Basic Concept of Function

Constant: A constant is a value or number that never changes in expression i.e. which retains the same value throughout a set of mathematical operations.

constant are of two types; Absolute and arbitrary.

- Constant of the type 15, 7, −500 etc. do not change whatever we may perform are known as absolute constants.

- In the equation $y = mx + c$ of a straight line, m and c are arbitrary constants as these remain fixed for a particular straight line but vary from straight line to straight line.

Variable: A variable is a quantity which is capable of taking various numerical values.

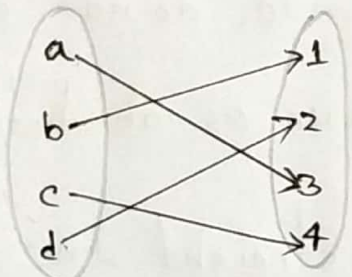
Variables are of two types: Dependent and Independent

consider the equation $y = x^2$ in which x can take any value, but for each value of x there exists a value of y.

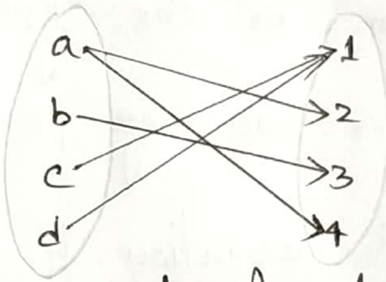
Here, x is called independent variable and y is called dependent variable.

Function: Let A ~~ab~~ and B be two non-empty sets. If each element of A is related to unique element of B, then it is called a function from the set A to B.

It is defined by, $f: A \rightarrow B$.



Function



Not a function.

For example;

① $y = x^2$ is a function.

For $x = \pm 1, \pm 2, \pm 3, \ldots$ we get $y = 1, 4, 9, \ldots$

Since we get unique y for x, so it is a function.

Explanation: suppose that $A = \{-1, 1, -2, 2, 3\}$ and $B = \{1, 4, 9, 16\}$ and the relation is, $y = f(x) = x^2$

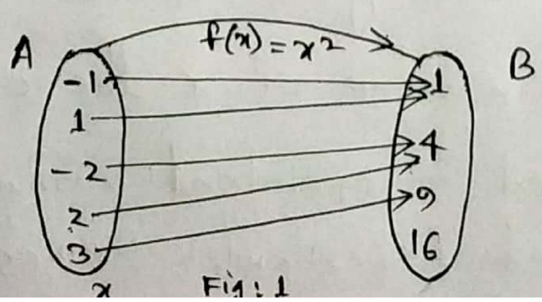


Fig: 1

(2) $y = \pm\sqrt{x}$ is not a function.

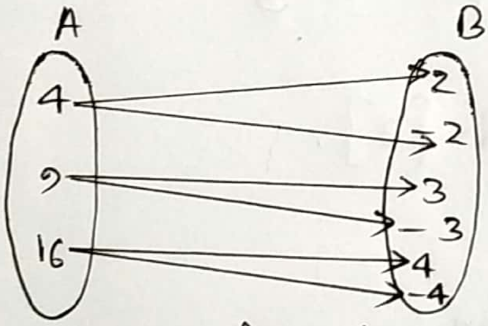
Because for each input x, we have two output values of y; which violates the definition of function.

Explan suppose that $A = \{4, 9, 16\}$ and $B = \{2, -2, 3, -3, 4, -4\}$

and the relation is defined by.

$$f : A \to B$$

$$y = f(x) = \pm\sqrt{x}$$



Which is not a function.

Domain of a function:

Let $f : A \to B$ be a function. Then the set A is called the domain of the function.

It is usually denoted by.

$$D_f = A$$

where, the set B is called co-domain of the function.

Example:

① Let $y = f(x) = x^2$.

where for all real value of x, $y = x^2$ is also real. Thus the domain is the set of real numbers.

i.e. $D_f = \mathbb{R}$ (set of real numbers)

② Let $f(x) = \dfrac{1}{x-3}$

Where $f(x)$ obtains real values for all real input of x except $x = 3$.

$\therefore D_f = \mathbb{R} - \{3\}$.

Range of a function:

The set of all output values produced by f is called the range of the function or the image of the function.

It is denoted by D_f.

