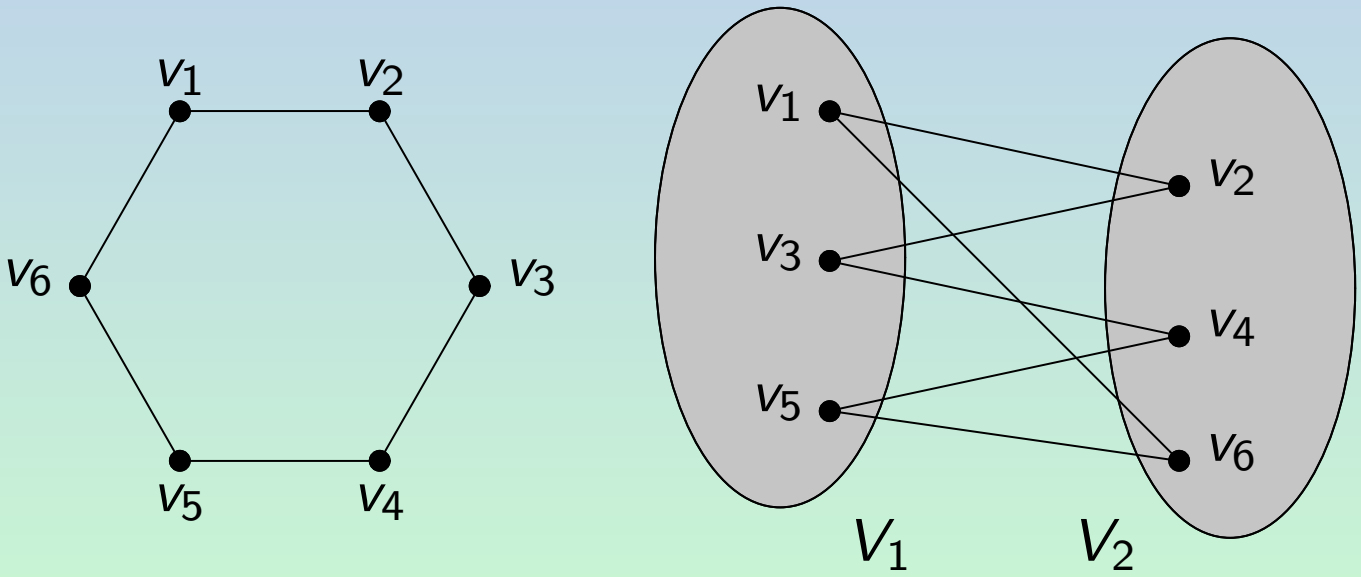
Bipartite Graphs
Discrete Mathematics

Definition: Bipartite Graphs

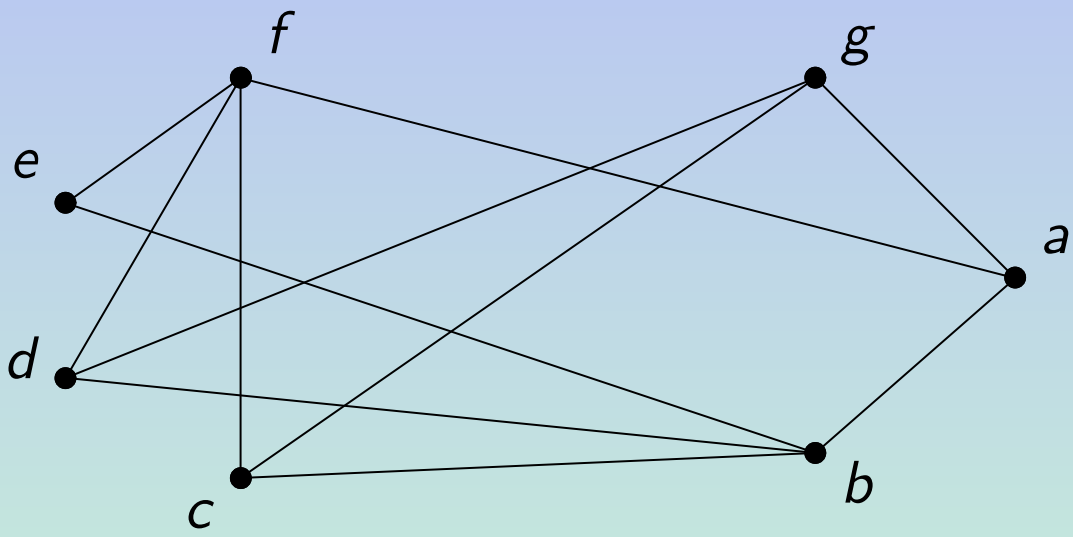
Definition

A simple graph G is called **bipartite** if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (or, there is no edge between vertices of subset V_1 and between vertices of subset V_2).



