# Microwave Engineering
## ETE 415

**LECTURE 4**

**SMITH CHART**

# The Smith Chart

- It was developed by **Phillip H. Smith** in the 1930s.
- Began its existence as a very useful **graphical calculator** for the analysis and design of TLs.
- Remains a useful tool today to visualize the results of TL analysis, oftentimes combined with computer analysis and visualization as an aid in design.
- Based on the **normalized** **TL impedance** defined as

$$\tilde{z} \equiv \frac{Z(z)}{Z_0} = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

  - where  is the total TL impedance at  and

$$\Gamma(z) = \frac{\tilde{z}-1}{\tilde{z}+1} = \Gamma_L e^{+j2\beta z}$$

**generalized reflection coefficient at** $z$**.**

- Substituting $\Gamma(z) \equiv \Gamma_r(z) + j\Gamma_i(z)$, gives

$$\tilde{z} = \frac{1+\Gamma(z)}{1-\Gamma(z)} \quad \Rightarrow \quad \tilde{z}(z) = \frac{1+(\Gamma_r + j\Gamma_i)}{1-(\Gamma_r + j\Gamma_i)}$$

# The Smith Chart

- Now, we will define $\tilde{z}(z) \equiv r + jx$ and separate the above equation into its real and imaginary parts

$$\tilde{z}(z) \equiv r + jx = \frac{1 + (\Gamma_r + j\Gamma_i)}{1 - (\Gamma_r + j\Gamma_i)} \cdot \frac{1 - (\Gamma_r + j\Gamma_i)^*}{1 - (\Gamma_r + j\Gamma_i)^*}$$

$$= \frac{1 + j2\Gamma_i - (\Gamma_r^2 + \Gamma_i^2)}{1 - 2\Gamma_r + \Gamma_r^2 + \Gamma_i^2}$$

- Equating the real and imaginary parts

$$r = \frac{1 - (\Gamma_r^2 + \Gamma_i^2)}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad \text{and} \quad x = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

- Rearranging both of these leads us to the final two equations

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$

**resistance circles**

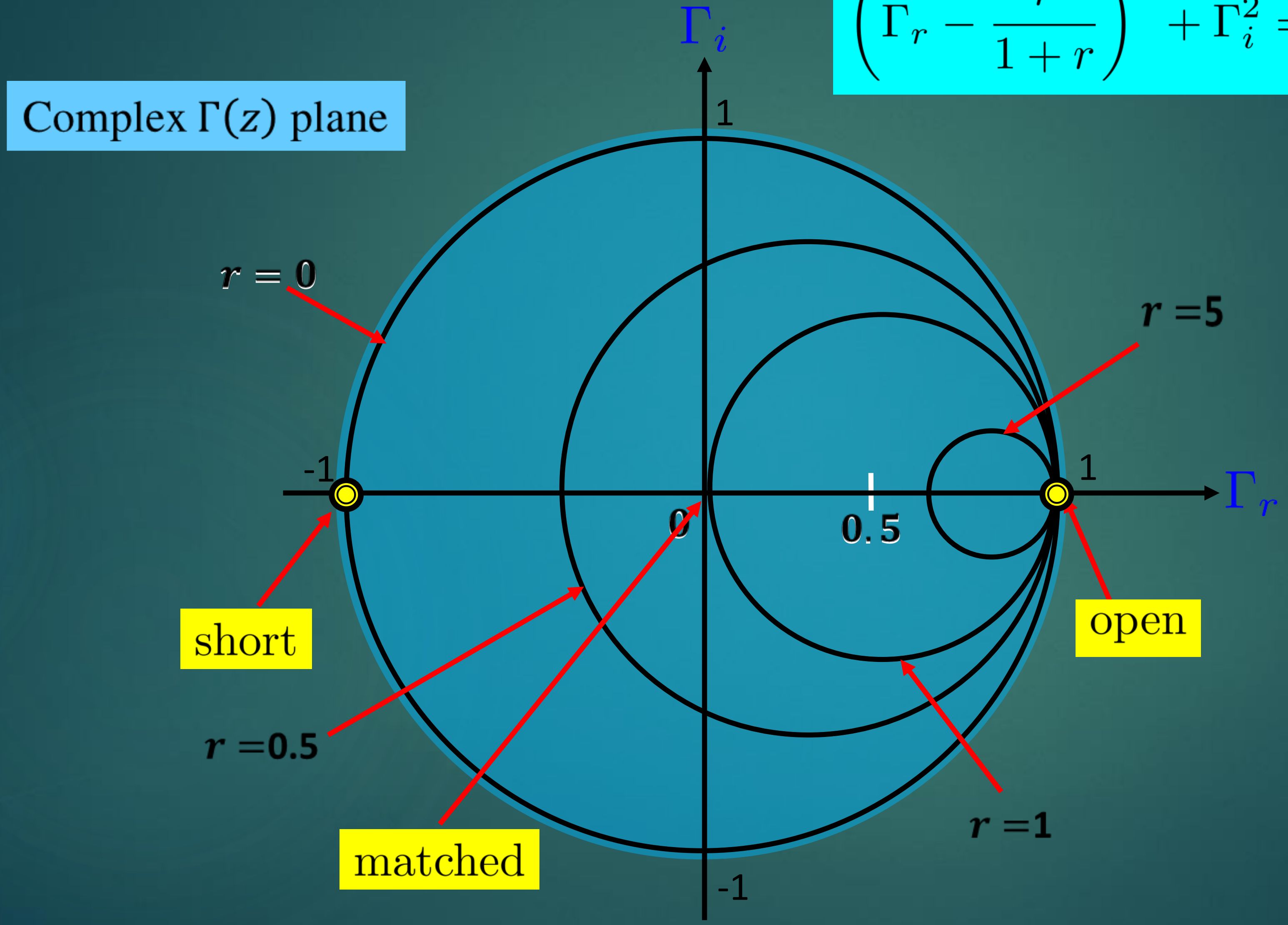
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

