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COMPUTER-AIDED GEOMETRIC DESIGN AND COMPUTER GRAPHICS:

LINE DRAWING ALGORITHMS

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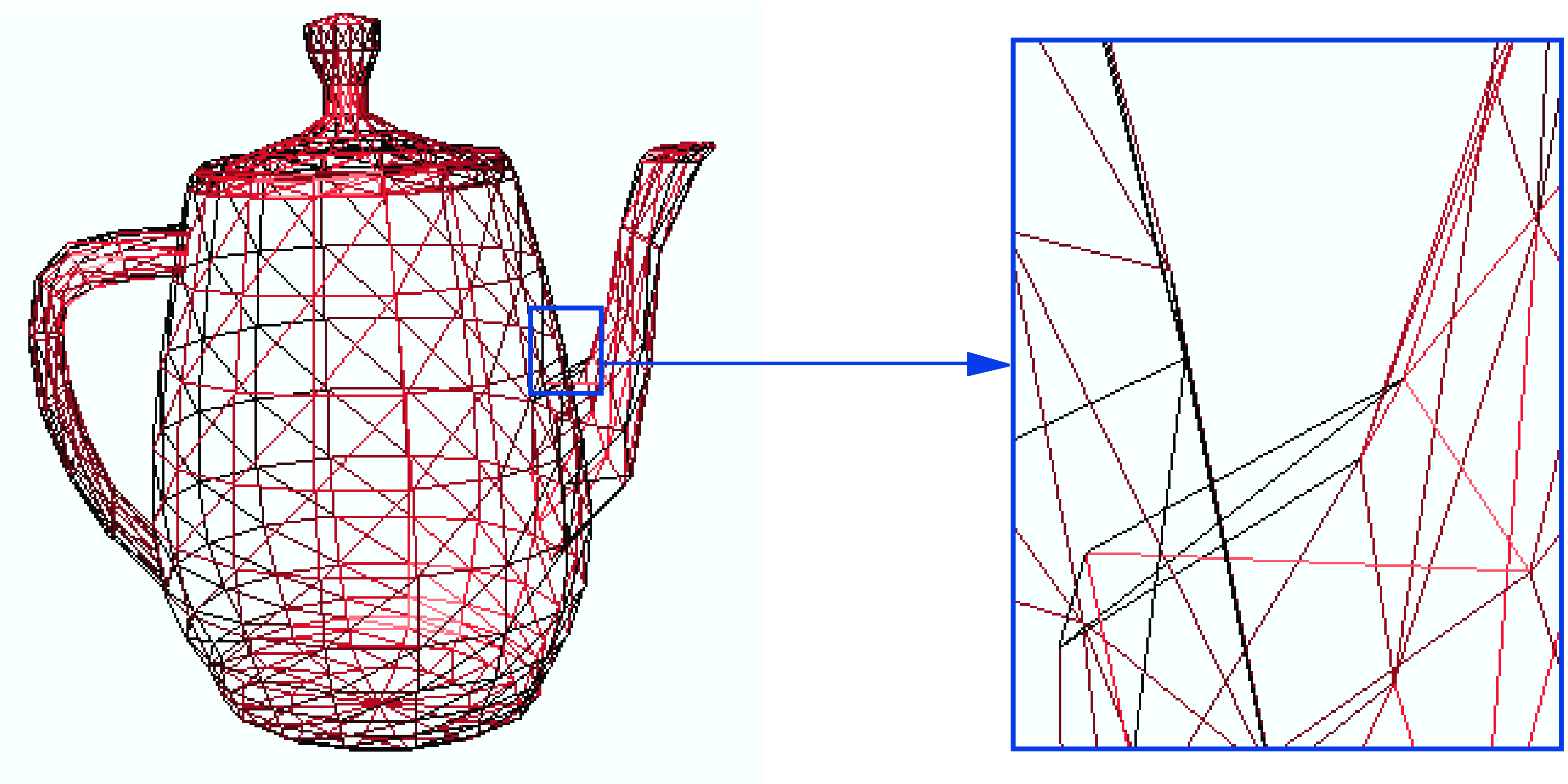
Line Drawing Algorithms



