**Probability** means possibility. It is a branch of mathematics that deals with the occurrence of a random event. The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen. The meaning of probability is basically the extent to which something is likely to happen. This is the basic probability theory, which is also used in the probability distribution, where you will learn the possibility of outcomes for a random experiment. To find the probability of a single event to occur, first, we should know the total number of possible outcomes.

Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only the chance of an event to occur i.e., how likely they are going to happen, using it. Probability can range from 0 to 1, where 0 means the event to be an impossible one and 1 indicates a certain event. The probability of all the events in a sample space adds up to 1.

For example, when we toss a coin, either we get Head OR Tail, only two possible outcomes are possible (H, T). But when two coins are tossed then there will be four possible outcomes, i.e  $\{(H, H), (H, T), (T, H), (T, T)\}$ .

### Formula for Probability

The probability formula is defined as the possibility of an event to happen is equal to the ratio of the number of favourable outcomes and the total number of outcomes.

# Probability of event to happen P(E) = Number of favourable outcomes/Total Number of outcomes

Sometimes students get mistaken for "favourable outcome" with "desirable outcome". This is the basic formula. But there are some more formulas for different situations or events.

### Examples

# 1) There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?

Ans: The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

# 2) There is a container full of coloured bottles, red, blue, green and orange. Some of the bottles are picked out and displaced. Sumit did this 1000 times and got the following results:

- No. of blue bottles picked out: 300
- No. of red bottles: 200
- No. of green bottles: 450

• No. of orange bottles: 50

### a) What is the probability that Sumit will pick a green bottle?

Ans: For every 1000 bottles picked out, 450 are green.

Therefore, P(green) = 450/1000 = 0.45

#### b) If there are 100 bottles in the container, how many of them are likely to be green?

Ans: The experiment implies that 450 out of 1000 bottles are green.

Therefore, out of 100 bottles, 45 are green.

#### **Probability Tree**

The **tree diagram** helps to organize and visualize the different possible outcomes. Branches and ends of the tree are two main positions. Probability of each branch is written on the branch, whereas the ends are containing the final outcome. Tree diagrams are used to figure out when to multiply and when to add. We can see below a tree diagram for the coin:



### **Types of Probability**

There are three major types of probabilities:

- Theoretical Probability
- Experimental Probability
- Axiomatic Probability
- Conditional Probability
- Marginal Probability

# **Theoretical Probability**

It is based on the possible chances of something to happen. The theoretical probability is mainly based on the reasoning behind probability. For example, if a coin is tossed, the theoretical probability of getting a head will be  $\frac{1}{2}$ .

# **Experimental Probability**

It is based on the observations of an experiment. The experimental probability can be calculated based on the number of possible outcomes by the total number of trials. For example, if a coin is tossed 10 times and head is recorded 6 times then, the experimental probability for heads is 6/10 or, 3/5.

# **Axiomatic Probability**

In axiomatic probability, a set of rules or axioms are set which applies to all types. These axioms are set by Kolmogorov and are known as **Kolmogorov's three axioms**. With the axiomatic approach to probability, the chances of occurrence or non-occurrence of the events can be quantified.

# **Conditional Probability**

Conditional Probability is the likelihood of an event or outcome occurring based on the occurrence of a previous event or outcome.

It represents the **likelihood** of an **event** under a **specific condition** or context. Conditional probability is denoted as P(A|B), where A is the event of interest, and B is the given condition or context.

The formula for conditional probability is as follows:

# P(A|B)=P(A and B)/P(B)



# Example:

you draw two cards from the deck without replacement. Find the conditional probability of drawing a red card on the second draw (event A) given that you drew a red card on the first draw (event B).

- 1. The probability of drawing a red card on the first draw (P(B)) is 26/52 because there are 26 red cards in a standard deck of 52 cards.
- 2. The probability of drawing a red card on the second draw (P(A)) is 25/51 because there are now 25 red cards left out of 51 cards after the first draw.
- 3. The joint probability of drawing a red card on both draws (P(A and B)) is (26/52) \* (25/51)

You can then find the conditional probability:

P(A|B)=P(A and B)/P(B)

=(26/52)\*(25/51)/(26/52)

=25/51

# Marginal probability

the probability of a single event or outcome without considering other events or outcomes.

It is derived from a joint probability distribution that involves multiple variables or events.

In the context of a probability distribution with multiple variables, the marginal probability of a specific variable is calculated by **summing** or integrating over all other variables, effectively "**marginalizing**" the distribution to **focus** on a **single** variable.