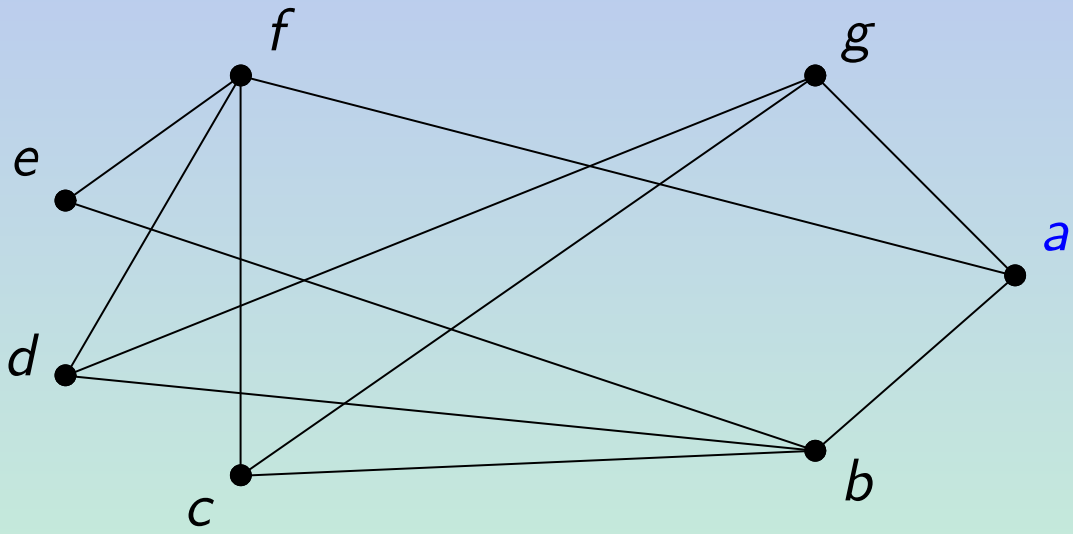
Showing that C_6 is bipartite

Is This Graph Bipartite?



Answer: Step 1 of 3

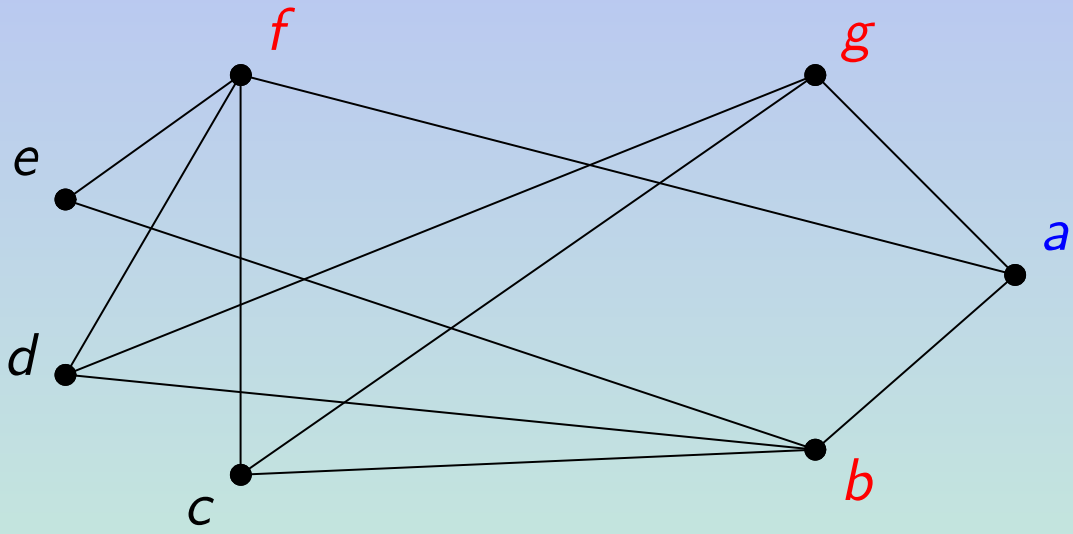
We label one vertex of the graph with the color blue. How to choose this first vertex? Simply choose the first one in lexicographic order.