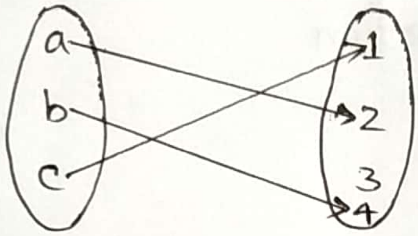
Range is a subset of co-domain. $(R_f \subseteq B)$

In fig: 1 we have $B = \{1, 4, 9, 16\}$ and $R_f = \{1, 4, 9\}$

$\therefore R_f \subset B$

One-one function (Injective):

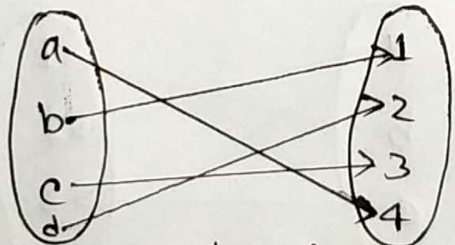
Let $f: A \to B$ be a function. If distinct element of A has distinct images in B then the function is called one-one function.



one-one function.

Onto function (Surjective):

Let $f: A \to B$ be a function. If all elements of the set B are images of the elements of A then the function is called onto function.

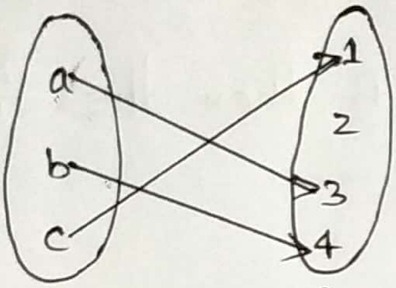


onto function (also one-one)

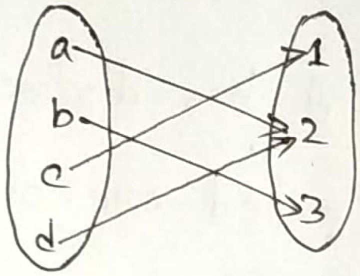
One-to-one correspondence (Bijective):

The function $f: A \to B$ is a one-to-one correspondence if it is both one-one and onto.

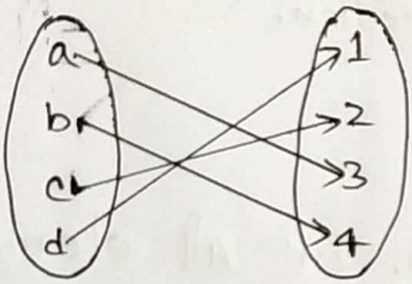
<u>Some different types of function:</u>



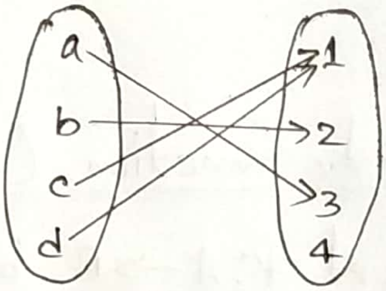
one-one function but not onto.



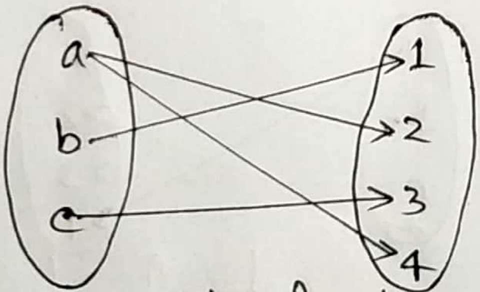
Onto function but not one-one.



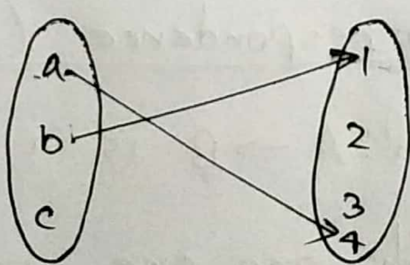
one-one and onto.



Neither one-one nor onto.



Not a function.



Not a function

Domain and Range of a function :-

1. The values of x for which the function $y=f(x)$ is of the form $\frac{a}{0}, \frac{0}{0}, 1^{\infty}, 0^{0}, \infty^{0}, \infty-\infty$ (Indeterminate form) then $x \notin D_f$.

