Microwave Engineering
ETE 415

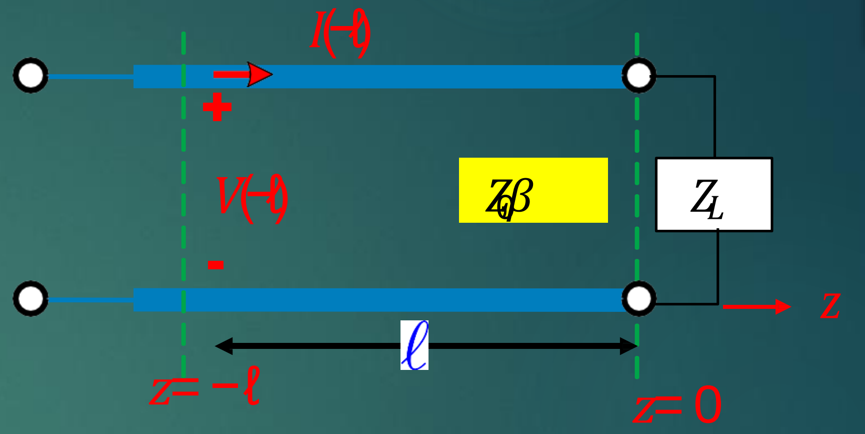
LECTURE 3
TRANSMISSION-LINE THEORY-II

Voltage Standing Wave Ratio

(lossless TL)

- The total phasor voltage as a function of position on a line connected to a load at $z=0$ is

$$V(-\ell) = V_o^+ e^{j\beta\ell}\left(1+\Gamma_L e^{-j2\beta\ell}\right)$$



- Let $\Gamma_L = |\Gamma_L| e^{j\theta}$

$$V(-\ell) = V_o^+ e^{j\beta\ell}\left(1+|\Gamma_L| e^{-j(2\beta\ell-\theta)}\right) \quad \& \quad V^*(-\ell) = V_o^{+*} e^{-j\beta\ell}\left(1+|\Gamma_L| e^{+j(2\beta\ell-\theta)}\right)$$

$$|V(-\ell)| = \sqrt{V(-\ell)V^*(-\ell)}$$

$$= |V_o^+|\sqrt{\left(1+|\Gamma_L| e^{-j(2\beta\ell-\theta)}\right)\left(1+|\Gamma_L| e^{+j(2\beta\ell-\theta)}\right)}$$

$$= |V_o^+|\sqrt{1+|\Gamma_L| e^{+j(2\beta\ell-\theta)}+|\Gamma_L| e^{-j(2\beta\ell-\theta)}+|\Gamma_L|^2}$$

$$= |V_o^+|\sqrt{1+2\Re e\{|\Gamma_L| e^{+j(2\beta\ell-\theta)}\}+|\Gamma_L|^2}$$

$$= |V_o^+|\sqrt{1+|\Gamma_L|^2+2|\Gamma_L|\cos(2\beta\ell-\theta)}$$

The amplitude of voltage as a function of $z=-\ell$

Voltage Standing Wave Ratio

(lossless TL)

$$|V|_{\max} = |V_o^+|\sqrt{1+|\Gamma_L|^2+2|\Gamma_L|} = |V_o^+|(1+|\Gamma_L|)$$

$$2\beta\ell_{\max,n} - \theta = 2n\pi$$

n is an integer

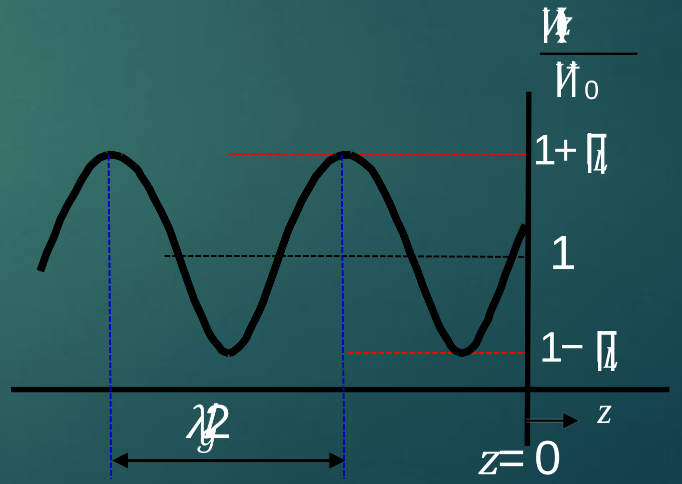
$$|V|_{\min} = |V_o^+|\sqrt{1+|\Gamma_L|^2-2|\Gamma_L|} = |V_o^+|(1-|\Gamma_L|)$$