Level 1 (Blue) : a

Answer: Step 2 of 3

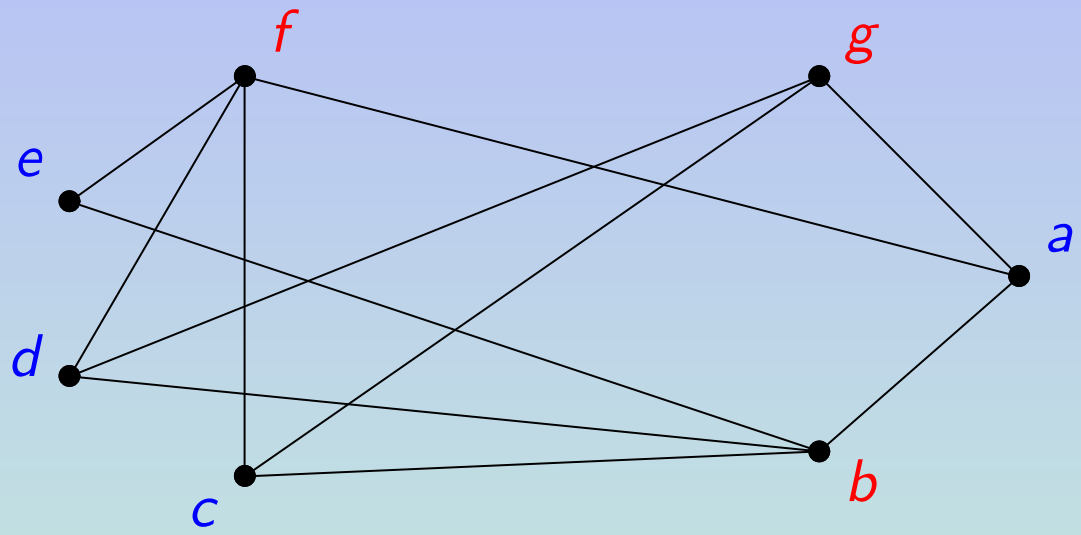
The adjacent vertices b, f and g of the first vertex a must be of the other color. We label them with the red color.



Level 1 (Blue) : a
Level 2 (Red) : b, f and g

Answer: Step 3 of 3

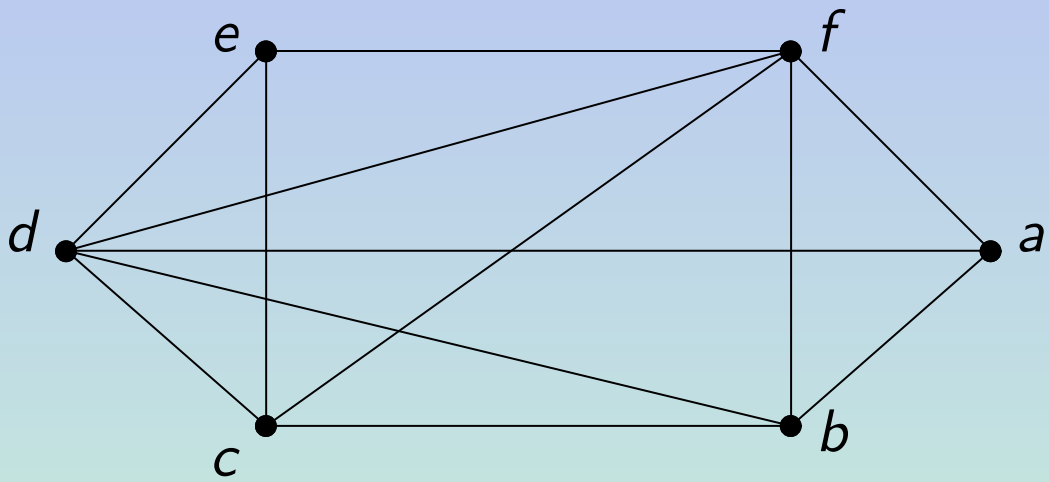
The adjacent vertices to b, f and g must be of the other color. We label them with the blue color.



