Graph Isomorphism
Discrete Mathematics

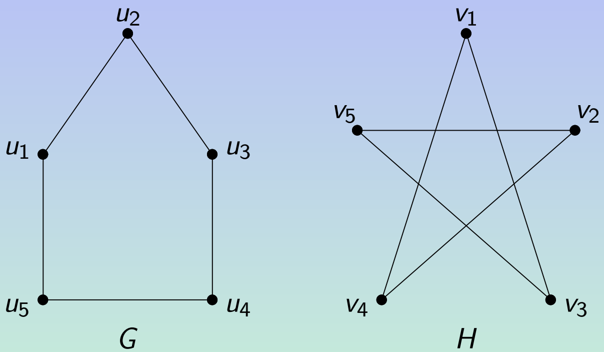
Definition: Isomorphism of Graphs

Definition

The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are **isomorphic** if there is an injective (one-to-one) and surjective (onto) function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called an **isomorphism**.

In other words, when two simple graphs are **isomorphic**, there is a bijection (one-to-one correspondence) between vertices of the two graphs that preserves the adjacency relationship.

Example of Isomorphic Graphs



$f(u_1) = v_1$, $f(u_2) = v_3$, $f(u_3) = v_5$, $f(u_4) = v_2$ and $f(u_5) = v_4$.

Example of Isomorphic Graphs

G	u_1	u_2	u_3	u_4	u_5
u_1	0	1	0	0	1
u_2	1	0	1	0	0
u_3	0	1	0	1	0
u_4	0	0	1	0	1
u_5	1	0	0	1	0

H	v_1	v_2	v_3	v_4	v_5
v_1	0	0	1	1	0
v_2	0	0	0	1	1
v_3	1	0	0	0	1
v_4	1	1	0	0	0
v_5	0	1	1	0	0

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u_2	1	0	1	0	0
u_3	0	1	0	1	0
u_4	0	0	1	0	1
u_5	1	0	0	1	0

H	v_1	v_3	v_5	v_2	v_4
v_1	0	1	0	0	1
v_3	1	0	1	0	0
v_5	0	1	0	1	0
v_2	0	0	1	0	1
v_4	1	0	0	1	0

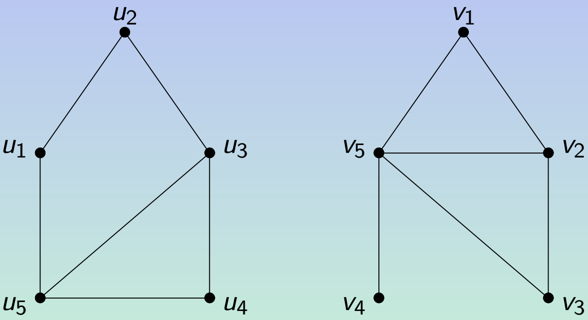
Isomorphic Graph's Invariant

We can tell if two graphs are invariant or not using **graphs invariant**. For example, two simple isomorphic graphs must :

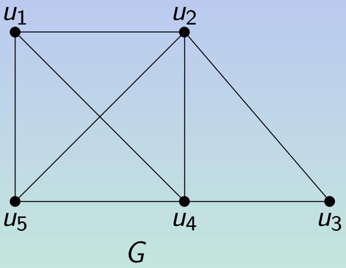
- have the same number of vertices,
- have the same number of edges,
- have the same degrees of vertices.

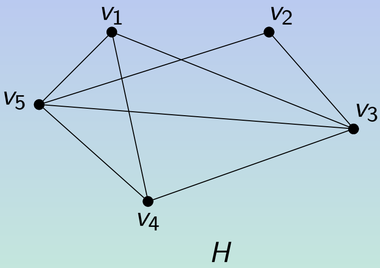
Note 1: These conditions are necessary but **not sufficient** to show that two graphs are isomorphics.
Note 2: The breaking of one of these conditions is sufficient but not necessary to show that two graphs are not isomorphic.

Example of Non-Isomorphic Graphs

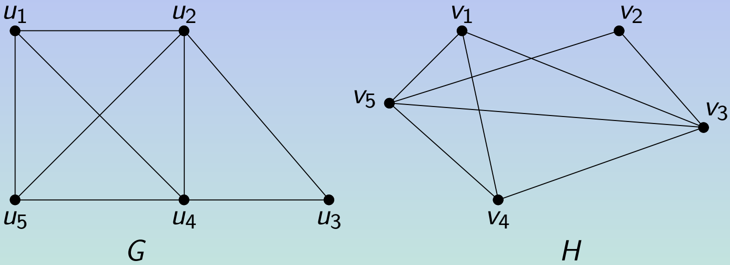


Are These Graphs Isomorphic?



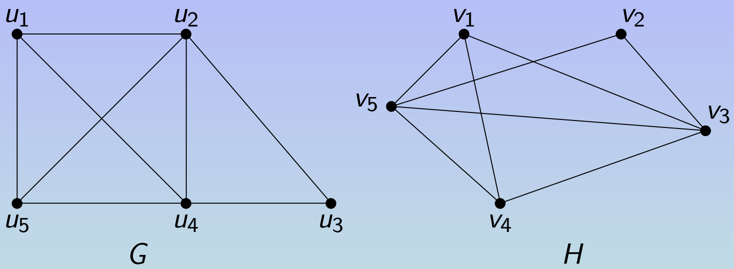


Are These Graphs Isomorphic?



$f(u_3) = v_2$, $f(u_4) = v_3$, $f(u_2) = v_5$, $f(u_5) = v_4$ and $f(u_1) = v_1$.

These Two Graphs Are Isomorphic



G	u_1	u_2	u_3	u_4	u_5
u_1	0	1	0	1	1
u_2	1	0	1	1	1
u_3	0	1	0	1	0
u_4	1	1	1	0	1
u_5	1	1	0	1	0

H	v_1	v_5	v_2	v_3	v_4
v_1	0	1	0	1	1
v_5	1	0	1	1	1
v_2	0	1	0	1	0
v_3	1	1	1	0	1
v_4	1	1	0	1	0