2. If a function is of the form $\sqrt{f(x)}$ then $x \in D_f$ for which the values of x, $f(x) \geq 0$.

3. If a function is of the form $\frac{\sqrt{g(x)}}{\sqrt{h(x)}}$ then $x \in D_f$ for which the values of x, $g(x) \geq 0$ and $h(x) > 0$.

4. If 'f' and 'g' are two functions then the domain of '$f \pm g$' is $D_f \cap D_g$.

5. For all values in the domain, the values of the function constitute the range of the function.

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Ex: $f(x) = x^2 + 2x+3$, $g(x) = \frac{1}{x}$,

$h(x) = f(x) + g(x) = x^2+2x+3 + \frac{1}{x}$; $D_h = ?$

Soln:- Here, $D_f = \mathbb{R}$, $D_g = \mathbb{R} - \{0\}$

So, $D_h = D_f \cap D_g$

$= \mathbb{R} \cap \mathbb{R} - \{0\}$

$= (-\infty, 0) \cup (0, \infty)$

[Ans]

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Find domain and range of the following functions:

(i) $f(x) = \dfrac{1}{x-2}$

(ii) $f(x) = \dfrac{1}{2x+1}$

(i) <u>solution:</u>

clearly, $f(x) = \dfrac{1}{x-2}$ is not defined when $x=2$

$\therefore$ dom $f(x) = \mathbb{R} - \{2\}$ = set of all real numbers except 2.

Let $y = f(x) = \dfrac{1}{x-2}$

$\Rightarrow x - 2 = \dfrac{1}{y}$

$\Rightarrow x = \dfrac{1}{y} + 2$

$\Rightarrow x = \dfrac{1+2y}{y}$

So, x is not defined for $y=0$

$\therefore$ range of $f(x) = \mathbb{R} - \{0\}$

= set of all real numbers except 0.

(ii) Solution:

Clearly, $f(x) = \dfrac{1}{2x+1}$ is undefined when,

$$2x + 1 = 0$$

$$\Rightarrow 2x = -1$$

$$\Rightarrow x = -\frac{1}{2}$$

$\therefore$ Domain of $f(x)$ is,

$$D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

Let $y = f(x) = \dfrac{1}{2x+1}$

$$\Rightarrow 2x+1 = \frac{1}{y}$$

$$\Rightarrow 2x = \frac{1}{y} - 1$$

$$\Rightarrow 2x = \frac{1-y}{y}$$

$$\Rightarrow x = \frac{1}{2}\left(\frac{1-y}{y}\right)$$

So, x is not defined for $y = 0$.

$\therefore$ range of $f(x)$ is,

$$R_f = \mathbb{R} - \{0\}$$

$=$ set of all real numbers except 0.

Find domain and range for the following functions:

(i) $f(x) = \sqrt{4-x^2}$

(ii) $f(x) = \sqrt{x^2-25}$

(iii) $f(x) = \dfrac{1}{\sqrt{9-x^2}}$

(iv) $f(x) = |x+1| + |x-1|$

(v) $f(x) = \dfrac{x}{|x|}$

Solution:

(i) Given that,

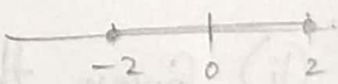
$$f(x) = \sqrt{4-x^2}$$

Here the value of $f(x)$ will be real if

$$4-x^2 \geq 0$$

$$\Rightarrow x^2 \leq 4$$

$$\Rightarrow -2 \leq x \leq 2$$



$$\therefore D_f = \{x : -2 \leq x \leq 2\}$$
$$= [-2, 2]$$

Again, $y = f(x) = \sqrt{4-x^2}$ ——— (i)

In (i) the value of y can not be negative, that is, the value of y will be positive or zero.