**reactance circles**

# The Smith Chart: Resistance Circles

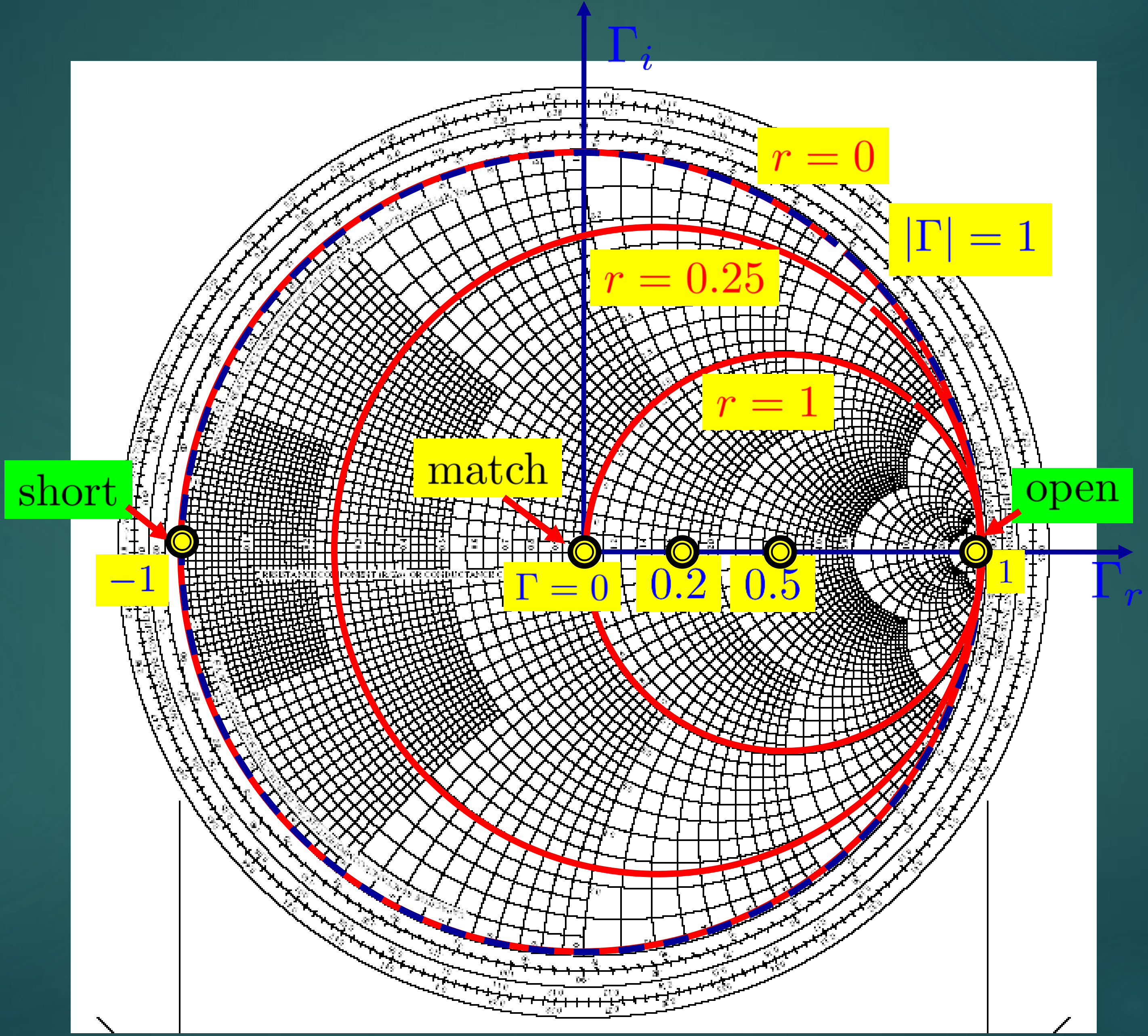
- **Plot (4) in the $\Gamma_r - \Gamma_i$ plane:**

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$




- **resistance circles** have centers lying on the $\Gamma_r$ axis (with $\Gamma_i = 0$)

