Introduction to Partial Differential Equations

Partial Differential Equations (PDE's)

PDE's describe the behavior of many engineering phenomena:

- Wave propagation
- Fluid flow (air or liquid)
  - Air around wings, helicopter blade, atmosphere
  - Water in pipes or porous media
  - Material transport and diffusion in air or water
  - Weather: large system of coupled PDE's for momentum, pressure, moisture, heat, …
- Vibration
- Mechanics of solids:
  - stress-strain in material, machine part, structure
- Heat flow and distribution
- Electric fields and potentials
- Diffusion of chemicals in air or water
- Electromagnetism and quantum mechanics

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Weather Prediction

- heat transport & cooling
- advection & dispersion of moisture
- radiation & solar heating
- evaporation
- air (movement, friction, momentum, coriolis forces)
- heat transfer at the surface

To predict weather one need "only" solve a very large systems of coupled PDE equations for momentum, pressure, moisture, heat, etc.

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Partial Differential Equations (PDE's):

A partial differential equation is an equation involving a function of two or more variables and some of its partial derivatives. Therefore a partial differential equation contains one dependent variable and one independent variable.

Here z will be taken as the dependent variable and x and y the independent variable so that $z = f(x, y)$.

We will use the following standard notations to denote the partial derivatives.

$$\frac{\partial z}{\partial x} = p, \frac{\partial z}{\partial y} = q, \frac{\partial^2 z}{\partial x^2} = r, \frac{\partial^2 z}{\partial x \partial y} = s, \frac{\partial^2 z}{\partial y^2} = t$$

The order of partial differential equation is that of the highest order derivative occurring in it.

Example: 1. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

2. $\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = v$

Order and Degree: The order of the highest ordered derivative involving in a partial differential equation is called the order of the partial differential equation.

Again, the exponent of the highest ordered derivative involving in a partial differential equation is called the degree of the partial differential equation, after freed from radicals and fractions in its derivatives.

Example: Consider the following partial differential equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

The order of this equation is 2 and the degree is 1.