$$2\beta\ell_{\min,n} - \theta = (2n+1)\pi$$

- The voltage standing wave ratio (VSWR) is defined as

$$\text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \geq 1$$

- For two adjacent maxima at, say, n=1 and n=0 we can write



$$2\beta\ell_{\max,n=1} - \theta = 2\pi \text{ and } 2\beta\ell_{\max,n=0} - \theta = 0$$

$$\Delta\ell = \ell_{\max,n=1} - \ell_{\max,n=0} = \frac{\pi}{\beta} = \frac{\pi}{\frac{2\pi}{\lambda_g}} = \frac{\lambda_g}{2}$$

- Voltage maxima and minima repeat every $\lambda_g/2$.

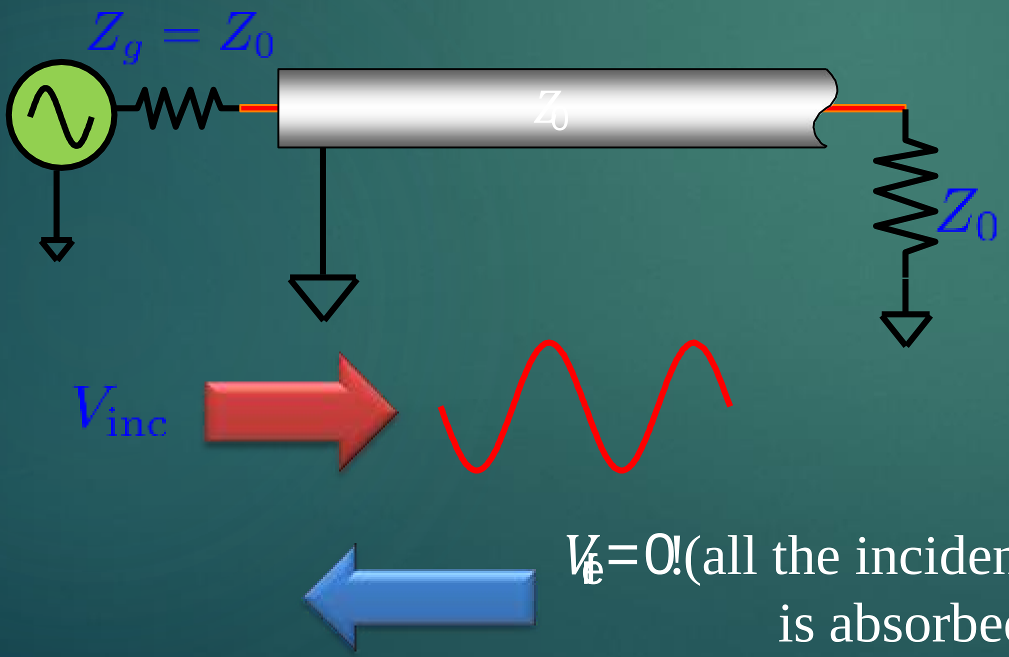
Special cases

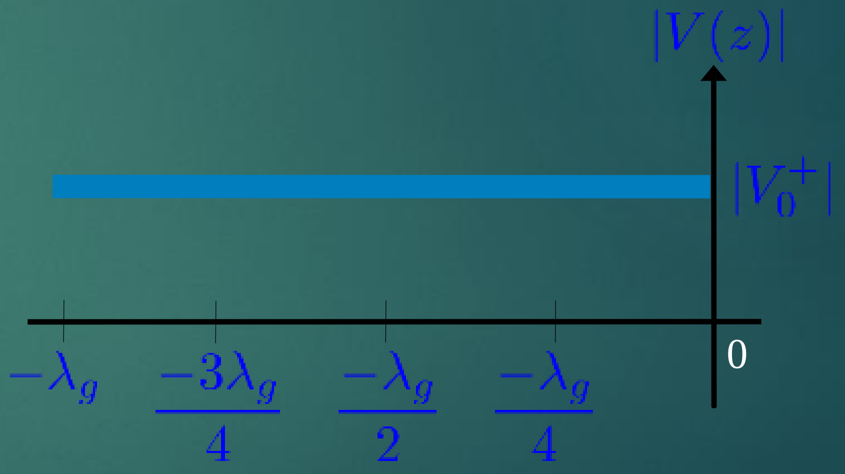
1. If $Z_L = Z_0$ (matched load) then $\Gamma_L = 0$. Consequently, $|\Gamma_L| = 0 \implies \text{VSWR} = 1$.