From (i),

$y^2 = 4 - x^2$ where $y \nless 0$

$\Rightarrow x^2 = 4 - y^2$, $y \nless 0$

$\Rightarrow x = \pm\sqrt{4 - y^2}$, $y \nless 0$

Here, the value of x will be exist if

$4 - y^2 \geq 0$ and $y \nless 0$

$\Rightarrow y^2 \leq 4$ and $y \nless 0$

$\Rightarrow -2 \leq y \leq 2$ and $y \nless 0$

$\Rightarrow 0 \leq y \leq 2$

$\therefore R_f = \{y : 0 \leq y \leq 2\}$

$= [0, 2]$ (ans)

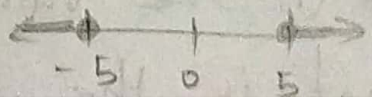
(ii) Given that,

$f(x) = \sqrt{x^2 - 25}$

Here the value of $f(x)$ will be ~~exist~~ real

if $x^2 - 25 \geq 0$

$\Rightarrow x \leq -5$ and $x \geq 5$



$\therefore D_f = \{x : x \leq -5\} \cup \{x : x \geq 5\}$

$= (-\infty, -5] \cup [5, \infty).$

Now, $y = f(x) = \sqrt{x^2 - 25}$ ——— (1)

In (1), the value of y only be positive or zero

from (1) $\Rightarrow y^2 = x^2 - 25$ where $y \nless 0$

$\Rightarrow x^2 = y^2 + 25$ where $y \nless 0$

$\Rightarrow x = \pm\sqrt{y^2 + 25}$, $y \nless 0$

Here x is defined for $y \geq 0$

$\therefore R_f = \{y : y \geq 0\}$

$= [0, \infty)$ (ans)

(iii) Given that,

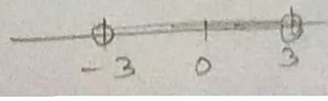
$$f(x) = \frac{1}{\sqrt{9 - x^2}}$$

Here the value of $f(x)$ will be real if $9 - x^2 > 0$
not equal

$x^2 - 9 < 0$

$\Rightarrow x^2 < 9$

$\Rightarrow -3 < x < 3$



$\therefore D_f = \{x : -3 < x < 3\}$

$= (-3, 3)$

Now, $y = f(x) = \dfrac{1}{\sqrt{9 - x^2}}$ ——(1)

In (1) the value of y can not be negative or zero, i.e., the value of y will be positive.

$\therefore y^2 = \dfrac{1}{9 - x^2}$ where $y \nleq 0$

$\Rightarrow 9 - x^2 = \dfrac{1}{y^2}$, $y \nleq 0$

$\Rightarrow x = \pm\sqrt{9 - \dfrac{1}{y^2}}$, $y \nleq 0$

Here, the value of x will ~~be~~ exist if

$9 - \dfrac{1}{y^2} \geq 0$ and $y \nleq 0$

$y^2 - 1 \geq 0$

$\Rightarrow -\dfrac{1}{y^2} \geq -9$ and $y \nleq 0$

$\Rightarrow y^2 \geq \dfrac{1}{9}$ and $y \nleq 0$

$\Rightarrow y \leq -\dfrac{1}{2}$ or $y \geq \dfrac{1}{3}$ and $y \nleq 0$

$\Rightarrow y \geq \dfrac{1}{3}$

$\therefore R_f = \{y : y \geq \frac{1}{3}\} = [\frac{1}{3}, \infty)$

(iv) Given that,

$$f(x) = |x-1| + |x+3|$$
$$= |x+3| + |x-1|$$

For $x < -3$, we can write,

$$f(x) = -(x+3) - (x-1)$$
$$= -2x - 2$$

For $-3 \le x < 1$, we write

$$f(x) = (x+3) - (x-1)$$
$$= 4$$

For, $x \geq 1$, we can write

$$f(x) = (x+3) + (x-1)$$
$$= 2x+2$$

$$\therefore y = f(x) = \begin{cases} -2x-2 & \text{when } x < -3 \\ 4 & \text{when } -3 \le x < 1 \\ 2x+2 & \text{when } x \geq 1 \end{cases}$$

$$\therefore D_f = \{x : x < -3\} \cup \{x : -3 \le x < 1\} \cup \{x : x \geq 1\}$$
$$= (-\infty, -3) \cup [-3, 1] \cup [1, \infty)$$
$$= \mathbb{R}$$

CamScanner

For $x < -3 \Rightarrow y > 4$

$-3 \leq x < 1 \Rightarrow y = 4$

$x \geq 1 \Rightarrow y \geq 4$

$\therefore R_f = \{y : y > 4\} \cup \{4\}$ ~~$\cup \{ y : y$~~

$= [4, \infty)$

(v) Given, $f(x) = \dfrac{x}{|x|}$

For, $x < 0 \Rightarrow f(x) = \dfrac{x}{-x} = -1$

For, $x > 0 \Rightarrow f(x) = \dfrac{x}{x} = 1$

$$\therefore y = f(x) = \begin{cases} -1 & \text{when } x < 0 \\ 1 & \text{when } x > 0 \end{cases}$$

$\therefore D_f = \{x : x < 0\} \cup \{x : x > 0\}$

$= \mathbb{R} - \{0\}$

and $R_f = \{-1, 1\}$.