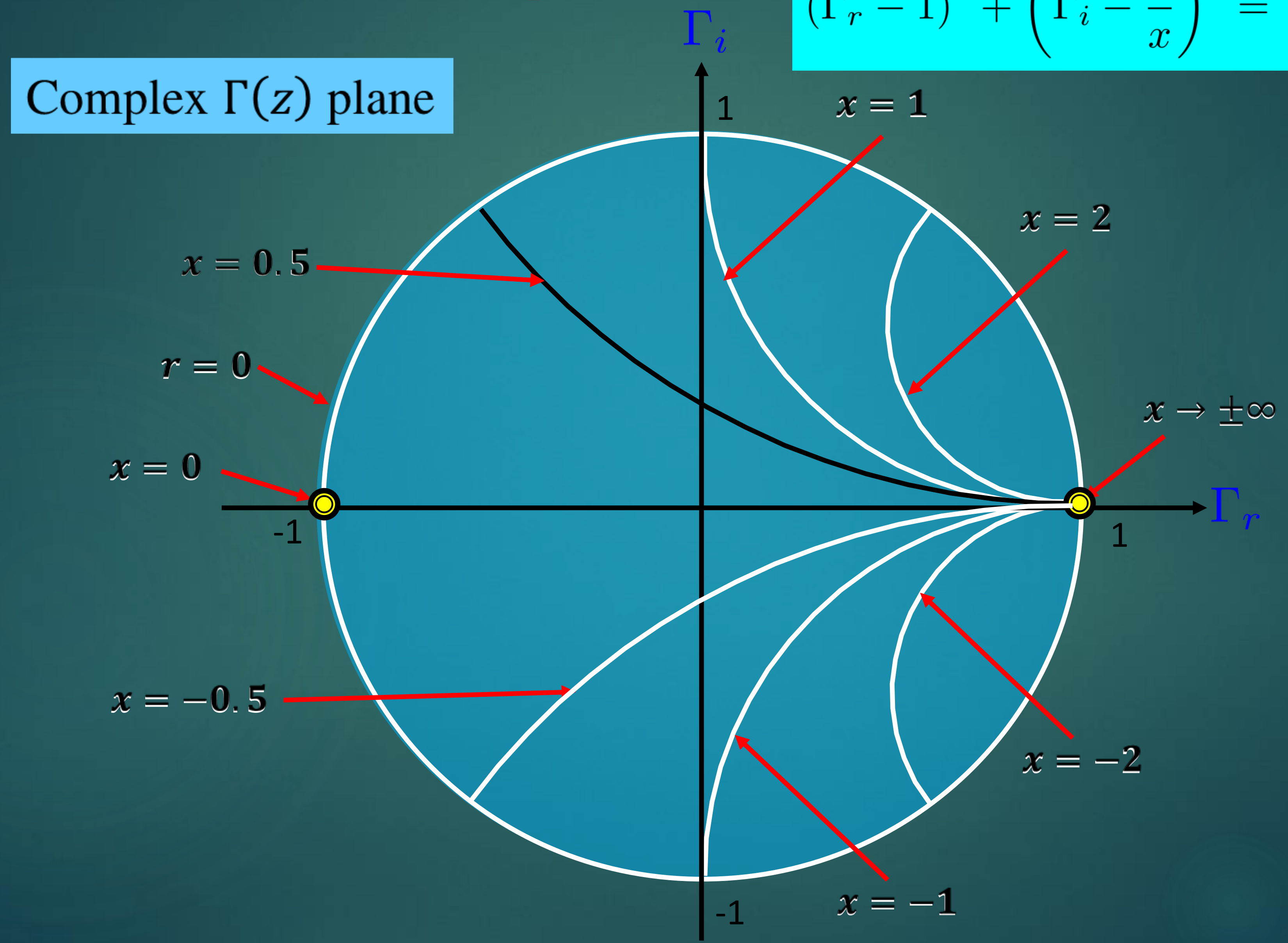
# The Smith Chart: Resistance Circles



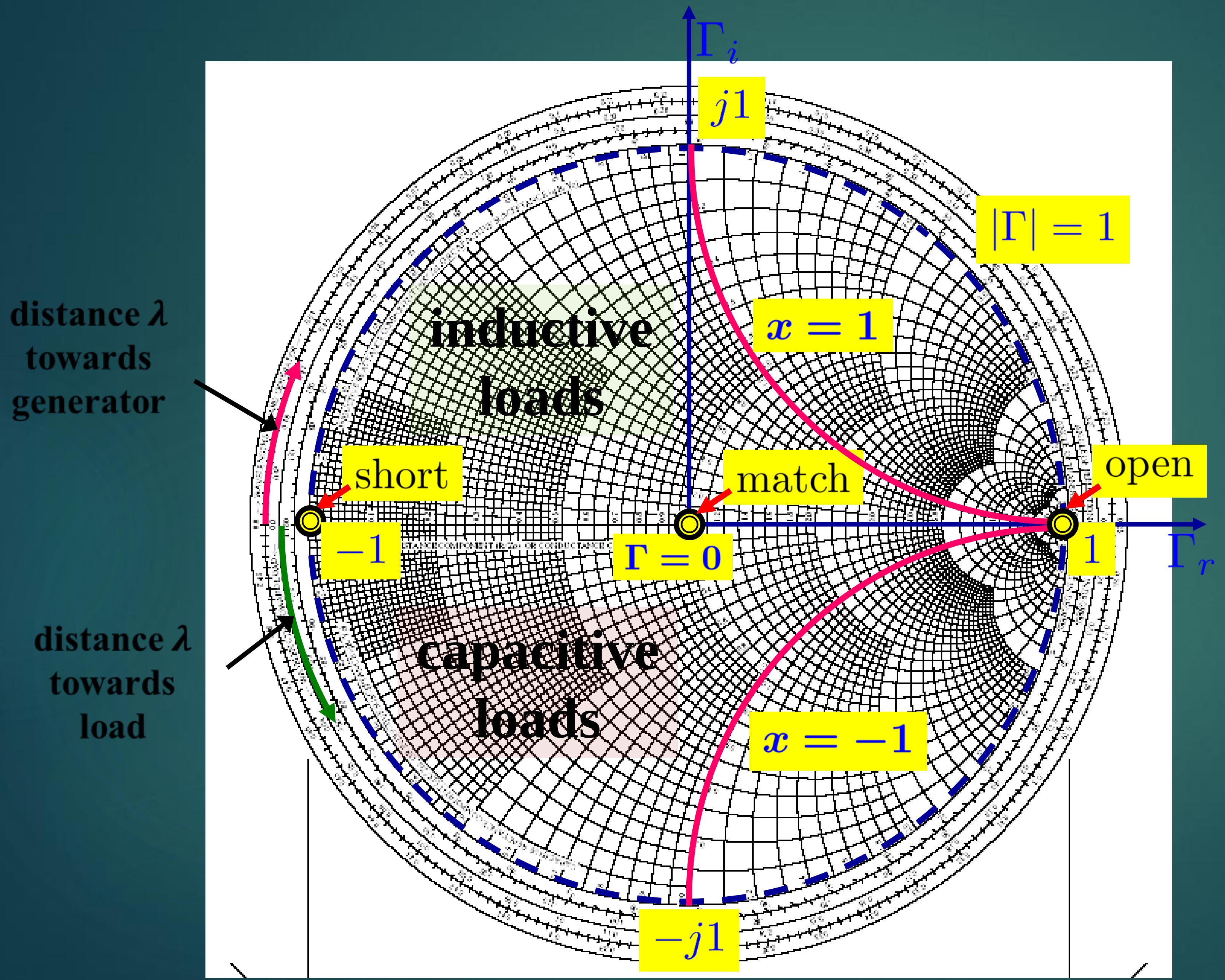
# The Smith Chart: Reactance Circles

**Plot (5) in the $\Gamma_r - \Gamma_i$ plane:**

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$



# The Smith Chart: Reactance Circles

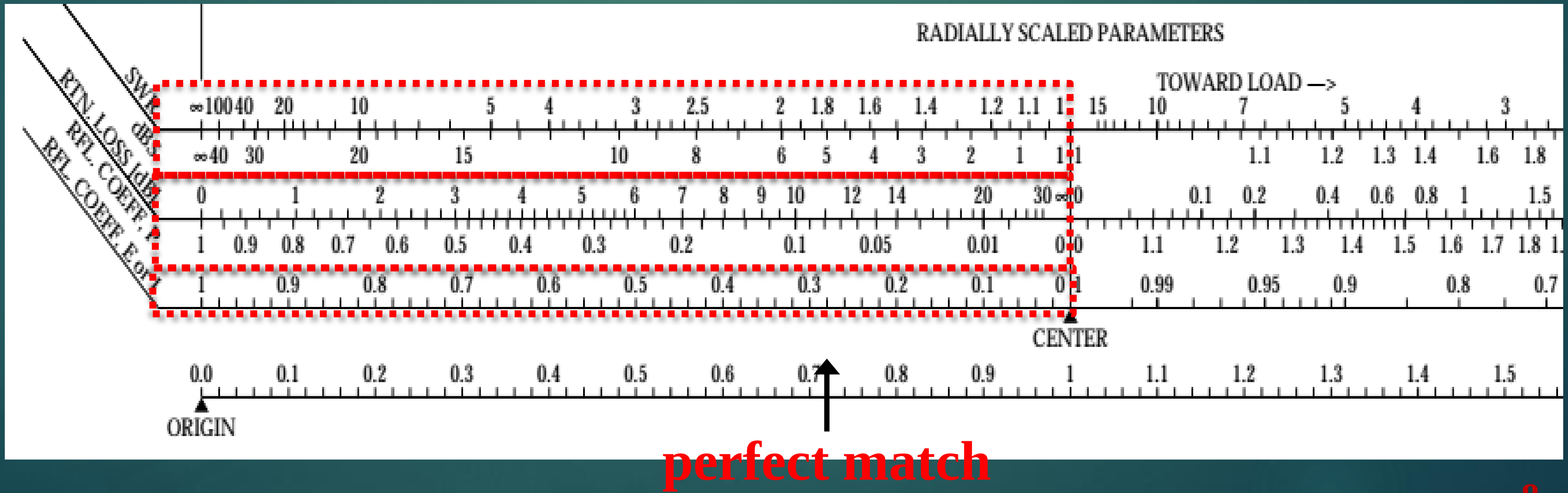


# The Smith Chart: Nomographs

- At the bottom of Smith chart (**left side**), nomograph is added to read out with a ruler the following
- **(1st ruler) above**: SWR

  **below**: SWR in dB, $20\log_{10}\mathrm{SWR}$
- **(2nd ruler) above**: return loss in dB, $-20\log_{10}|\Gamma|$

  **below**: power reflection $|\Gamma|^2$(P)
- **(3rd ruler) above**: reflection coefficient $|\Gamma|$ (E or I)



# Given $Z(-\ell) \Rightarrow$ Find $\Gamma(-\ell)$

- Normalize the impedance

$$\tilde{z}(-\ell) = \frac{Z(-\ell)}{Z_o} = \frac{R}{Z_0} + j\frac{X}{Z_0} = r + jx$$

- Find the circle of constant **normalized resistance *r***.
- Find the arc of constant **normalized reactance *x*** .
- The **intersection** of the two curves indicates the reflection coefficient in the complex plane.

The chart provides directly the magnitude and the phase angle of $\Gamma(-\ell)$ .