$$V(z) = V_o^+ e^{-j\beta z} \quad \Rightarrow \quad |V(z)| = |V_o^+|$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} \quad \Rightarrow \quad |I(z)| = \frac{|V_o^+|}{Z_0}$$

for all values of z





Matched line

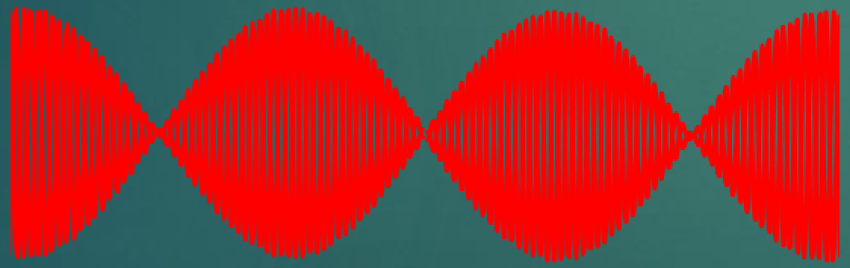
$V_{re} = 0$! (all the incident power is absorbed in the load)

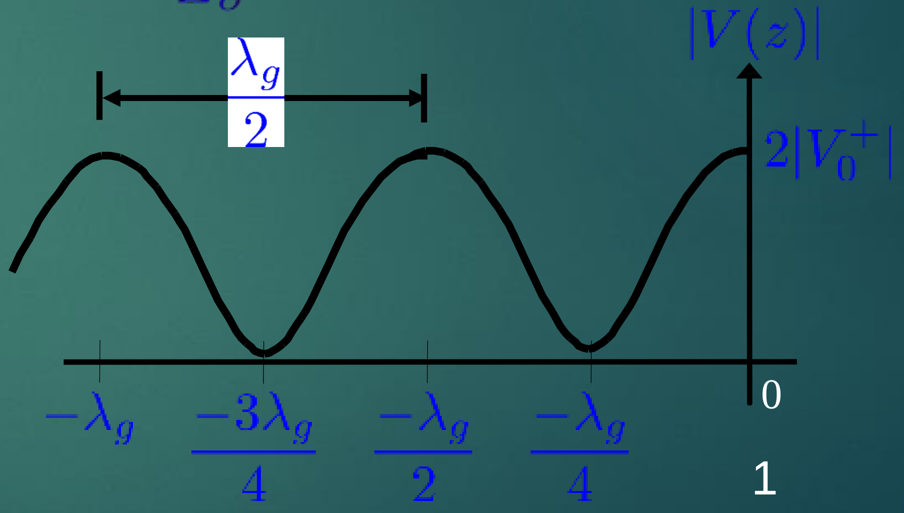
Special cases

2. If $Z_L = \infty$ (open-circuit load) then $\Gamma_L = 1$. Consequently, $|\Gamma_L| = 1 \implies \text{VSWR} = \infty$,

$$V(z) = V_o^+ \left(e^{-j\beta z} + \underbrace{\frac{V_o^-}{V_o^+}}_{\equiv \Gamma_L = 1} e^{j\beta z} \right) = 2V_o^+ \cos(\beta z)$$

$$I(z) = \frac{V_o^+}{Z_0} \left(e^{-j\beta z} - \underbrace{\frac{V_o^-}{V_o^+}}_{\equiv \Gamma_L = 1} e^{j\beta z} \right) = -\frac{j2V_o^+}{Z_o} \sin(\beta z)$$