The lines of this object appear ***continuous***

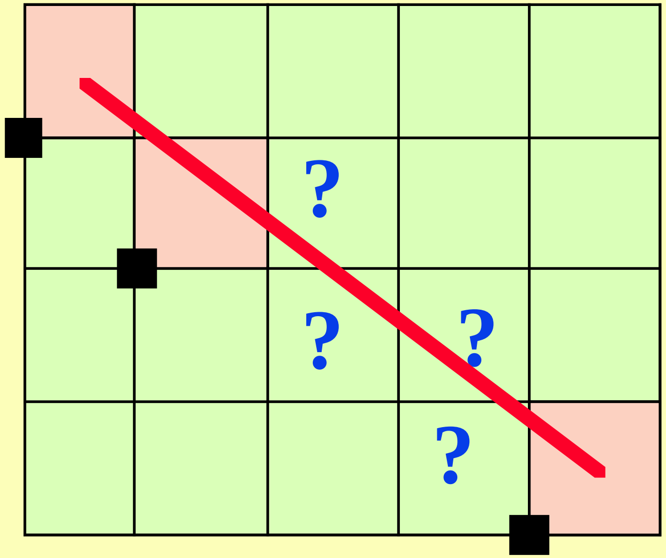
However, they are ***made of pixels***

Line Drawing Algorithms

We are going to analyze how this process is achieved.

Some useful definitions

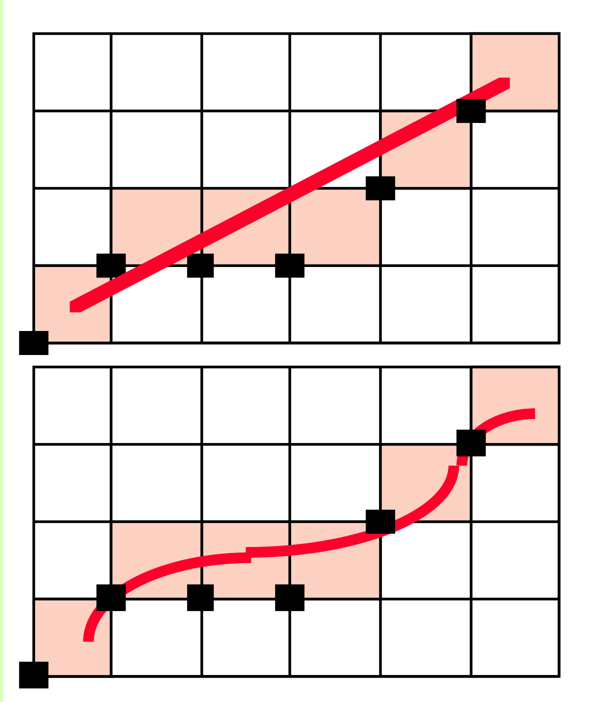
Rasterization: Process of determining which pixels provide the best approximation to a desired line on the screen.



Scan Conversion: Combination of rasterization and generating the picture in scan line order.

General requirements

• Straight lines must appear as straight lines.



