moment of inertia:

The **moment of inertia** has various practical applications in physics, engineering, and everyday life, especially in situations involving rotational motion. Here are some key applications:

1. Flywheels

- ➢ Energy Storage

2. Rotational Mechanics of Vehicles

- ➢ Cars and Motorcycles
- ➢ Bicycles

3. Balancing and Stability

- ➢ Aircraft and Spacecraft

- Skateboarding and Gymnastics

4. Structural Engineering
- Beams and Columns

5. Robotics
- Robotic Arms

6. Rotational Kinetic Energy in Mechanical Systems
- Machinery

7. Sports and Physical Activities
- Gymnastics and Diving
- Golf, Baseball, and Tennis

8. Wheels and Rolling Objects
- Moment of Inertia in Rolling

9. Pendulums
- Clock Mechanisms

10. Wind Turbines:
- Energy Generation

11. Medical Applications
- Prosthetics and Orthopedics

***** Practice more related mathematical problems as shown in the lecture for the final exam**