- **Example:** Find $\Gamma(-\ell)$ , given

$$Z(-\ell) = 25 + j100\,\Omega \text{ with } Z_0 = 50\,\Omega$$

**1. Normalization**

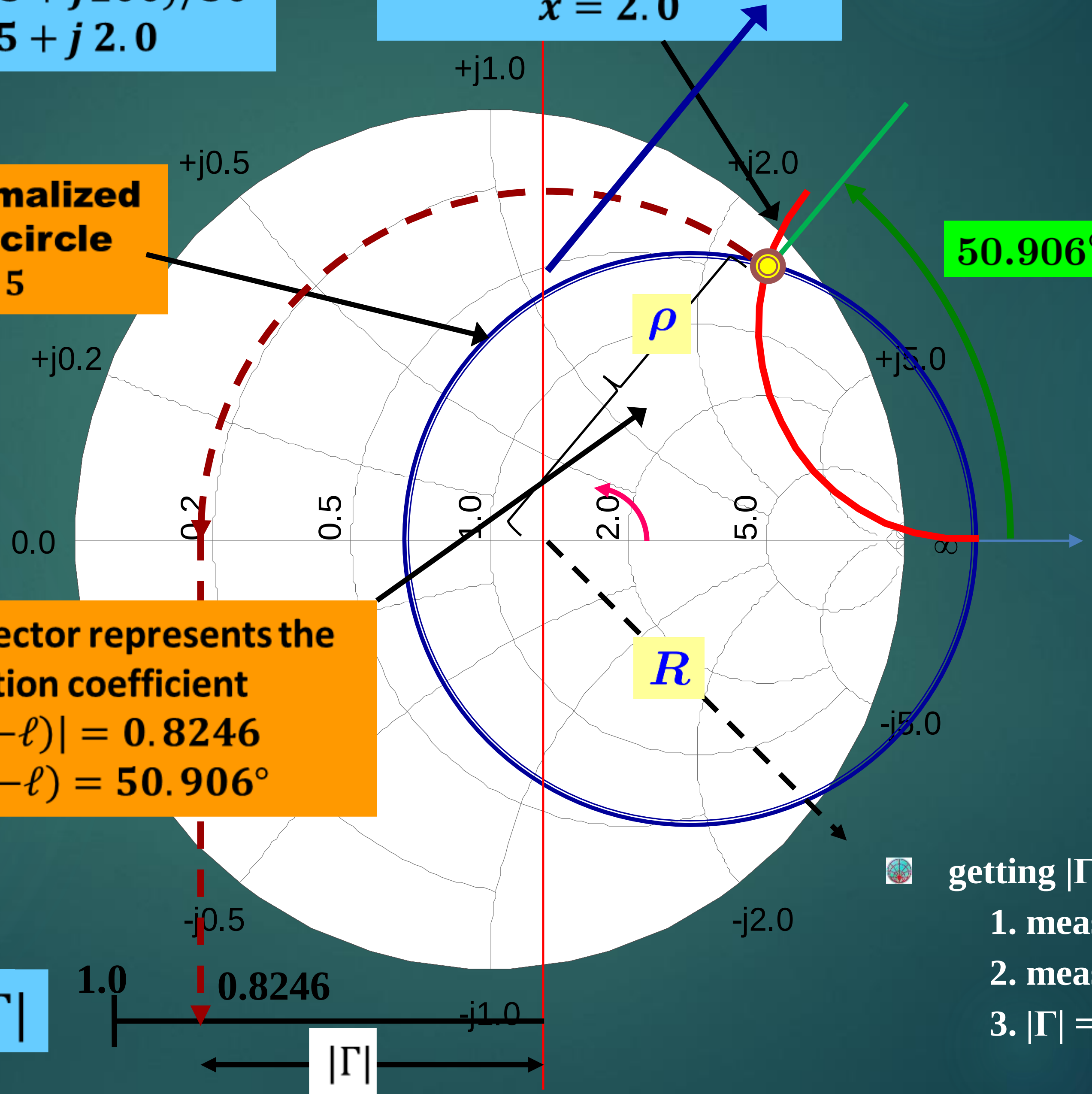
$$\tilde{z}(-\ell) = (25 + j100)/50 = 0.5 + j\,2.0$$

**2. Find normalized resistance circle**

$$r = 0.5$$

**3. Find normalized reactance arc**

$$x = 2.0$$




**4. This vector represents the reflection coefficient**

$$|\Gamma(-\ell)| = 0.8246$$

$$\angle\Gamma(-\ell) = 50.906°$$

3rd ruler $|\Gamma|$

- getting $|\Gamma|$ with a ruler:
  1. measure R
  2. measure ρ
  3. $|\Gamma| = \rho/R$

# Plotting Γ and Reading Out Impedance

- Determine the complex point representing the given reflection coefficient $\Gamma(-\ell)$ on the chart.

- Read the values of the normalized resistance $r$ and of the normalized reactance $x$ that correspond to the reflection coefficient point.