#### Marginal probabilities Total Fail Pass Males 56 P(male) = 0.5146 102 Females P(female) = 0.4968 30 98 Total 86 200 114 P(passed) P(failed = 0.57= 0.43)

# **Probability of an Event**

Assume an event E can occur in r ways out of a sum of n probable or possible **equally likely ways**. Then the probability of happening of the event or its success is expressed as;

P(E) = r/n

The probability that the event will not occur or known as its failure is expressed as:

P(E') = (n-r)/n = 1-(r/n)

E' represents that the event will not occur.

Therefore, now we can say;

# P(E) + P(E') = 1

This means that the total of all the probabilities in any random test or experiment is equal to 1.

# What are Equally Likely Events?

When the events have the same theoretical probability of happening, then they are called equally likely events. The results of a sample space are called equally likely if all of them have the same probability of occurring. For example, if you throw a die, then the probability of getting 1 is 1/6. Similarly, the probability of getting all the numbers from 2,3,4,5 and 6, one at a time is 1/6. Hence, the following are some examples of equally likely events when throwing a die:

- Getting 3 and 5 on throwing a die
- Getting an even number and an odd number on a die

• Getting 1, 2 or 3 on rolling a die are equally likely events, since the probabilities of each event are equal.

### **Complementary Events**

The possibility that there will be only two outcomes which states that an event will occur or not. Like a person will come or not come to your house, getting a job or not getting a job, etc. are examples of complementary events. Basically, the complement of an event occurring in the exact opposite that the probability of it is not occurring. Some more examples are:

- It will rain or not rain today
- The student will pass the exam or not pass.
- You win the lottery or you don't.

# **Probability Theory**

Probability theory had its root in the 16th century when J. Cardan, an Italian mathematician and physician, addressed the first work on the topic, The Book on Games of Chance. After its inception, the knowledge of probability has brought to the attention of great mathematicians. Thus, Probability theory is the branch of mathematics that deals with the possibility of the happening of events. Although there are many distinct probability interpretations, probability theory interprets the concept precisely by expressing it through a set of axioms or hypotheses. These hypotheses help form the probability in terms of a possibility space, which allows a measure holding values between 0 and 1. This is known as the probability measure, to a set of possible outcomes of the sample space.

# **Probability Density Function**

The Probability Density Function (PDF) is the probability function which is represented for the density of a continuous random variable lying between a certain range of values. Probability Density Function explains the normal distribution and how mean and deviation exists. The standard normal distribution is used to create a database or statistics, which are often used in science to represent the real-valued variables, whose distribution is not known.

# **Probability Terms and Definition**

Some of the important probability terms are discussed here:

Term	Definition	Example
Sample Space	The set of all the possible outcomes to occur in any trial	<ol> <li>Tossing a coin, Sample Space (S) = {H,T}</li> <li>Rolling a die, Sample Space (S) = {1,2,3,4,5,6}</li> </ol>
Sample Point	It is one of the possible results	<ul> <li>In a deck of Cards:</li> <li>4 of hearts is a sample point.</li> <li>The queen of clubs is a sample point.</li> </ul>
Experiment or Trial	A series of actions where the outcomes are always uncertain.	The tossing of a coin, Selecting a card from a deck of cards, throwing a dice.
Event	It is a single outcome of an experiment.	Getting a Heads while tossing a coin is an event.
Outcome	Possible result of a trial/experiment	T (tail) is a possible outcome when a coin is tossed.
Complimentary event	The non-happening events. The complement of an event A is the event, not A (or A')	In a standard 52-card deck, A = Draw a heart, then A' = Don't draw a heart
Impossible Event	The event cannot happen	In tossing a coin, impossible to get both head and tail at the same time

### **Applications of Probability**

- It is used for risk assessment and modelling in various industries
- Weather forecasting or prediction of weather changes
- Probability of a team winning in a sport based on players and strength of team
- In the share market, chances of getting the hike of share prices

### **Problems on Probability**

Question 1: Find the probability of 'getting 3 on rolling a die'.

**Question 2:** Draw a random card from a pack of cards. What is the probability that the card drawn is a face card?

**Question 3:** A vessel contains 4 blue balls, 5 red balls and 11 white balls. If three balls are drawn from the vessel at random, what is the probability that the first ball is red, the second ball is blue, and the third ball is white?

Question 4: Two dice are rolled, find the probability that the sum is:

- equal to 1
- equal to 4
- less than 13