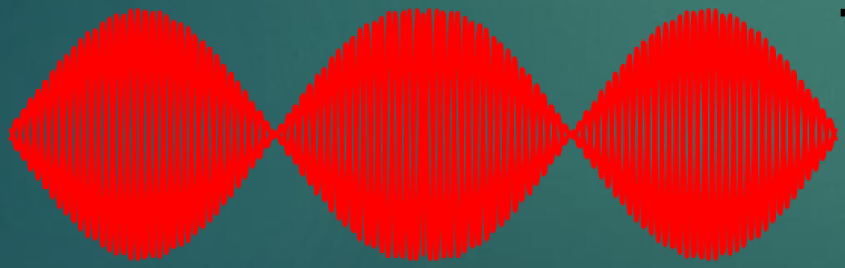
Open circuit

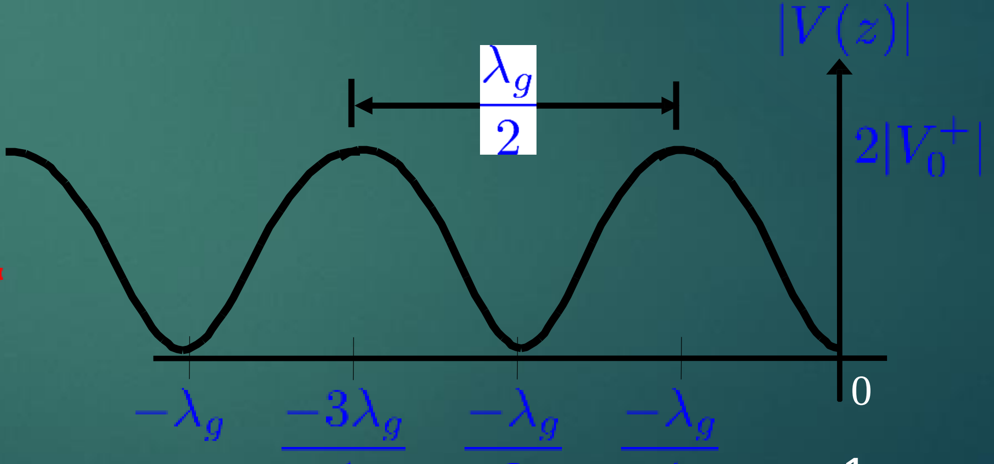
Special cases

3. If $Z_L = 0$ (short-circuit load) then $\Gamma_L = -1$. Consequently, $|\Gamma_L| = 1 \Longrightarrow$ VSWR $= \infty$,

$$V(z) = V_o^+ \left(e^{-j\beta z} + \underbrace{\frac{V_o^-}{V_o^+}}_{\equiv \Gamma_L = -1} e^{j\beta z} \right) = -j2V_o^+ \sin(\beta z)$$

$$I(z) = \frac{V_o^+}{Z_0} \left(e^{-j\beta z} - \underbrace{\frac{V_o^-}{V_o^+}}_{\equiv \Gamma_L = -1} e^{j\beta z} \right) = \frac{2V_o^+}{Z_o} \cos(\beta z)$$