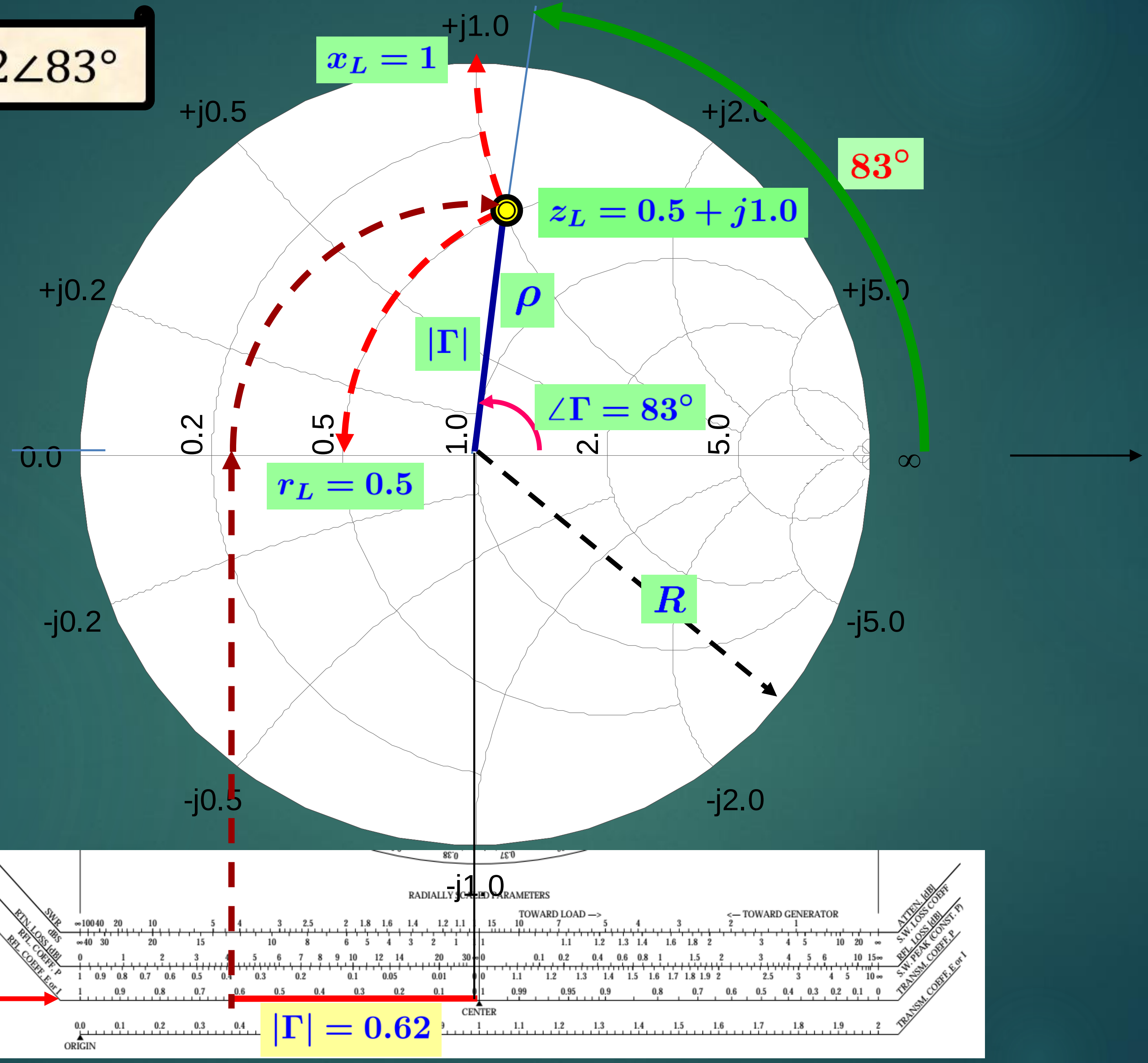
- The normalized impedance is

$$\tilde{z}(-\ell) = r + jx$$

and the actual impedance is

$$Z(-\ell) = Z_0 \tilde{z}(-\ell) = Z_0 (r + jx)$$

**Example:** $\Gamma = 0.62\angle 83°$




3rd ruler

# Tracking Impedance Changes with $\ell$

- At $z = -\ell, \Gamma(-\ell) = \Gamma_L e^{-j2\beta\ell}$
- **NOTE:** the magnitude of the reflection coefficient is constant along a loss-less TL terminated by a specified load, since


$$|\Gamma(-\ell)| = |\Gamma_L e^{-j2\beta\ell}| = |\Gamma_L|$$

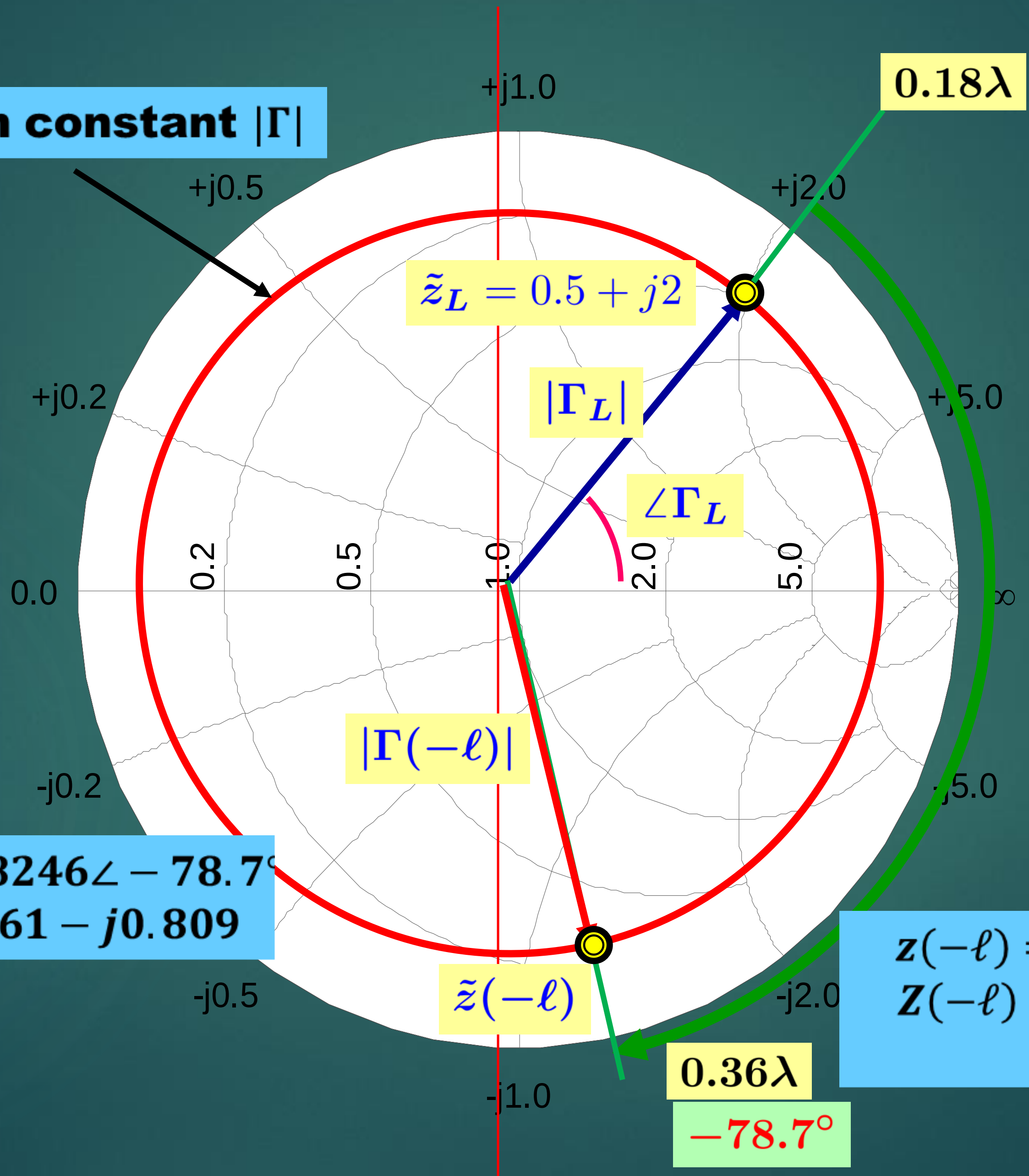
$$Z(-\ell) = Z_0 \frac{1 + \Gamma_L e^{-j2\beta\ell}}{1 - \Gamma_L e^{-j2\beta\ell}} \quad \Rightarrow \quad \tilde{z}(-\ell) = \frac{1 + \Gamma_L e^{-j2\beta\ell}}{1 - \Gamma_L e^{-j2\beta\ell}}$$

- compare with at load ($z = 0$)

$$\tilde{z}_L(0) = \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

- on Smith chart, the point corresponding to $\tilde{z}(-\ell)$ is rotated by $-2\beta\ell$ (**decreasing angle, clockwise rotation**) with respect to the point corresponding to $\tilde{z}(0)$ along the circle of $|\Gamma(-\ell)| = |\Gamma_L|$ (**toward generator**).