Level 1 (Blue) : a
Level 2 (Red) : b, f and g
Level 3 (Blue) : c, d and e

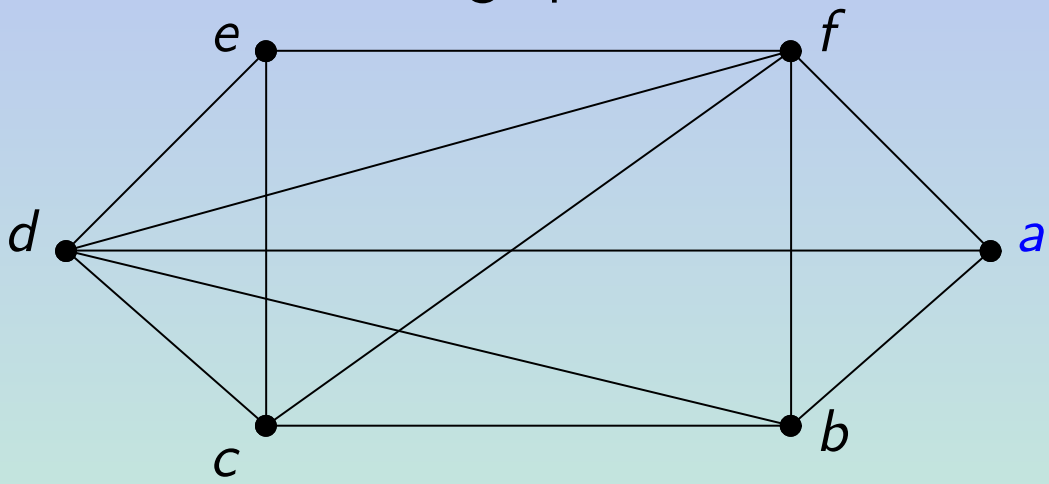
At the end of the process, if all the vertices have one **unique** label, then the graph is bipartite.

Is This Graph Bipartite?



Answer: Step 1 of 3

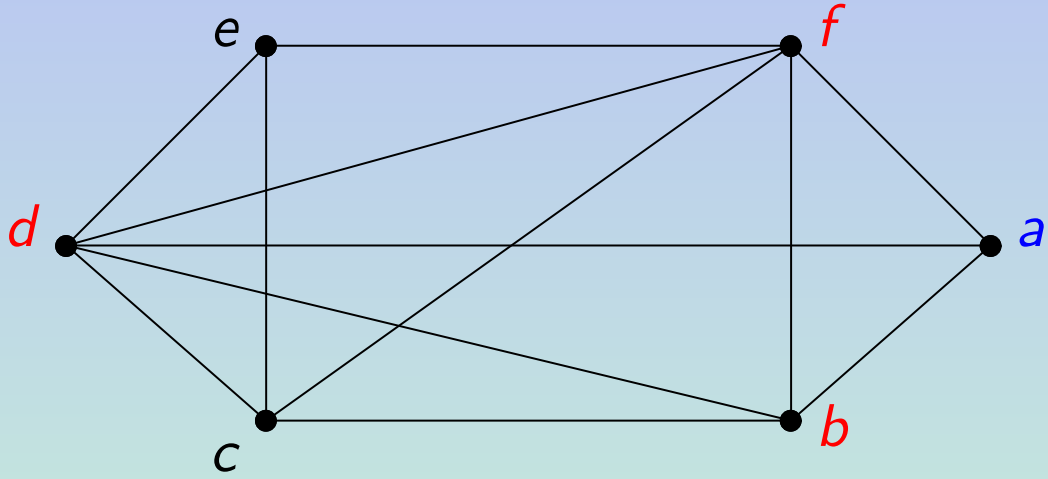
We label one vertex of the graph with the color blue. For this, we choose the first vertex in lexicographic order.