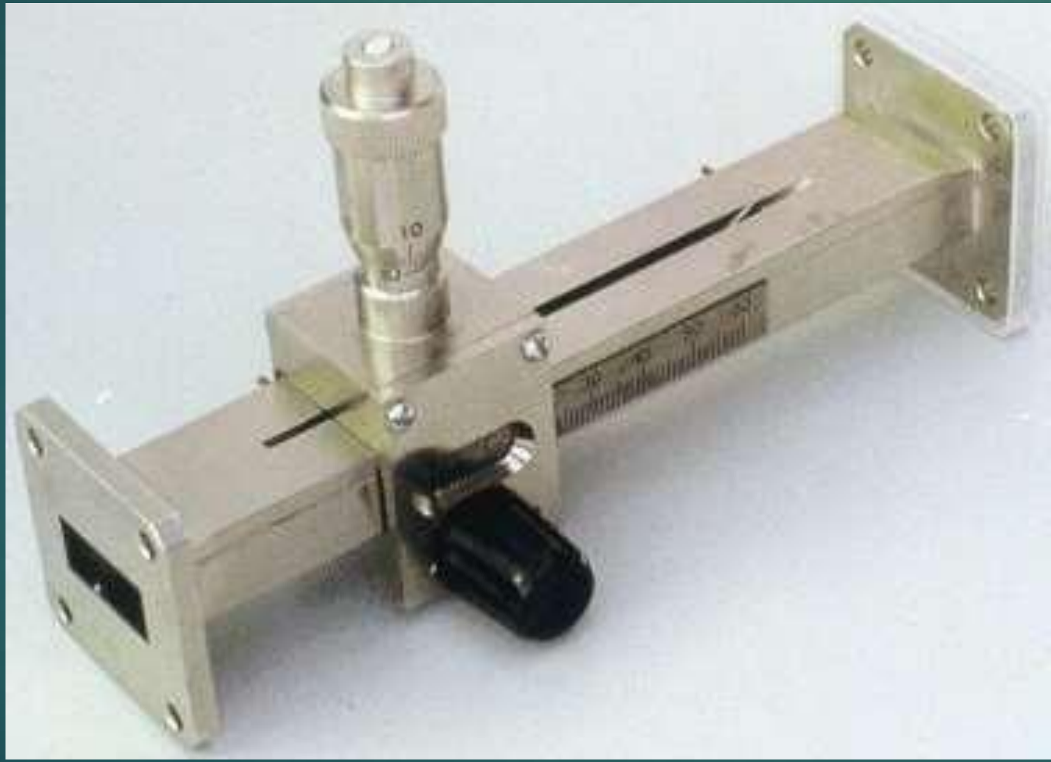




Short circuit

The Slotted Line

- A **slotted line** is a TL configuration (usually a waveguide or coaxial line) that allows the sampling of the electric field amplitude of a standing wave on a terminated line.



- Used to measure:
  - SWR
  - λ
  - ℓ_{min} ℓ_{max}
  - Z_L

Slotted Line Measurement Procedure

- Calculate reflection coefficient magnitude from SWR

$$\text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \quad \Rightarrow \quad \boxed{|\Gamma_L| = \frac{\text{VSWR}-1}{\text{VSWR}+1}}$$

- locations of voltage minima respective to load terminals are:

$$\boxed{2\beta\ell_{\min,n} - \theta = (2n+1)\pi}$$

$$\ell_{\min,n} = (2n+1)\frac{\lambda_g}{4} + \frac{\theta}{\pi}\cdot\frac{\lambda_g}{4}, n = 0, 1, \ldots$$

$$\Rightarrow \theta = 2\beta\ell_{\min,n} - (2n+1)\pi, n = 0, 1, \ldots$$

$$\boxed{2\beta = \frac{4\pi}{\lambda_g}}$$

- For $0 \le \ell_{min} \le \frac{\lambda_g}{2}$ (1st minimum) and $-\pi \le \theta < \pi$

$$\boxed{\theta = 2\beta\ell_{\min} - \pi}$$

See example 2.4 pp. 70-72

- load impedance is calculated as

$$Z_L = Z_0\frac{1+\Gamma_L}{1-\Gamma_L} = \boxed{Z_0\frac{1+|\Gamma_L|e^{j\theta}}{1-|\Gamma_L|e^{j\theta}}}$$

Time-Average Power Flow on TLs

- A hugely important part of MW engineering is **delivering signal power to a load**.
- **Time average power** $(P_{in})_{av}$ delivered to input of TL, i.e., $z = -\ell$

$$(P_{\text{in}})_{\text{av}} = \frac{1}{2}\Re e\left[V(-\ell)I^*(-\ell)\right]$$

similar to that used in circuit analysis.

$$V(-\ell) = V_0^+\left(e^{\alpha\ell}e^{j\beta\ell} + \Gamma_L e^{-\alpha\ell}e^{-j\beta\ell}\right) \ \& \ I(-\ell) = \frac{V_0^+}{Z_0}\left(e^{\alpha\ell}e^{j\beta\ell} - \Gamma_L e^{-\alpha\ell}e^{-j\beta\ell}\right)$$

$$I^*(-\ell) = \frac{V_0^{+*}}{Z_0}\left(e^{\alpha\ell}e^{-j\beta\ell} - \Gamma_L^* e^{-\alpha\ell}e^{+j\beta\ell}\right)$$

Assume Z_o is real

$$(P_{\text{in}})_{\text{av}} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\Re e\left[e^{2\alpha\ell} - |\Gamma_L|e^{+j(2\beta\ell-\theta} + |\Gamma_L|e^{-j(2\beta\ell-\theta)} - |\Gamma_L|^2 e^{-2\alpha\ell}\right]$$

$$= \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\Re e\left[e^{2\alpha\ell} - j2|\Gamma_L|\sin(\beta\ell-\theta) - |\Gamma_L|^2 e^{-2\alpha\ell}\right]$$

$$= (P_i)_{\text{av}}\left[e^{2\alpha\ell} - |\Gamma_L|^2 e^{-2\alpha\ell}\right] \quad [\text{W}]$$

Forward ($e^{2\alpha\ell}$) **Backward** ($|\Gamma_L|^2 e^{-2\alpha\ell}$)

$$(P_i)_{\text{av}} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}$$

Time-Average Power Flow on TLs

- Power delivered to load may be found by setting $\ell = 0$

$$(P_L)_{\rm av} = (P_i)_{\rm av}\left(1 - |\Gamma_L|^2\right) \quad [\rm W]$$

- For **entirely reactive** load, $|\Gamma_L| = 1$, then $(P_L)_{av} = 0$ and no time average power is delivered to the load, as expected.