**Example:** $Z_L = 25 + j100\ \Omega,\ Z_0 = 50\ \ \Omega$. Find $Z(-\ell), \Gamma(-\ell)$ for $\ell = 0.18\lambda$



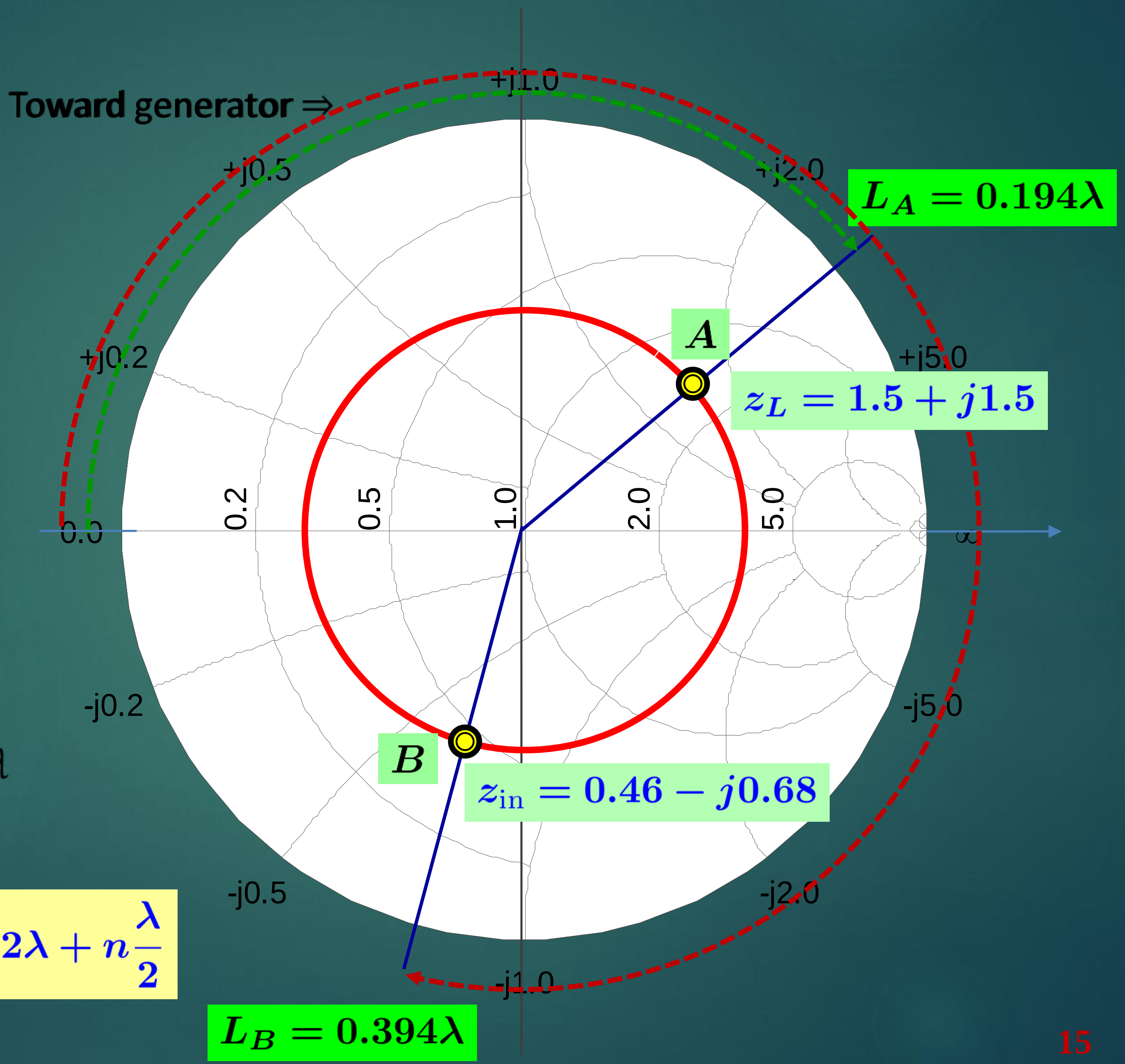

$$\theta = 2\beta\ell = 2\frac{2\pi}{\lambda}0.18\lambda = 2.262 \text{ rad} = 129.6^\circ$$

$|\Gamma(-\ell)| = 0.8246\angle - 78.7^\circ$
$= 0.161 - j0.809$

$z(-\ell) = 0.236 - j1.192$
$Z(-\ell) = z(-\ell) \times Z_0$
$= 11.79 - j59.6\ \ \Omega$

# Read Out Distance to Load

- Known load $Z_L = 75 + j75\ \Omega$
- Known $Z_0 = 50\ \Omega$
- Measured $Z_{in} = 23 - j34\ \Omega$




- Unknown distance to load in terms of $\lambda$ $D_n = D/\lambda$

$$D_n = L_B - L_A = 0.2\lambda + n\frac{\lambda}{2}$$

# Given $\Gamma_L$ and $Z_L$ $\Rightarrow$ Find VSWR

- The **VSWR** is defined as

$$\text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

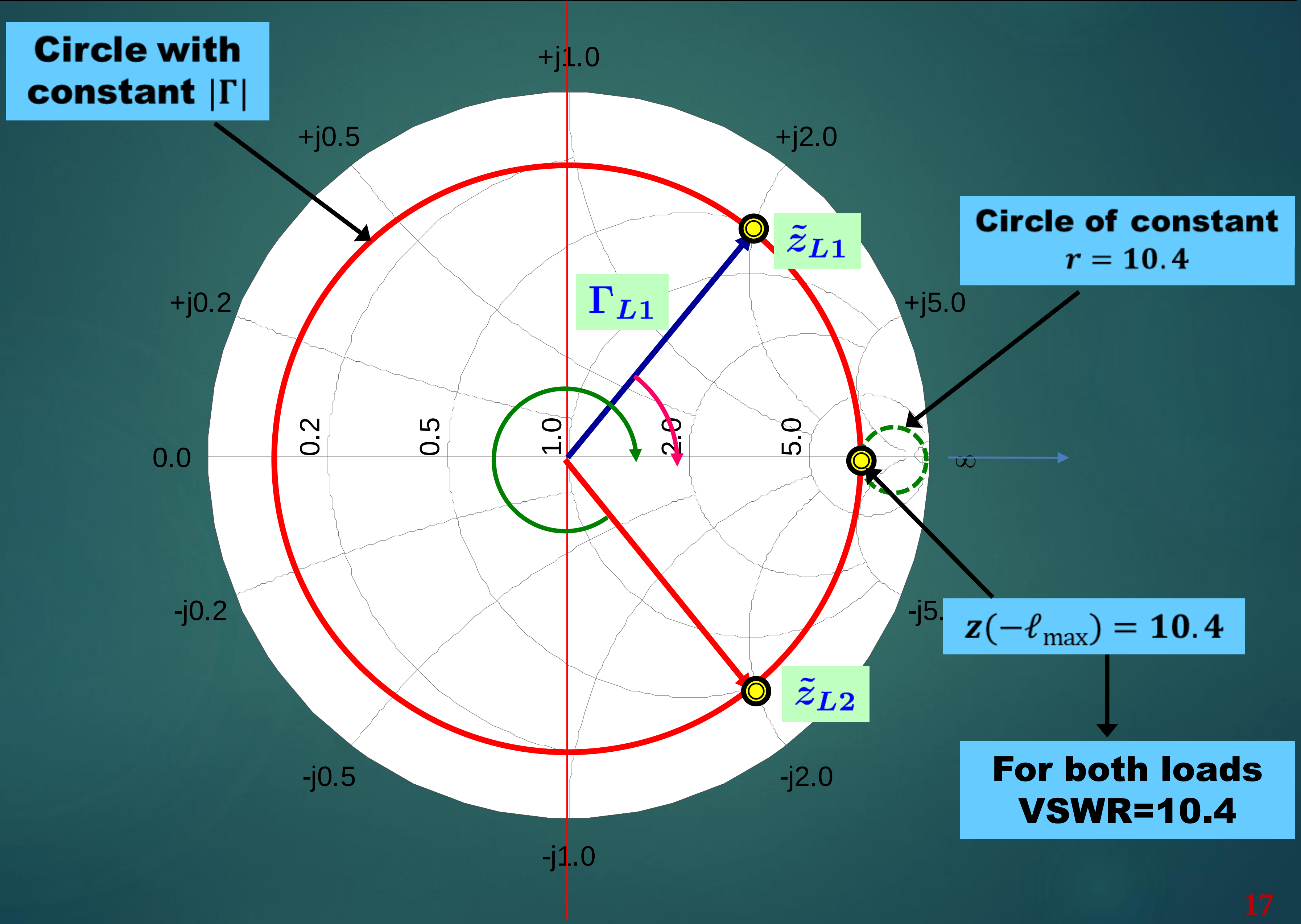
- The normalized impedance at a maximum location of the standing wave pattern is given by

$$\tilde{z}(-\ell_{\max}) = \frac{1+\Gamma(-\ell_{\max})}{1-\Gamma(-\ell_{\max})} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \text{VSWR}$$

- This quantity is always real and $\geq 1$ .

