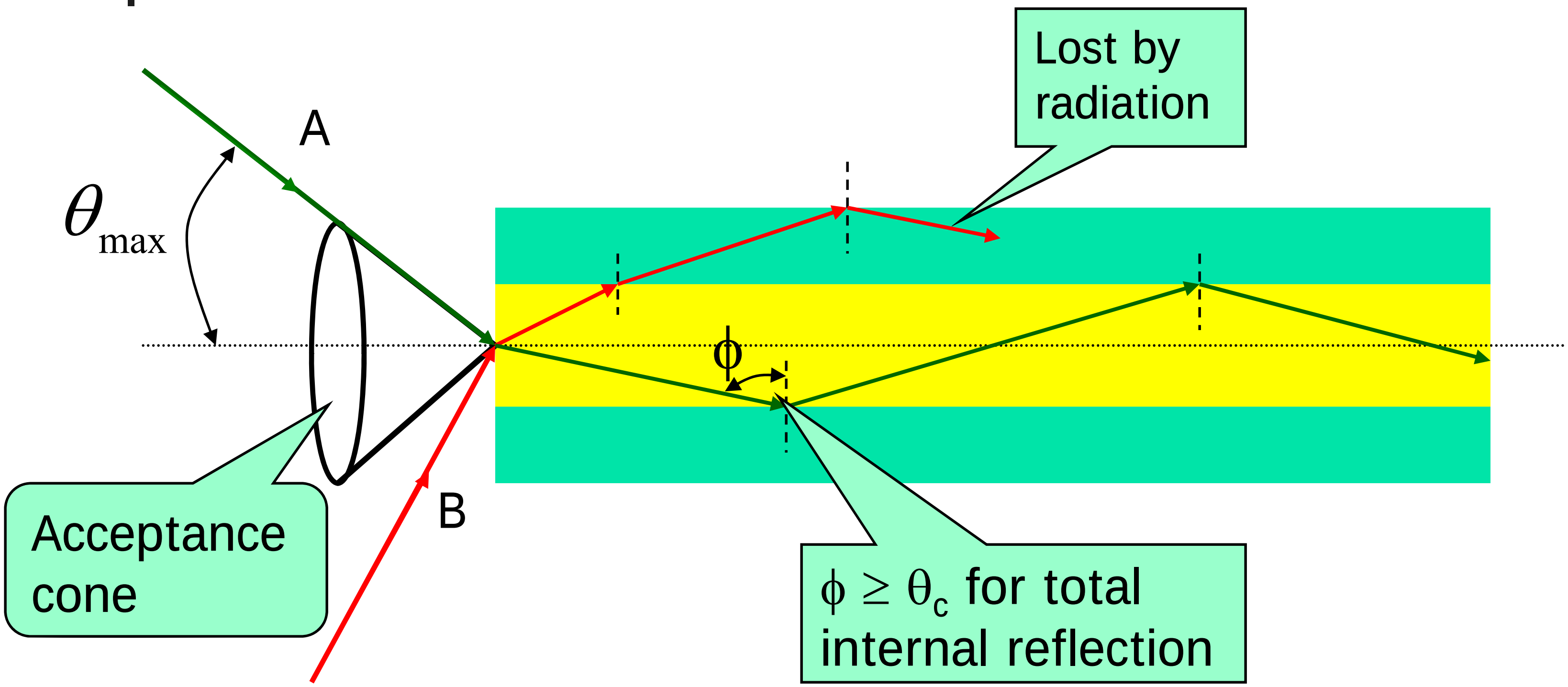
Propagation of Light Through Optical Fiber