  For all other passive loads, $(P_L)_{av} > 0$.

- the power dissipated in TL is the difference between the input power and the power delivered to the load

$$P_{\rm loss} = (P_{\rm in})_{\rm av} - (P_{\rm L})_{\rm av}$$

$$= (P_i)_{\rm av}\left[|\Gamma_L|^2(1 - e^{-2\alpha\ell}) + e^{2\alpha\ell} - 1\right]$$

- For lossless TL, i.e., $\alpha = 0$

$$(P_{\rm in})_{\rm av} = (P_{\rm L})_{\rm av} = (P_i)_{\rm av}\left(1 - |\Gamma_L|^2\right) \quad \Rightarrow \quad P_{\rm loss} = 0$$

Time-Average Power Flow on TLs

Return loss (RL)

- The **relative reflected time average power** from an arbitrary load on a lossless TL $= |\Gamma_L|^2$.
- The relative time average power that is not delivered to the load can be considered a **"loss"**
  - since the signal from the generator was intended to be completely transported – not returned to the generator.
- This **return loss (RL)** is defined as

$$\mathrm{RL} = -10\log_{10}\left(|\Gamma_L|^2\right) = -20\log_{10}\left(|\Gamma_L|\right) \quad \mathrm{dB}$$

- The two extremes for return loss with a passive load are:
  1. **A matched load** where $\Gamma_L = 0$ and RL=∞ dB (no reflected power)
  2. **A reactive load** where $|\Gamma_L| = 1$ and RL=0 dB (all power reflected).

Dispersion

(lossy TL)

- From before, we know $\beta = \frac{2\pi}{\lambda_g} = \frac{\omega}{v_p} \Rightarrow$ $v_p = \frac{w}{\beta}$ $\gamma = \alpha + j\beta$
- For a lossy line, $\beta = \mathrm{Im}\{\sqrt{(R' + jwL')(G' + jwC')}\}$
- Then, $v_p = \dfrac{w}{\mathrm{Im}\{\sqrt{(R' + jwL')(G' + jwC')}\}}$

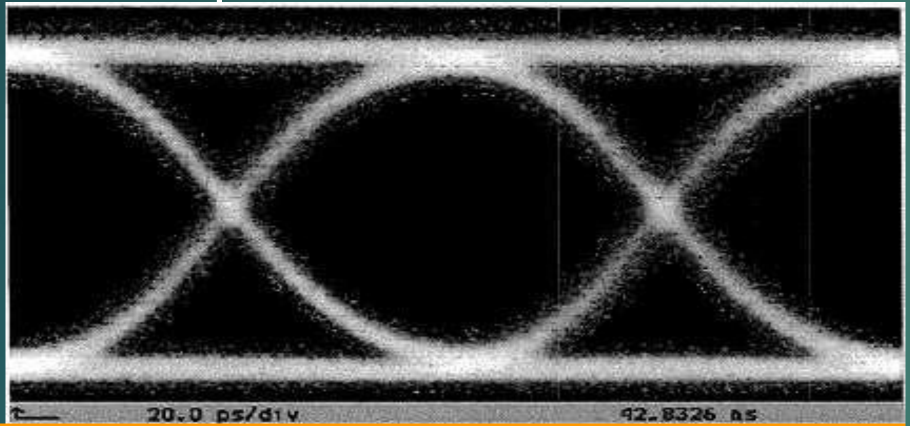
v_p is dependent on frequency!!

Is this a problem?