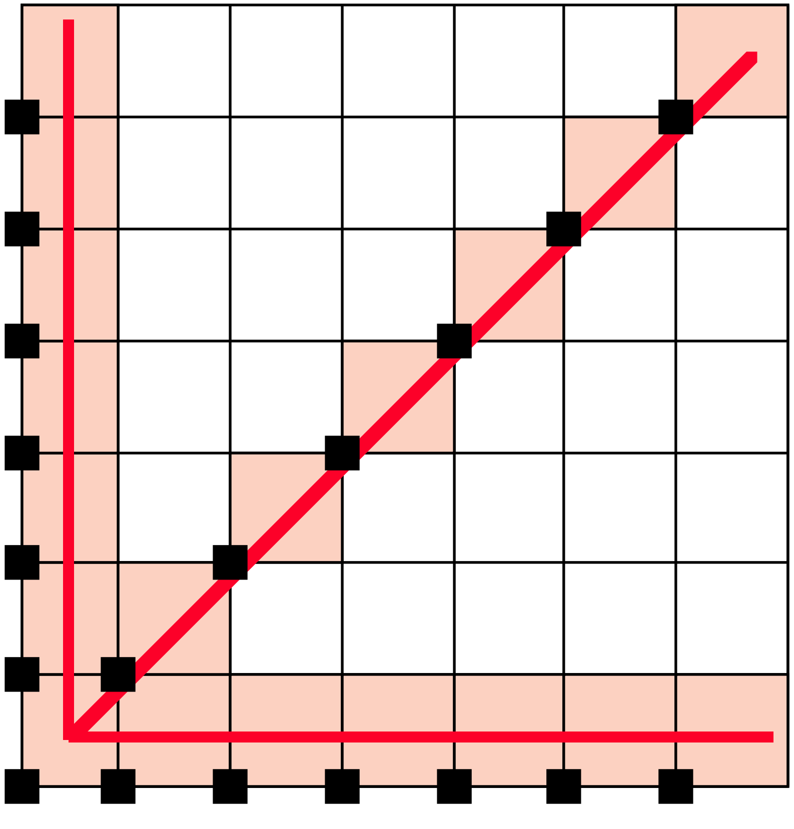
• They must start and end accurately

• Lines should have constant brightness along their length

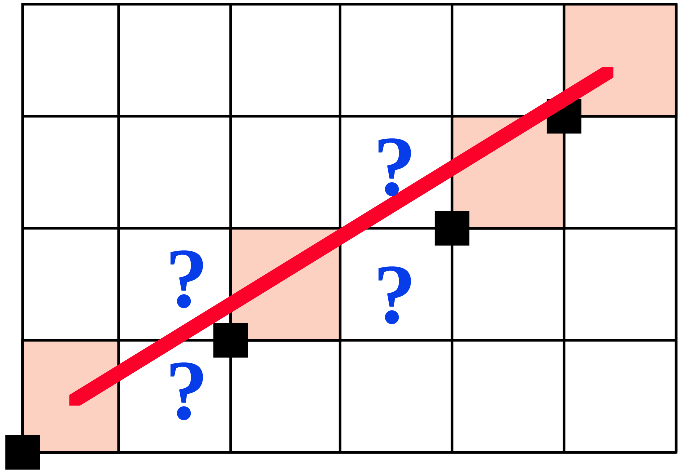
•Lines should drawn rapidly

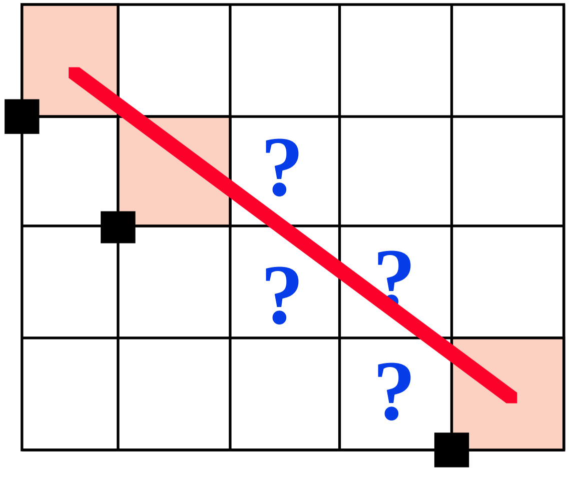
Line Drawing Algorithms

For horizontal, vertical and 45º lines, the choice of raster elements is obvious. This lines exhibit constant brightness along the length:



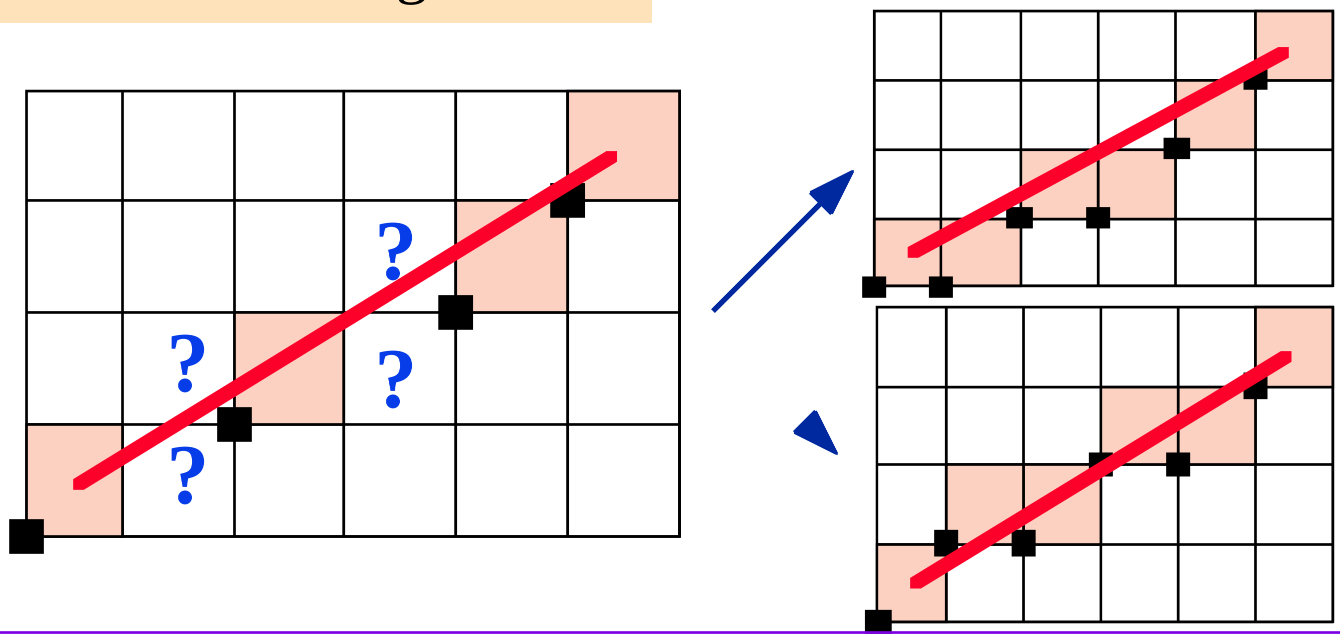
For any other orientation the choice is more difficult:





Line Drawing Algorithms

Rasterization of straight lines.



Rasterization yields uneven brightness: Horizontal and vertical lines appear brighter than the 45º lines.

For fixing so, we would need:

1. Calculation of square roots (increasing CPU time)
2. Multiple brigthness levels

=>

Compromise:

1. Calculate only an approximate line
2. Use integer arithmetic
3. Use incremental methods

Line Drawing Algorithms

The equation of a straight line is given by: $\boxed{y = m.x + b}$

Algorithm 1: Direct Scan Conversion

1. Start at the pixel for the left-hand endpoint x1
2. Step along the pixels horizontally until we reach the right-hand end of the line, xr
3. For each pixel compute the corresponding y value
4. round this value to the nearest integer to select the nearest pixel

```
x = xl;

while (x <= xr){

      ytrue = m*x + b;
      y = Round (ytrue);
      PlotPixel (x, y);
        /* Set the pixel at (x,y) on */
      x = x + 1;

}
```

The algorithm performs a floating-point multiplication for every step in *x*. This method therefore requires an enormous number of floating-point multiplications, and is therefore expensive.