Outline of Talk

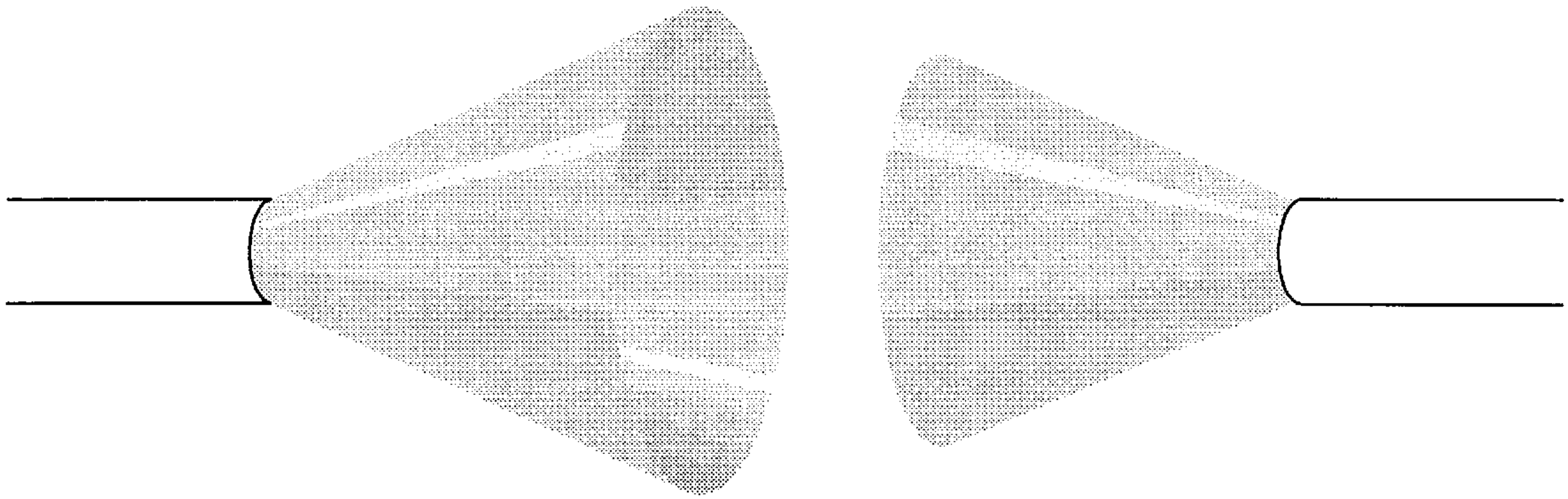
- Acceptance angle
- Numerical aperture
- Phase velocity
- Group velocity

Acceptance angle



θ_{max} is the maximum acceptance angle to the axis of the fiber at which light may enter into the fiber in order to propagate

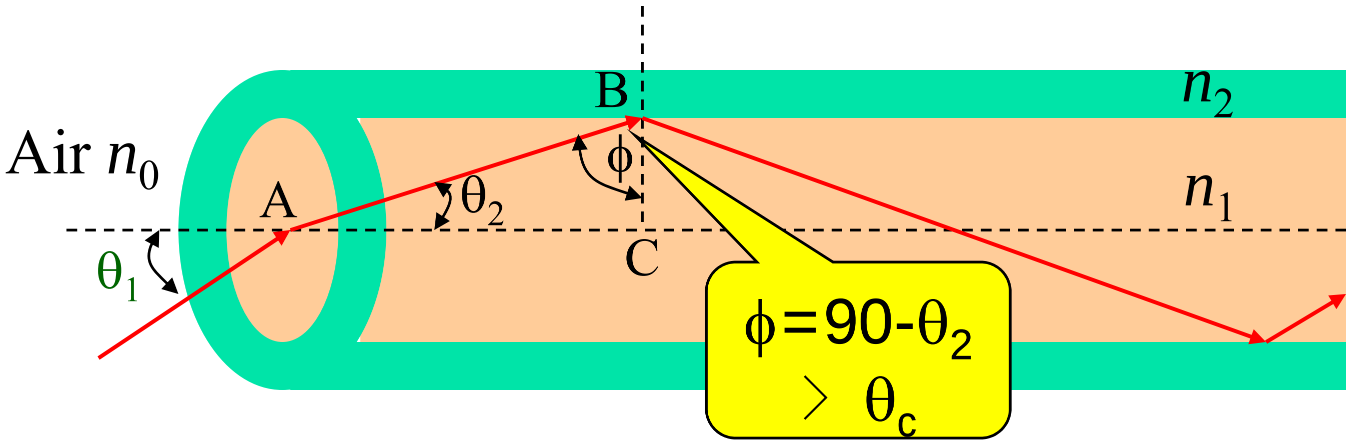
Different cones of acceptance



Large diameter fiber **Small diameter fiber**

Numerical aperture (NA)