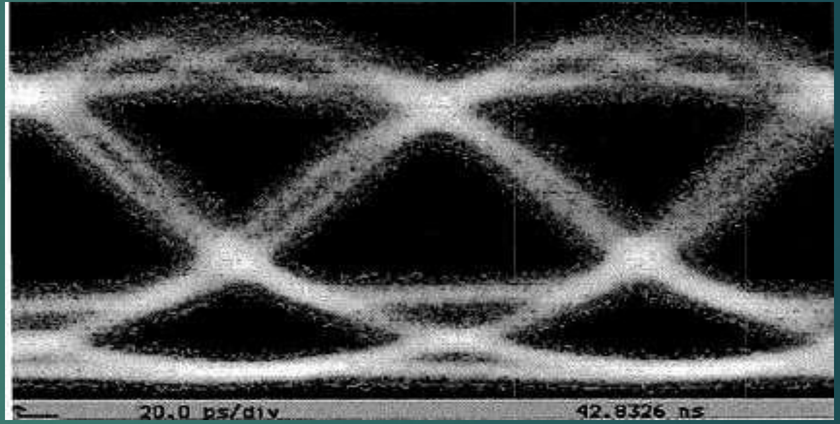
- Signals that carry any information must occupy some bandwidth.
  - FM broadcast (stereo) ⇒occupies bandwidth ≈ 100 kHz
  - AM analog modulation ⇒occupies bandwidth ≈ 6 kHz
- What about digital signals?
  - A high-speed digital signal has t_r & t_f on the order of 10's of ps.

Dispersion

- Many high frequency components make up the edges.
- What if each frequency travels at a different velocity?
  - When each frequency making up an edge travels at a different speed, the waveform edges get ―smeared‖ over time and space. Dispersion!

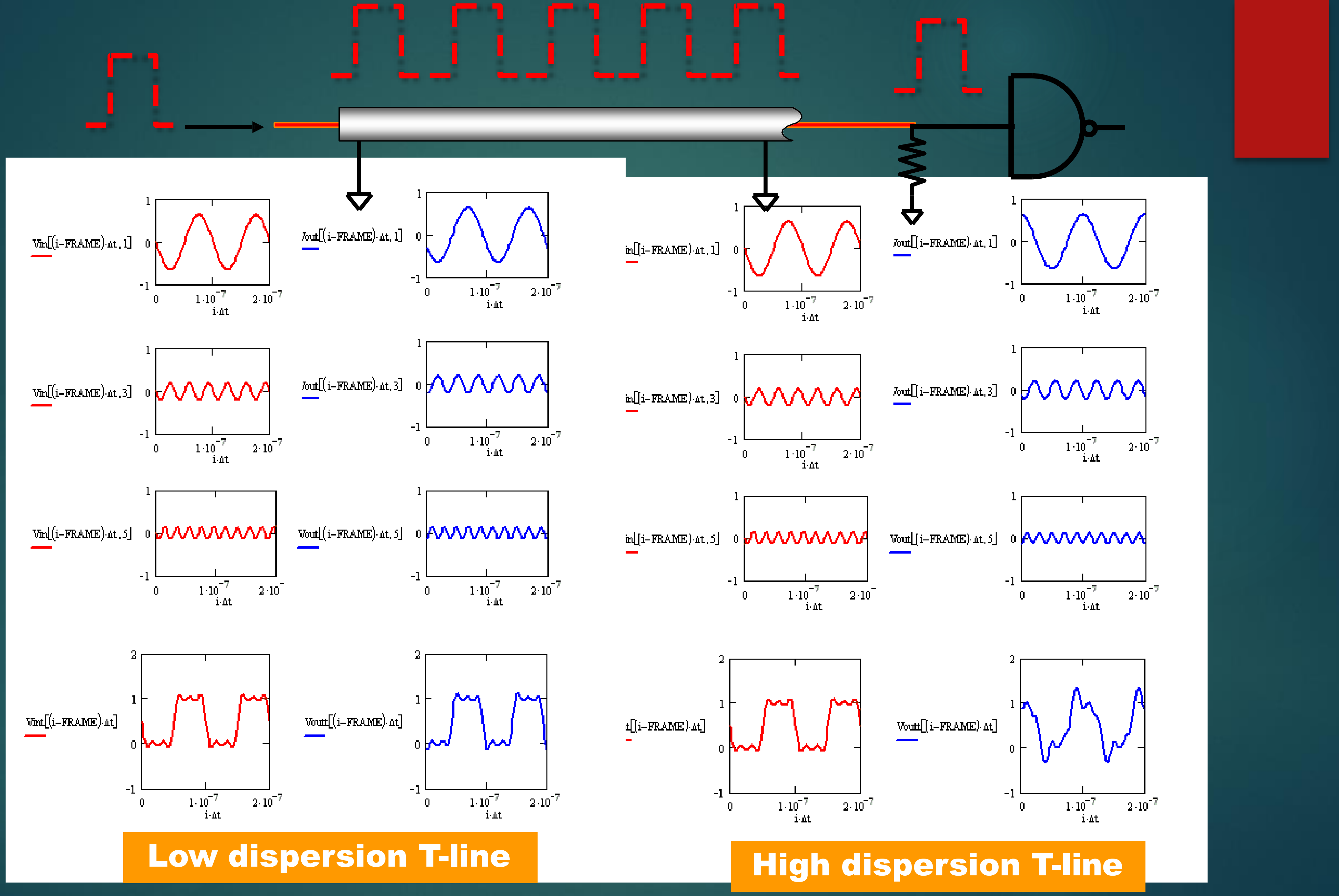


Transmitted signal @ ≈ 5 GHz "A very high-speed digital signal"



Received signal after travelling down a lossy TL with loss.