Line Drawing Algorithms

Algorithm 2: Digital Differential Analyzer (DDA)

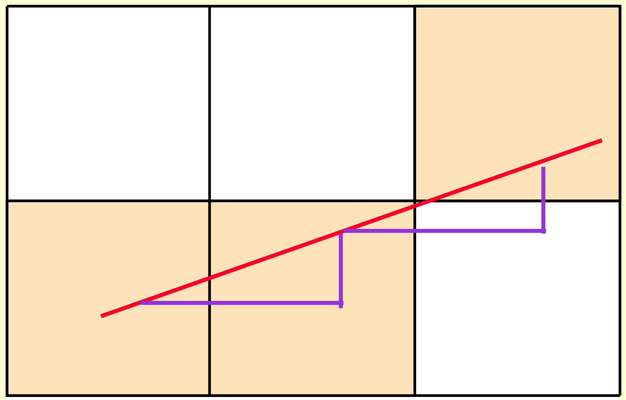
The differential equation of a straight line is given by:

$$\frac{dy}{dx} = constant \quad \text{or} \quad \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The solution of the finite difference approximation is:

$$x_{i+1} = x_i + \Delta x$$

$$y_{i+1} = y_i + \frac{y_2 - y_1}{x_2 - x_1} \Delta y$$



DDA uses repeated addition

We need only compute *m* once, as the start of the scan-conversion.

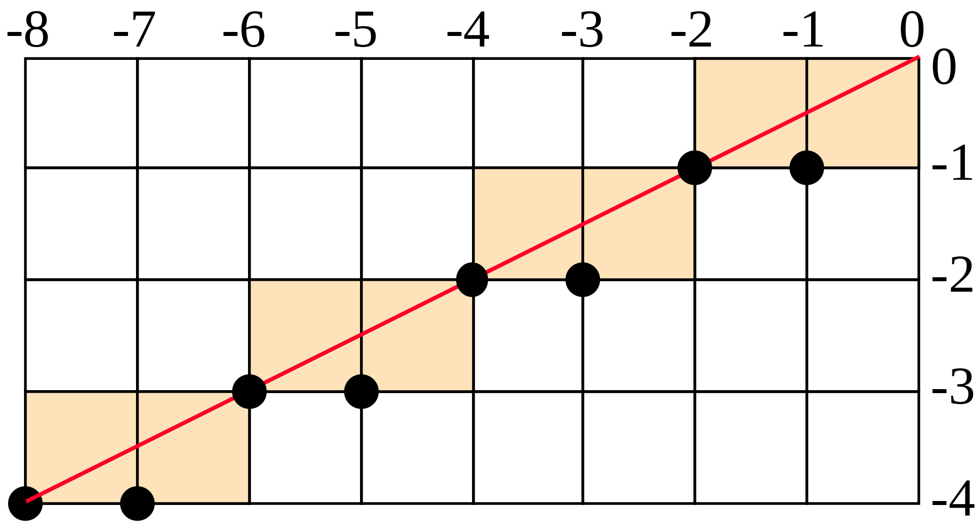
The DDA algorithm runs rather slowly because it requires real arithmetic (floating-point operations).

Line Drawing Algorithms

DDA algorithm for lines with $-1 < m < 1$

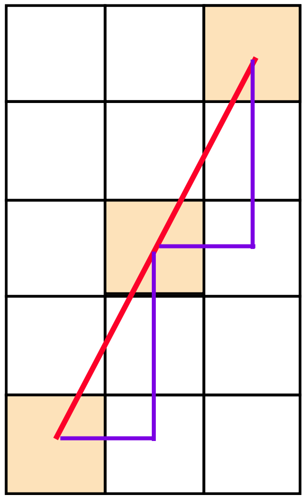
```
x = xl;
ytrue = yl;
while (x <= xr){
      ytrue = ytrue + m;
      y = Round (ytrue);
      PlotPixel (x, y);
      x = x + 1;
}
```