The NA defines a cone of acceptance for light that will be guided by the fiber



At the air core interface $n_o \sin\theta_1 = n_1 \sin\theta_2$

From the triangle ABC $\phi = \dfrac{\pi}{2} - \theta_2$

Numerical aperture (NA)

$$n_o \sin\theta_1 = n_1 \cos\phi$$

Using trigonometric relationship $n_o \sin\theta_1 = n_1(1-\sin^2\phi)^{\frac{1}{2}}$

For total internal reflection, $\theta_1 = \theta_a$, and $\phi \geq \theta_c$

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

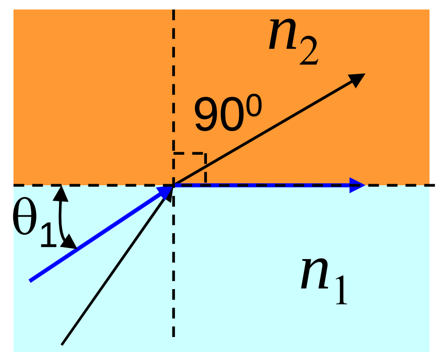
$$\Delta \approx \frac{n_1 - n_2}{n_1}$$

$$n_o \sin\theta_a = n_1\left(1 - \frac{n_2^2}{n_1^2}\right)^{1/2}$$

$$n_o \sin\theta_a = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$NA = n_o \sin\theta_a = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$NA = n_1(2\Delta)^{1/2}$$



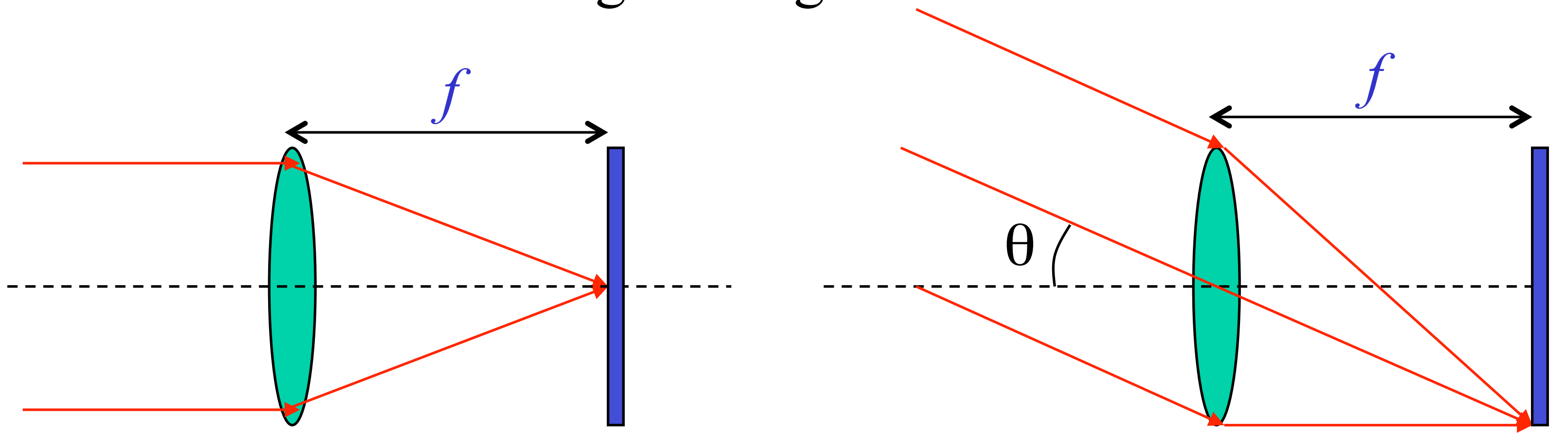
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Exmp. 2.1 and 2.2

Numerical aperture

- An important characteristic of an optic system is its ability to collect light incident over a wide range of angles.



