Limit of a Function

What is Limit?

Limits are defined as the values that a function approaches the output for the given input values

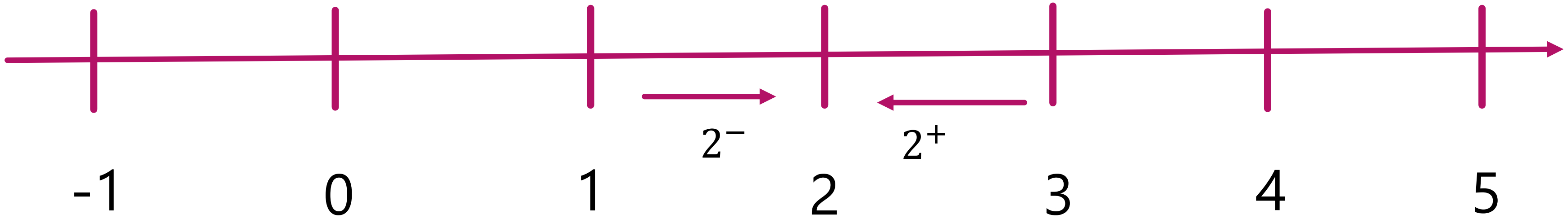
The Concept of Limits

$f(x) = \frac{x^2-4}{x-2}$ Values of a function cannot find directly any value then take limit to get idea of the value of y at that point $x = 2$

$$f(x) = \frac{x^2 - 4}{x - 2},$$

$$x = 2, f(x) = \frac{2^2 - 4}{2 - 2} = \frac{0}{0}$$

$x = 1.99$	$f(x) = 3.99$	$x = 2.01,$	$f(x) = 4.01$
$x = 1.999,$	$f(x) = 3.999$	$x = 2.001,$	$f(x) = 4.001$
$x = 1.9999,$	$f(x) = 3.9999$	$x = 2.0001,$	$f(x) = 4.0001$



Example of Limit

- $Find \lim_{x \to -1} (3x^3 - 4x + 8)$
- Solution
- Apply the properties of limits to obtain
- $\lim_{x \to -1} (3x^3 - 4x + 8)$
- $= 3 \lim_{x \to -1} x^3 - 4 \lim_{x \to -1} x + \lim_{x \to -1} 8$
- $= 3(-1)^3 - 4(-1) + 8$
- $= 9$

- Find $\lim_{x \to 1} \frac{3x^3-8}{x-2}$
- Solution: Since $\lim_{x \to 1}(x-2) \neq 0$ you can use the quotient rule for limits to get
- $\lim_{x \to 1} \frac{3x^3-8}{x-2} = \frac{\lim_{x \to 1}(3x^3-8)}{\lim_{x \to 1} x-2} = \frac{3\lim_{x \to 1} x^3 - \lim_{x \to 1} 8}{\lim_{x \to 1} x - \lim_{x \to 1} 2} = \frac{3-8}{1-2} = 5$

- Find $\lim_{x\to 2}\frac{x+1}{x-2}$
- Solution:
- X approaches 2 from the left side
- $\lim_{x\to 2^-}\frac{x+1}{x-2}=\frac{2+1}{2-2}=-\infty$
- X approaches 2 from the right side
- $\lim_{x\to 2^+}\frac{x+1}{x-2}=\frac{2+1}{2-2}=+\infty$
- f(x) increases without bound as x approaches 2 from the right and decreases without bound as x approaches 2 from the left. You can conclude that the limit of the function does not exist.

$$\lim_{x\to 2^-}\frac{x+1}{x-2}=\frac{1.999+1}{1.999-2}=\frac{2.999}{-0.001}=-2999/-\infty$$

$$\lim_{x\to 2^+}\frac{x+1}{x-2}=\frac{2.001+1}{2.001-2}=\frac{3.001}{0.001}=3001/\infty$$

- Find $\lim_{x \to 1} \frac{x^2-1}{x^2-3x+2}$
- As x approaches 1, both the numerator and the denominator approach zero, and you can draw no conclusion about the size of the quotient. To proceed, observe that the given function is not defined when x=1 but that for all other values of x, you can cancel the common factor x -1 to obtain.
- $\frac{x^2-1}{x^2-3x+2} = \frac{x^2-1}{x^2-2x-x+2} = \frac{(x+1)(x-1)}{(x-1)(x-2)} = \frac{(x+1)}{(x-2)}, x \neq 1$
- $\lim_{x \to 1} \frac{x^2-1}{x^2-3x+2} = \lim_{x \to 1} \frac{(x+1)}{(x-2)} = \frac{\lim_{x \to 1} (x+1)}{\lim_{x \to 1} (x-2)} = \frac{2}{-1} = -2$

Exercise

- Find $\lim_{x \to 4^+} (3x^2 - 9)$
- $\lim_{x \to 4^+} (3x^2 - 9) = 3 \lim_{x \to 4^+} x^2 - \lim_{x \to 4^+} 9$
- $= 3(4)^2 - 9 = 39$
- Find $\lim_{x \to 1^-} x(2 - x)$
- $\lim_{x \to 1^-} x(2 - x) = \lim_{x \to 1^-} 2x - \lim_{x \to 1^-} x^2$
- $= 2(1) - (1)^2 = 1$

- Find $\lim\limits_{x\to 2^-} \frac{x^2+4}{x-2}$
- Since the numerator is positive and the denominator x-2 approaches zero and less than zero for x near 2 to the left, the function decreases without bound.
- $\lim\limits_{x\to 2^-} \frac{x^2+4}{x-2} = \frac{2^2+4}{2-2} = -\infty$

$$\lim_{x\to 2^-} \frac{x^2+4}{x-2} = \frac{(1.999)^2+4}{1.999-2} = -\infty$$

- $\lim_{x \to 3^+} \sqrt{3x - 9}$
- X approaches 3 from the right side
- $\lim_{x \to 3^+} \sqrt{3x - 9}$
- $= \sqrt{3 \times 3 - 9} = 0$

$$\lim_{x \to 3^+} \sqrt{3x - 9}$$
$$= \sqrt{3 \times 3.001 - 9}$$
$$\sqrt{9.003 - 9}$$
$$= 0.003 = 0$$

- $\lim_{x \to 2^-} \sqrt{4 - 2x}$
- X approaches 2 from the left side
- $\lim_{x \to 2^-} \sqrt{4 - 2x} = \sqrt{4 - 2 \times 2} = 0$

$$\lim_{x \to 2^-} \sqrt{4 - 2x}$$
$$= \sqrt{4 - 2 \times 1.999}$$
$$\sqrt{4 - 3.998}$$
$$= 0$$