Example: Third quadrant

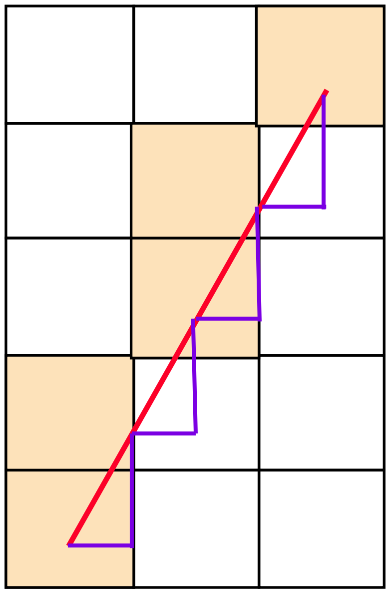


Switching the roles of *x* and *y* when $m>1$

Gaps occur when $m > 1$



Reverse the roles of *x* and *y* using a unit step in *y*, and $1/m$ for *x*.

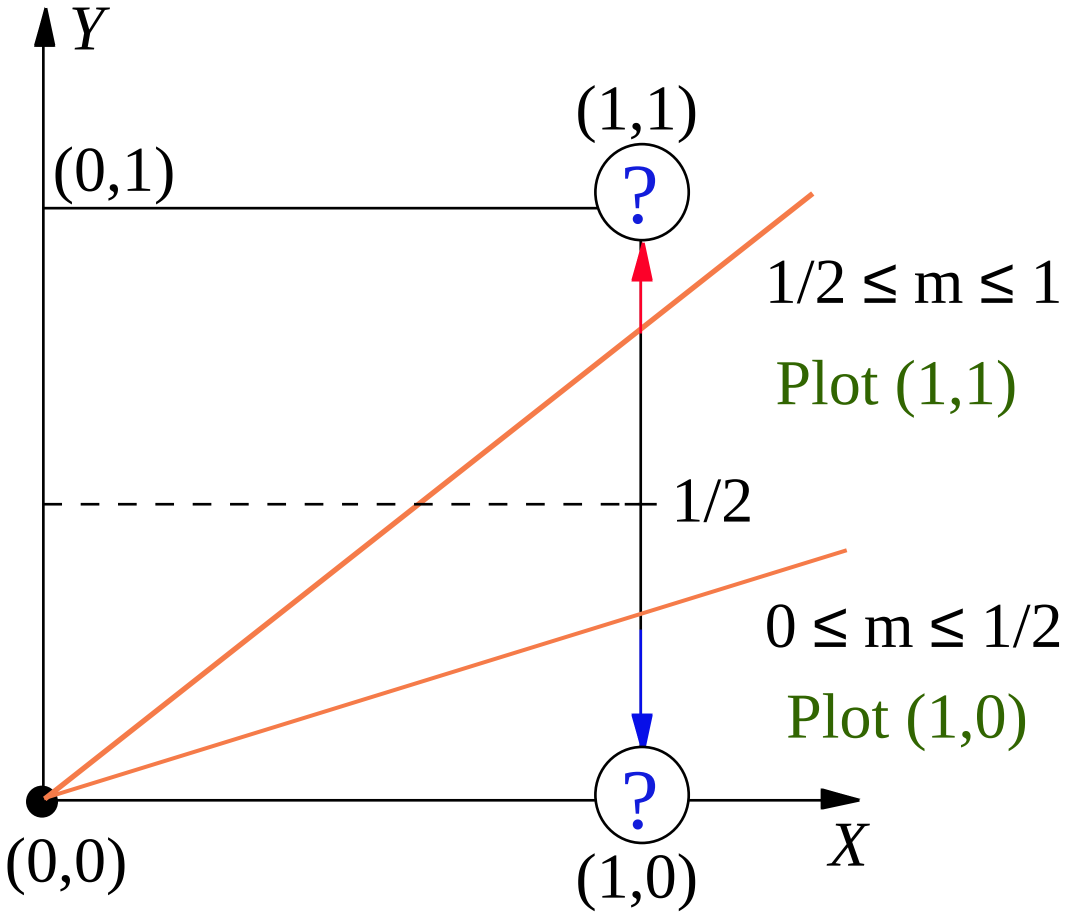


Line Drawing Algorithms

Algorithm 3: Bresenham's algorithm (1965)