Level 1 (Blue) : a

Answer: Step 2 of 3

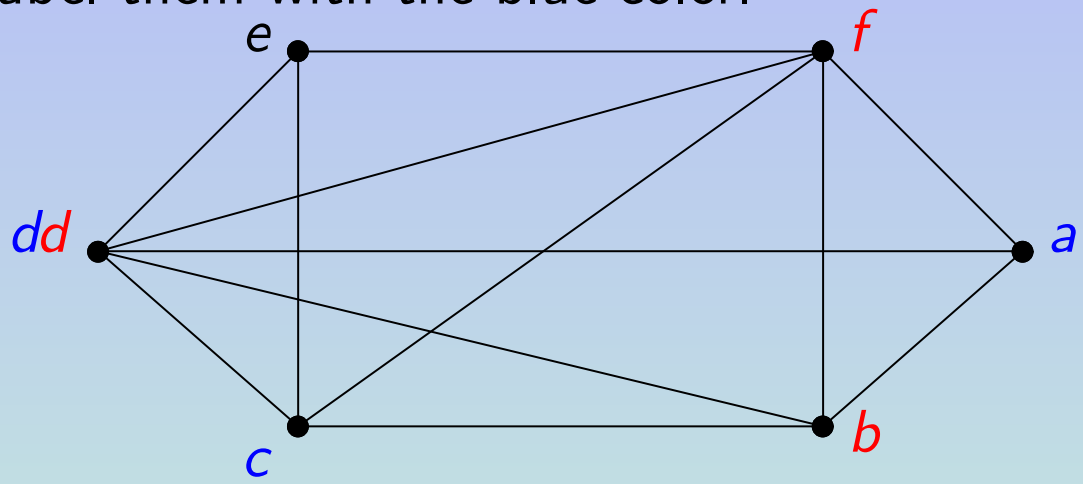
The adjacent vertices b, d and f of the first vertex a must be of the other color. We label them with the red color.



Level 1 (Blue) : a
Level 2 (Red) : b, d and f

Answer: Step 3 of 3

The adjacent vertices to vertices b, d and f must be of the other color. We label them with the blue color.



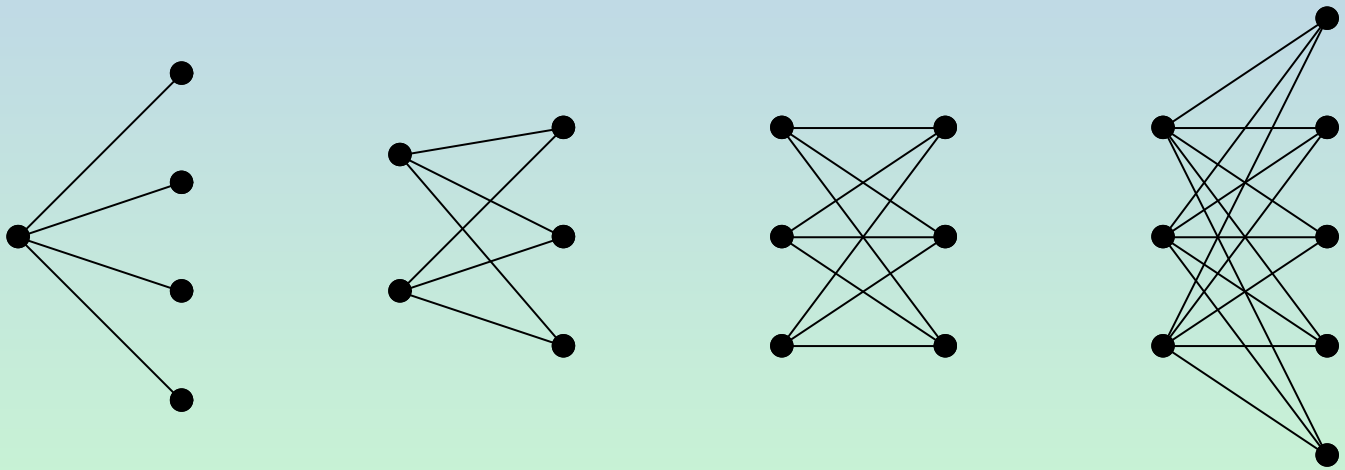
Level 1 (Blue) : a
Level 2 (Red) : b, d and f
Level 2 (Red) : c, d ???

As soon as one vertex must have two different colors, the graph is **not** bipartite.

Definition: Complete Bipartite Graph

Definition

The **complete bipartite graph** $K_{m,n}$ is the graph that has its vertex set partitioned into two subsets of m and n vertices, respectively. There is an edge between two vertices if and only if one vertex is in the first subset and the other vertex in the second subset.



$K_{1,4}$ $K_{2,3}$ $K_{3,3}$ $K_{3,5}$