The VSWR is simply obtained on the Smith chart, by reading the value of the **(real) normalized impedance**, at the **location** $\ell_{max}$ where is real and positive.

Find the VSWR for $Z_{L1} = 25 + j100\ \Omega$; $Z_{L2} = 25 - j100\ \Omega$, $Z_0 = 50\ \Omega$



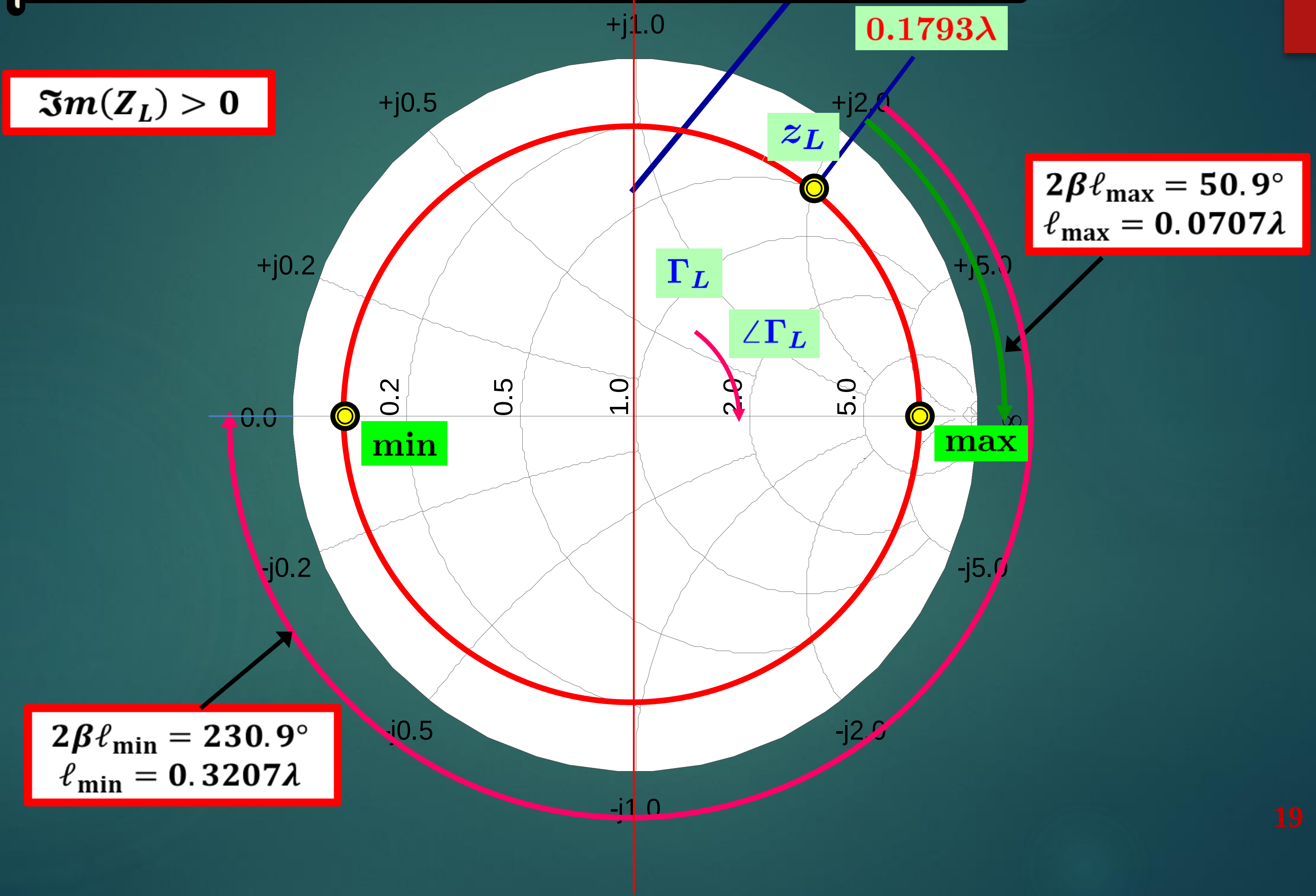
# Given $\Gamma_L$ and $Z_L \Rightarrow$ Find $\ell_{max}$ and $\ell_{min}$

- Identify on the Smith chart the **load reflection coefficient** $\Gamma_L$ or the **normalized load impedance** $Z_L$ .

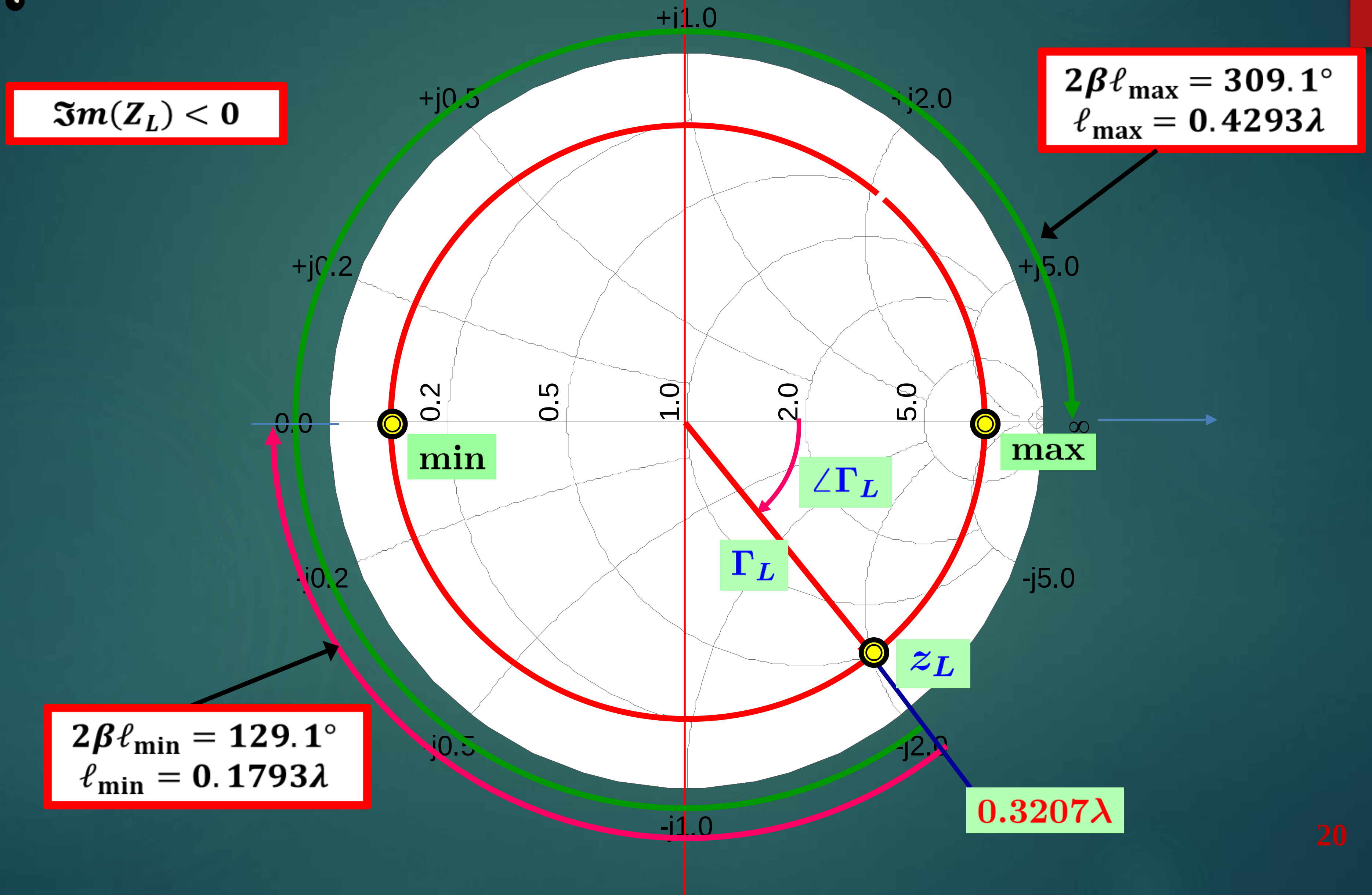
- Draw the circle of **constant reflection coefficient amplitude** $|\Gamma(-\ell)| = |\Gamma_L|$. The circle intersects the real axis of the reflection coefficient at two points which identify $\ell_{max}$ (when $\Gamma(-\ell)$ Real positive) and $\ell_{min}$ (when $\Gamma(-\ell)$ Real negative).