The numerical aperture (NA) is defined as:

$$NA = n_o \sin \theta$$

where n_o is the refractive index of the medium between the lens and the image plane (e.g. a photodetector) and θ is the maximum acceptance angle.

- The definition of numerical aperture applies to *all light-collecting systems*, including optical fibers.

e.g. Light rays incident at angles *outside* the collection cone for a fiber will *not* propagate along the fiber (*instead will attenuate rapidly*).

- The numerical aperture is often measured in air, $n_o = 1$

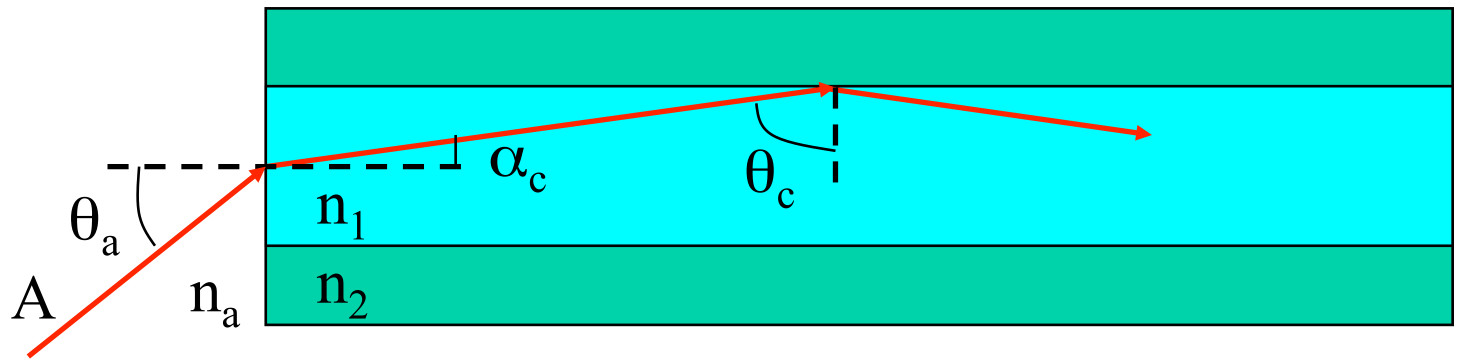
$$NA = \sin \theta$$

- A *low* NA indicates a *small* acceptance angle.