[গুরুত্ব]

Transcendental function :-

If a function is having at least one of the three elements : Trigonometric, Logarithmic, Exponential, such function is called Transcendental function.

Ex :- a^x (e^x), $\log_a x$ $(\ln x)$, $\sin x$, $\cos x$, $\tan x$, ——

$x^2 + \sin x$, $ax^2 + e^x$, $bx^2 + \log x$, ... ∞

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[গুরুত্ব]

Piece-wise function :-

A piece-wise function is a function that is defined on a sequence of intervals.

Ex :-

$$f(x) = \begin{cases} x^2+1 & \text{when } x \le 0 \\ x & \text{when } 0 < x < 2 \\ 1-x^2 & \text{when } x \ge 2 \end{cases}$$

↓ Range ↓ Domain

*** Absolute value function is an example of piece-wise function.

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[বিখ্যাত]

Inverse function :-

If a function $f : A \to B$ is one-one and onto then corresponding to each element y in B there exists a unique element x in A such that $f^{-1} : B \to A$: $f^{-1}(y) = x$, then function f^{-1} is called the inverse of f. Ex :- $f(x) = \log x = y$ or, $e^y = x = f^{-1}(y)$

$\therefore e^x = f^{-1}(x)$

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[বহুপদী]

Polynomial function :-

"A polynomial in x" is a function that is expressible as a sum of finitely many of the form cx^n ; where c is a constant and n is a non-negative integer.

such as :- $4 \rightarrow 4x^0$, $2x+4 \rightarrow 2x^1+4x^0$, $x^2+2x+4=0$

$1.x^2+2.x^1+4x^0=0x^0$

— 0 —

[মূলদ ফাংশন]

Rational function :-

A function that can be expressed as a ratio of two polynomials is called a rational function.

If $P(x)$ and $Q(x)$ be polynomials, then the domain of the rational function $f(x) = \frac{P(x)}{Q(x)}$ consists of all values of x such that $Q(x) \neq 0$.

such as :- $f(x) = \frac{x^2+2x}{x^2-1}$; Here, $x^2-1 \neq 0$ or $x \neq \pm 1$

So, $D_f = \mathbb{R} - \{-1, 1\}$

— 0 —

[বীজগাণিতিক ফাংশন]

Algebraic function :-

Functions that can be constructed from polynomials by applying finitely many algebraic operations $(+, -, *, \div, \sqrt{\ })$ are called algerbraic functions.

: addition
: subtraction
: multiplication
: division
: root extraction

such as :- $f(x) = \sqrt{x^2-4}$, $f(x) = 3(\sqrt[3]{x})(2+x)$,

$f(x) = x^{2/3}(x+2)^2$, ...

— 0 —

Absolute property

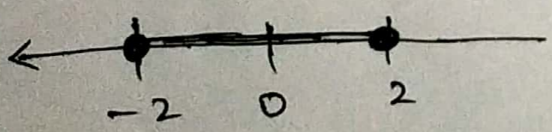
(mod x)

$$|x| = \begin{cases} -x & \text{when } x < 0 \\ x & \text{when } x > 0 \\ 0 & \text{when } x = 0 \end{cases}$$

$[1, 2]$

$(1, 2)$

$\{1, 2\}$

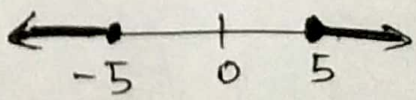


$$x^2 - 25 \geq 0$$

$$\Rightarrow x^2 - 5^2 \geq 0$$

$$\Rightarrow (x+5)(x-5) \geq 0$$

$$\Rightarrow x \leq -5 \text{ or } x \geq 5$$



$$\underline{x^2 \geq 25}$$