- The **angles**, between the vector $\Gamma_L$ and the real axis, also provide a way to compute $\ell_{max}$ and $\ell_{min}$ .

## Find $\ell_{max}$ and $\ell_{min}$ for $Z_L = 25 + j100\ \Omega,\ \ Z_0 = 50\ \Omega$



Find $\ell_{max}$ and $\ell_{min}$ for $Z_L = 25 - j100\ \Omega,\ \ Z_0 = 50\ \ \Omega$



# Switching Between Impedance and Admittance

- Consider the definition of the negative generalized reflection coefficient

$$-\Gamma(z) = \Gamma_L e^{j(2\beta z+\pi)} = \Gamma_L e^{j\left(2\beta z+2\beta\lambda/4\right)}$$

$$= \Gamma_L e^{j2\beta\left(z+\frac{\lambda}{4}\right)} = \Gamma\left(z+\frac{\lambda}{4}\right)$$

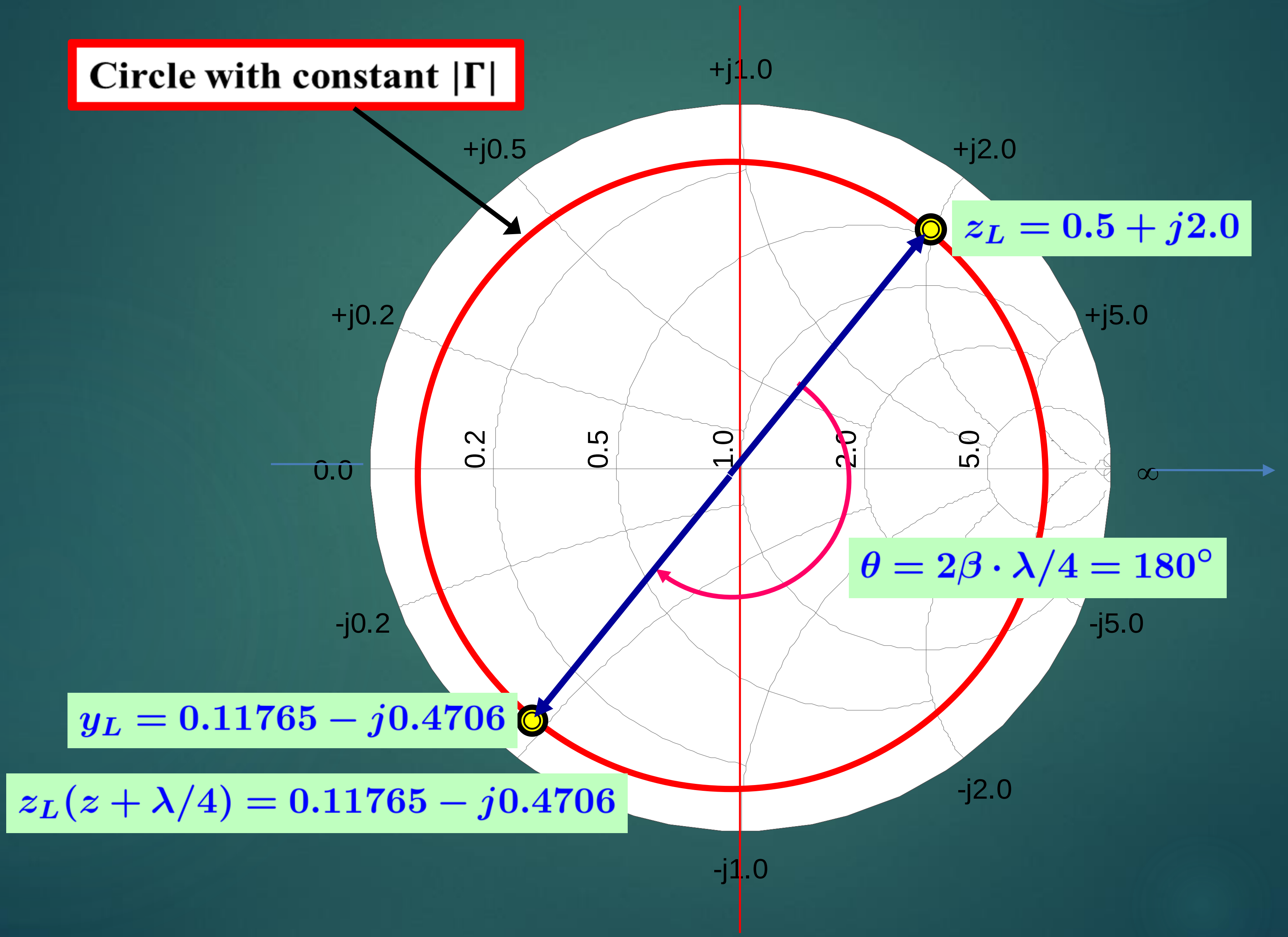
- The **normalized TL admittance**

$$\tilde{y}(z) \equiv \frac{1}{\tilde{z}(z)} = \frac{1-\Gamma(z)}{1+\Gamma(z)} = \frac{1+\Gamma\left(z+\frac{\lambda}{4}\right)}{1-\Gamma\left(z+\frac{\lambda}{4}\right)} = \tilde{z}\left(z+\frac{\lambda}{4}\right)$$

- But what is $z + {}^{\lambda}/_{4}$ ? It's a **half rotation** around the Smith chart.
- Keep in mind $\tilde{z}\left(z+\frac{\lambda}{4}\right) = \tilde{y}(z)$ is only valid for normalized impedance and admittance. The actual values are given by

$$Z\left(z+\frac{\lambda}{4}\right) = Z_0 \cdot \tilde{z}\left(z+\frac{\lambda}{4}\right) \text{ and } Y(z) = Y_0 \cdot \tilde{y}(z) = \frac{\tilde{y}(z)}{Z_0}$$

Example:- $Z_L = 25 + j100\ \Omega, \quad Z_0 = 50\ \Omega \quad$ Find $Y_L$



Thank you Very Much !!!