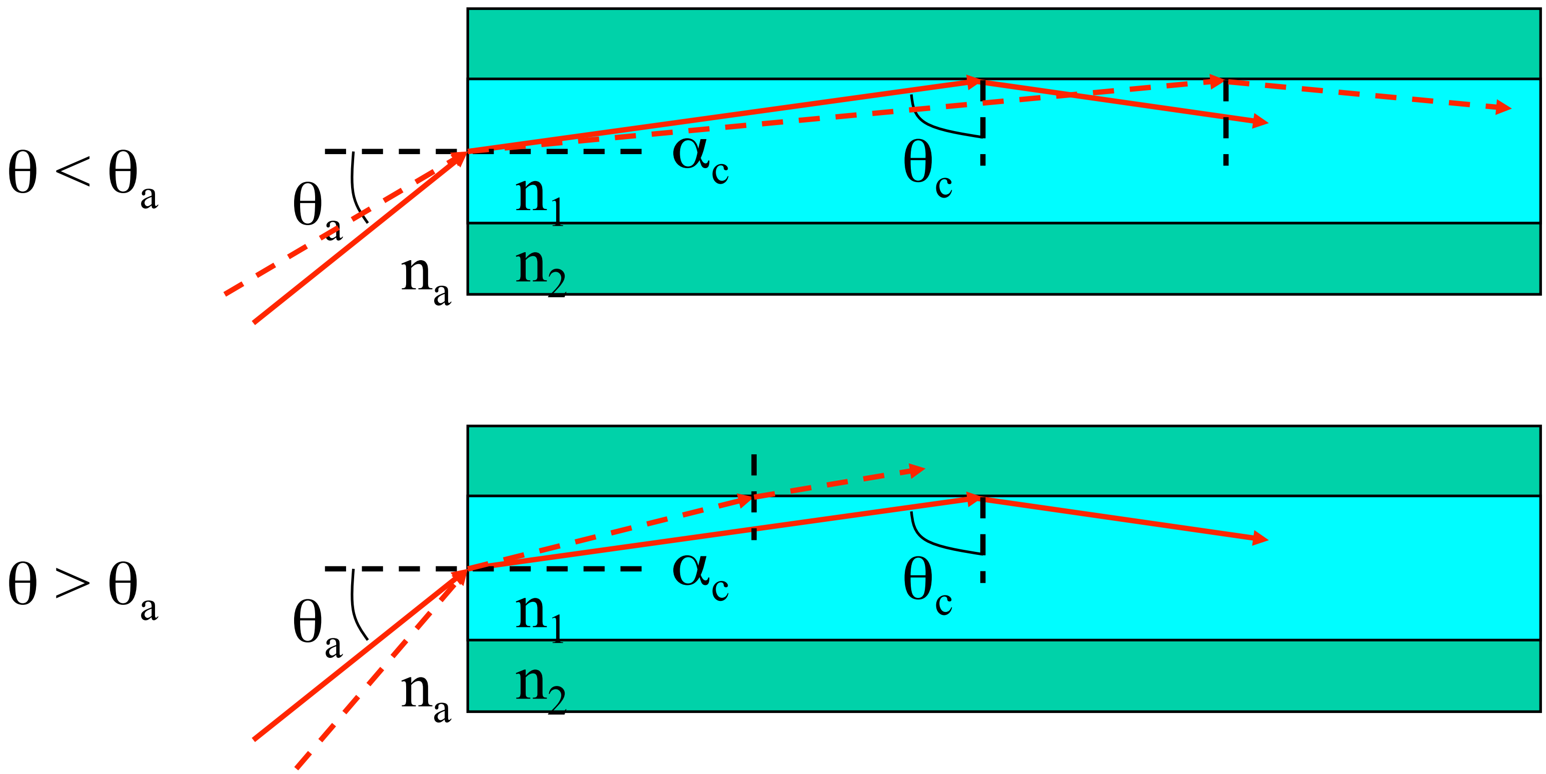
⇒Light coupling to a low-NA optical system (e.g. fiber) is more difficult (*alignment is more sensitive*) and less efficient (*some of the rays are outside the acceptance angle*) than is coupling to a high-NA optical system.

Acceptance angle

- Only rays with a sufficiently shallow grazing angle (i.e. with an angle to the normal greater than θ_c) at the core-cladding interface are transmitted by total internal reflection.



- Ray A incident at the critical angle θ_c at the core-cladding interface enters the fiber core at an angle θ_a to the fiber axis, and is refracted at the air-core interface.



• Any rays which are incident into the fiber core at an angle > θ_a have an incident angle less than θ_c at the core-cladding interface.

These rays will NOT be totally internal reflected, thus eventually loss to radiation (at the cladding-jacket interface).

• Light rays will be confined inside the fiber core if it is <u>input-coupled</u> at the fiber core end-face within the <u>acceptance angle</u> θ_a.

e.g. What is the fiber acceptance angle when $n_1 = 1.46$ and $n_2 = 1.44$?