- Lossy lines can strongly affect high-speed digital signals.
- What about lower frequencies/longer lines?



Low dispersion T-line
High dispersion T-line

Distortionless Line

- The complex propagation constant

$$\gamma = \sqrt{(R' + jwL')(G' + jwC')}$$

- Can be rearranged as

$$\gamma = \sqrt{(jwL')(jwC')\left(1 + \frac{R'}{jwL'}\right)\left(1 + \frac{G'}{jwC'}\right)}$$

$$= jw\sqrt{L'C'}\sqrt{1 - j\left(\frac{R'}{wL'} + \frac{G'}{wC'}\right) - \frac{R'G'}{w^2L'C'}}$$

- For $\frac{R'}{L'} = \frac{G'}{C'}$

$$\gamma = jw\sqrt{L'C'}\sqrt{1 - 2j\frac{R'}{wL'} - \left(\frac{R'}{wL'}\right)^2}$$

$$= jw\sqrt{L'C'}\left(1 - j\frac{R'}{wL'}\right)$$

$$= R'\sqrt{\frac{C'}{L'}} + jw\sqrt{L'C'} = \alpha + j\beta$$

$\beta = w\sqrt{L'C'}$ ⇒ **linear function of frequency**

$$v_p = \frac{w}{w\sqrt{L'C'}} = \frac{1}{\sqrt{L'C'}}$$

$\alpha = R'\sqrt{C'/L'}$ ⇒ **independent of frequency**

The Low-Loss Line

- In most practical microwave and RF transmission lines the loss is small—if this were not the case, the line would be of little practical value.
- When the loss is small, some approximations can be made to simplify the expressions for the general transmission line parameters of $\gamma=\alpha+\beta$ and Z_0.

$$\gamma = jw\sqrt{L'C'}\sqrt{1 - j\left(\frac{R'}{wL'} + \frac{G'}{wC'}\right) - \frac{R'G'}{w^2 L'C'}}$$

- For a low-loss line both conductor and dielectric loss will be small, and we can assume that $R' \ll \omega L'$ and $G' \ll \omega C'$. Then, $R'G' \ll \omega^2 L'C'$, and above equation reduces to

$$\gamma \simeq jw\sqrt{L'C'}\sqrt{1 - j\left(\frac{R'}{wL'} + \frac{G'}{wC'}\right)}$$

$$\alpha + j\beta \simeq jw\sqrt{L'C'}\left[1 - \frac{j}{2}\left(\frac{R'}{wL'} + \frac{G'}{wC'}\right)\right]$$

Taylor approximation:

$$\sqrt{1\pm x} \approx 1 \pm \frac{x}{2}, \quad |x| \ll 1$$

The Low-Loss Line

$$\alpha = \mathrm{Re}\{\gamma\}$$

$$= \frac{w}{2}\sqrt{L'C'}\left(\frac{R'}{wL'} + \frac{G'}{wC'}\right) = \frac{1}{2}\left(\frac{\sqrt{L'C'}R'}{L'} + \frac{\sqrt{L'C'}G'}{C'}\right)$$

$$= \frac{1}{2}\left(R'\sqrt{\frac{C'}{L'}} + G'\sqrt{\frac{L'}{C'}}\right) = \frac{1}{2}\left(\frac{R'}{Z_o} + G'Z_o\right)$$

Higher Z_o increases dielectric loss and reduces conduction loss

$$\beta = \mathrm{Im}\{\gamma\} = w\sqrt{L'C'}$$

Same as no loss line⇒ negligible dispersion

the characteristic impedance Z_0 can be approximated as a real quantity:

$$Z_0 = \sqrt{\frac{R' + jwL'}{G' + jwC'}} \simeq \sqrt{\frac{L'}{C'}}$$

Thank you Very Much !!!