Bresenham, J.E. ***Algorithm for computer control of a digital plotter*****, IBM Systems Journal, January 1965, pp. 25-30.**

This algorithm uses only integer arithmetic, and runs significantly faster.



Key idea: distance between the actual line and the nearest grid locations (error).

Initialize error:

$e=-1/2$

Error is given by:

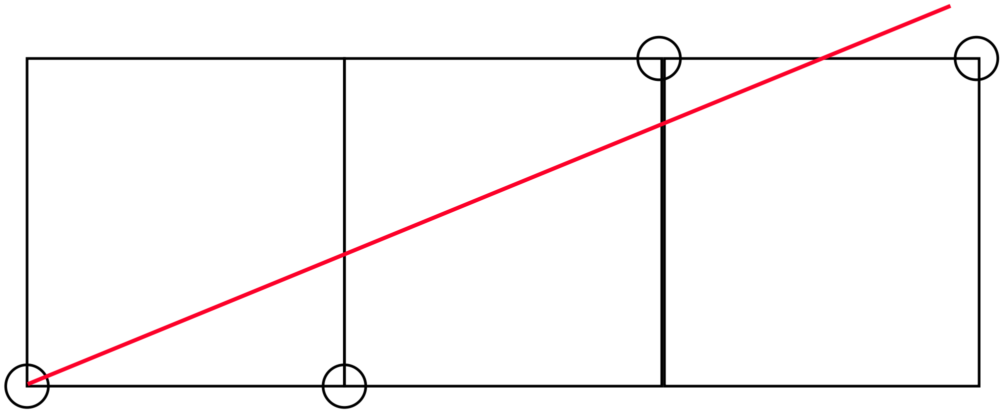
$e=e+m$

Reinitialize error:

when $e>0$

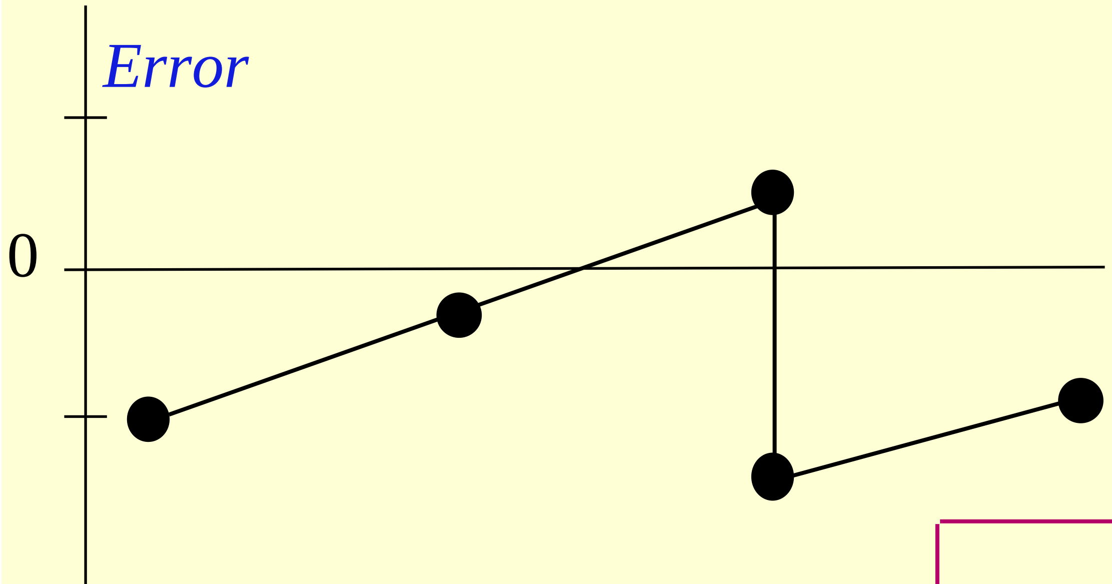
Line Drawing Algorithms

Example: *m=3/8*



If e<0 below
else above

below below above above



Reinitialize error:
e = 1/4 -1
= -3/4

Error: *e=e+m*

Initial value: *e = - 1/2*	*e = -1/2+3/8 =-1/8*	*e = -1/8+3/8 = 1/4*	*e = -3/4+3/8 = -3/8*

Line Drawing Algorithms

However, this algorithm does not lead to integer arithmetic. Scaling by: *2* dx*

```
void Bresenham (int xl, int yl, int xr, int yr)
    {
      int x,y;                    /* coordinates of pixel being drawn  */
      int dy, dx;
      int ne;                     /* integer scaled error term     */
      x = xl; y = yl;             /* start at left endpoint            */
      ie = 2 * dy - dx;           /* initialize the error term */
       while (x <= xr){           /* pixel-drawing loop */
          PlotPixel (x,y);        /* draw the pixel   */
          if (ie > 0) {
             y = y + 1;
             ne = ne - 2 * dx;    /* replaces e = e - 1  */
          }
          x = x + 1;
         ne = ne + 2 * dy;        /* replaces e = e + m  */
       }
    }
```

Line Drawing Algorithms

Bresenham's algorithm also applies for circles.

Bresenham, J.E. *A linear algorithm for incremental digital display of circular arcs* Communications of the ACM, Vol. 20, pp. 100-106, 1977.

Key idea: compute the initial octant only

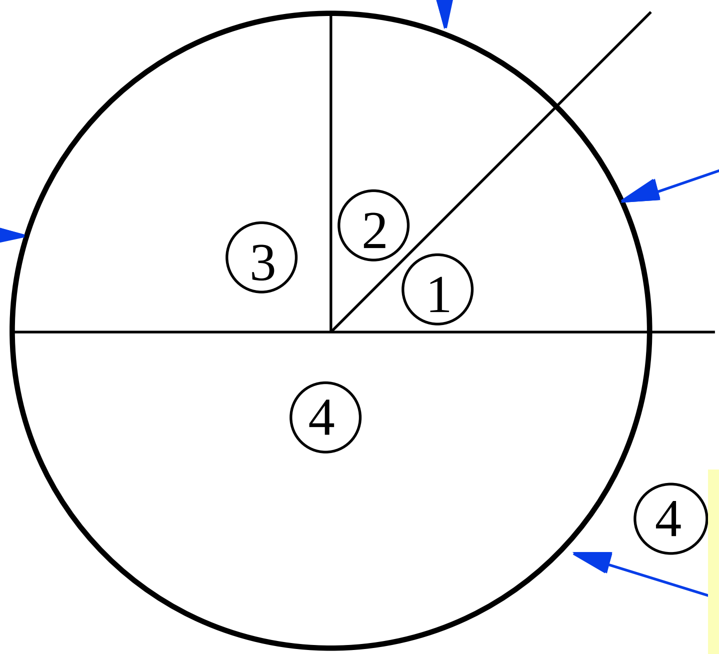
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2 Reflect first octant about $y=x$

1 Generate first octant

3 Reflect first quadrant about $x=0$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



4 Reflect upper semicircle about $y=0$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Line Drawing Algorithms

Bresenham’s incremental circle algorithm.

Example:

circle of radius 8

Bright pixels:

initial pixel → (0,8)
(1,8)
(2,8)
(3,7)
(4,7)
(5,6)
(6,5)
(7,4)
(7,3)
(8,2)
(8,1)
end pixel → (8,0)