$\theta_c = \sin^{-1}(n_2/n_1) = 80.5^o \Rightarrow \alpha_c = 90^o - \theta_c = 9.5^o$

using $\sin\theta_a = n_1 \sin\alpha_c$ (taking $n_a = 1$)

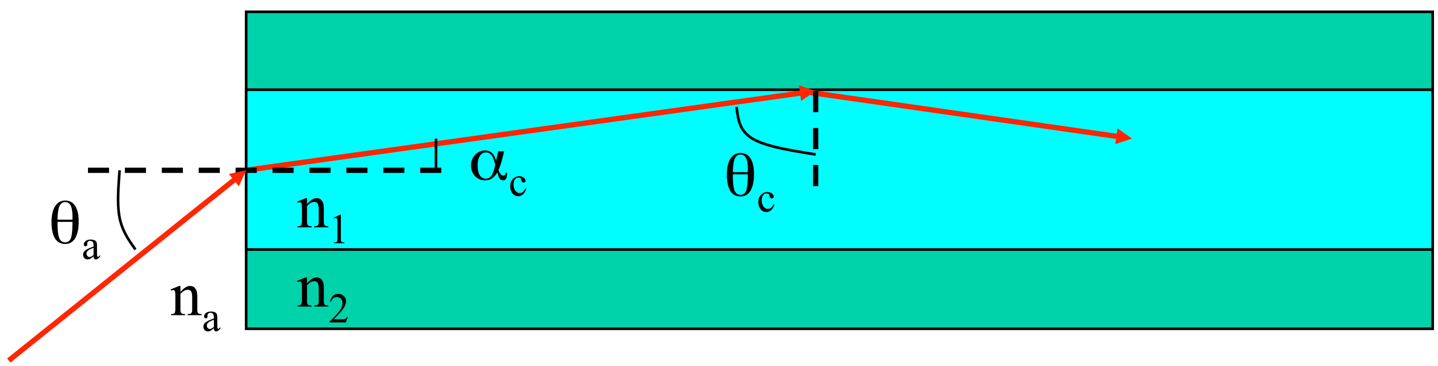
$\theta_a = \sin^{-1}(n_1 \sin\alpha_c) = \sin^{-1}(1.46 \sin 9.5^o) \sim 14^o$

=> the acceptance angle $\theta_a \sim 14^o$

Fiber numerical aperture

In fiber optics, we describe the fiber acceptance angle using **Numerical Aperture (NA):**

$$NA = n_a \sin \theta_a = \sin \theta_a = (n_1^2 - n_2^2)^{1/2}$$



- We can relate the <u>acceptance angle θ_a</u> and the <u>refractive indices of the core n_1, cladding n_2 and air n_a</u>.

• Assuming the end face at the fiber core is *flat* and *normal* to the fiber axis (when the fiber has a “nice” cleave), we consider the refraction at the air-core interface using Snell’s law:

$$\text{At } \theta_a: \quad n_a \sin\theta_a = n_1 \sin\alpha_c$$

launching the light from air: $(n_a \sim 1)$

$$\sin\theta_a = n_1 \sin\alpha_c$$

$$= n_1 \cos\theta_c$$

$$= n_1 (1 - \sin^2\theta_c)^{1/2}$$

$$= n_1 (1 - n_2^2/n_1^2)^{1/2}$$

$$= (n_1^2 - n_2^2)^{1/2}$$

- Fiber NA therefore characterizes the fiber's ability to gather light from a source and guide the light.

e.g. What is the fiber numerical aperture when $n_1 = 1.46$ and $n_2 = 1.44$?

$$NA = \sin\theta_a = (1.46^2 - 1.44^2)^{1/2} = 0.24$$

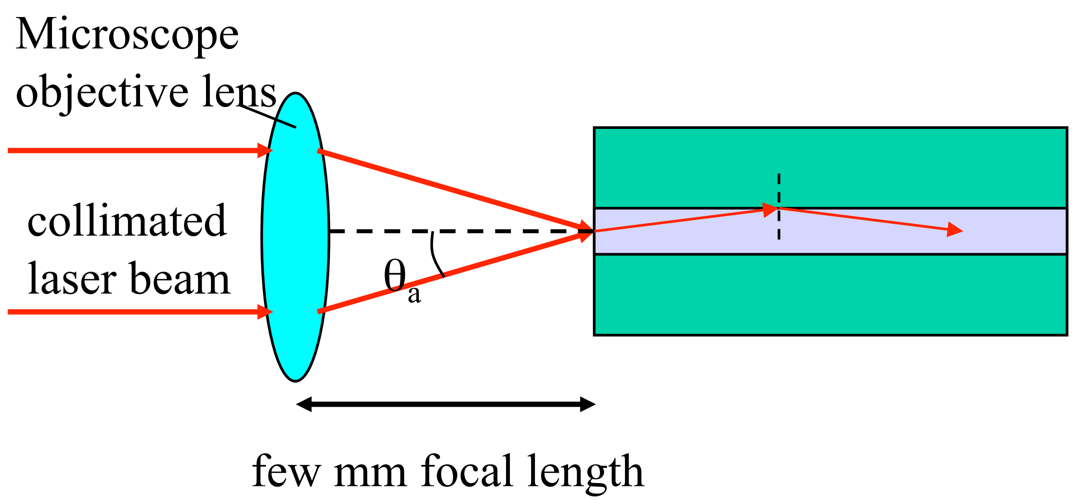
- It is a common practice to define a relative refractive index Δ as:

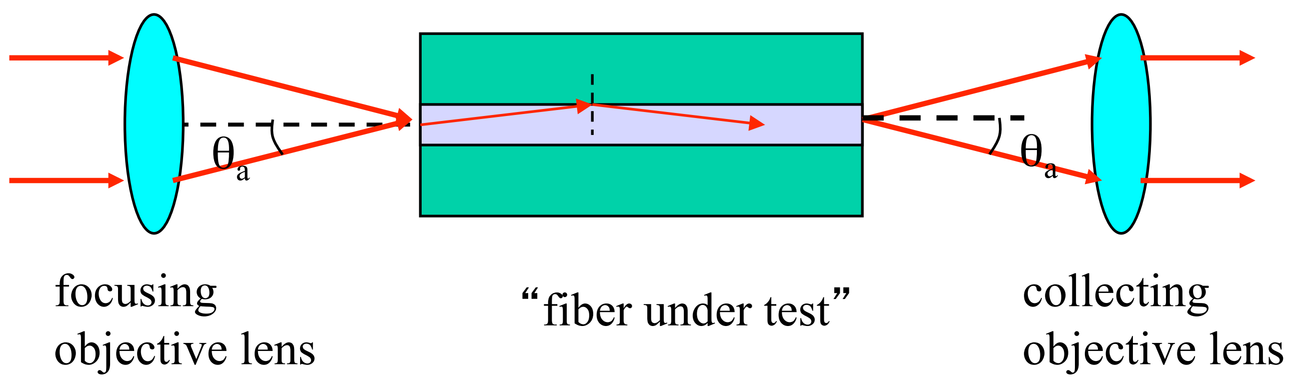
$$\Delta = (n_1 - n_2) / n_1$$

$$(n_1 \sim n_2) \quad => \quad NA = n_1 (2\Delta)^{1/2}$$

i.e. Fiber NA only depends on n_1 and Δ.

Lens coupling to fiber end faces





- By measuring the output couple ray cone angle, we can measure the fiber acceptance angle. (This is like part of Lab 1 but without using lenses.)

Large-NA fibers?

- Developing ways for fiber to collect light efficiently was an important early step in developing practical fiber optic communications (particularly in the 1970s)

- It seems logical to have optical fibers with NA as large as possible … with as large Δ as possible … in order to couple maximum amount of light into the fiber.

- Soon, we will find out that such large-NA fibers tend to be "multimode" and are *unsuitable* for high-speed communications because of a limitation known as modal dispersion.

- Relatively small-NA fibers are therefore used for high-speed optical communication systems.

Typical fiber NA

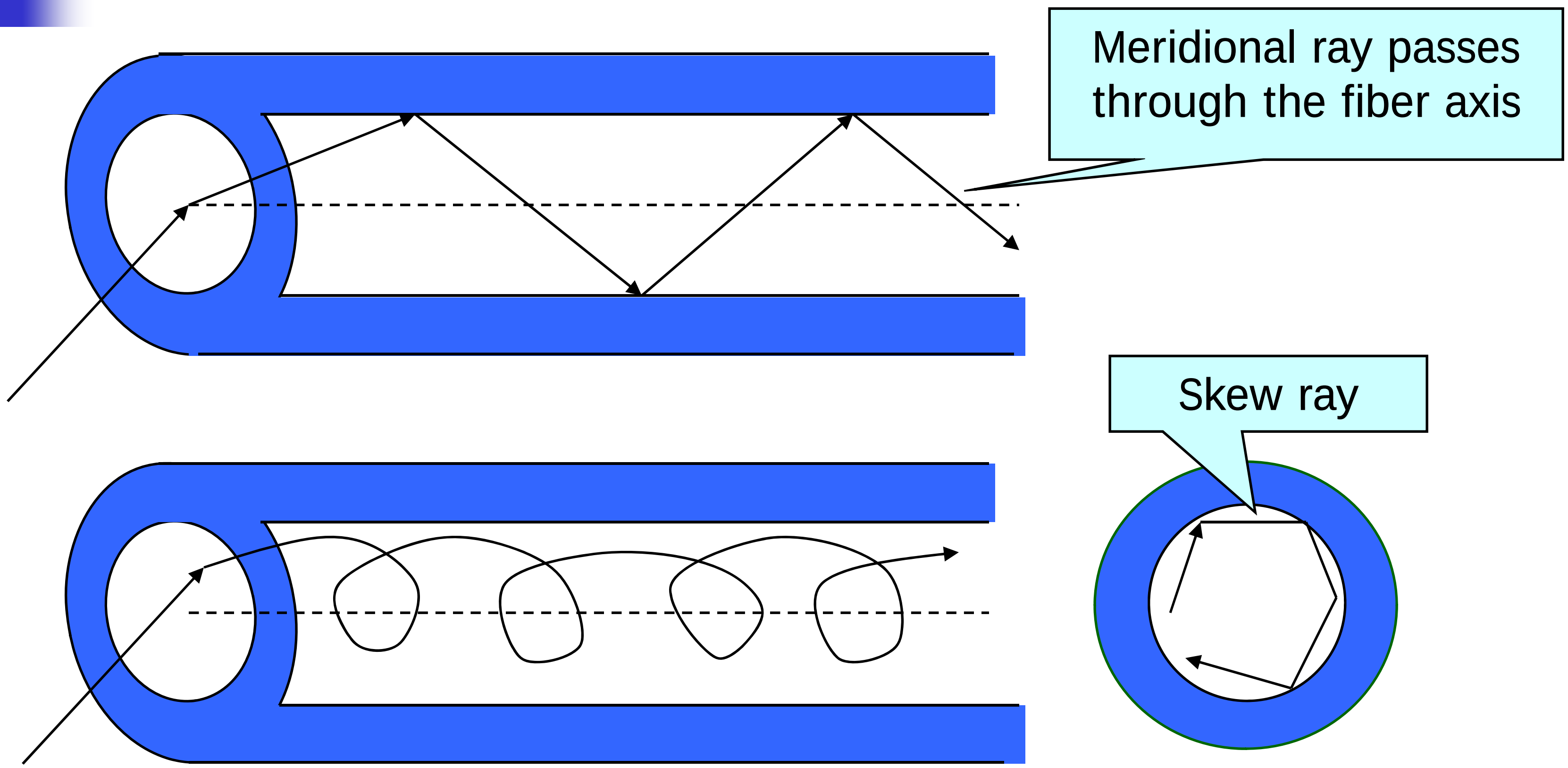
- Silica fibers for long-haul transmission are designed to have numerical apertures from about 0.1 to 0.3.

The low NA makes coupling efficiency tend to be poor, but turns out to improve the fiber's bandwidth! (details later)

- Plastic, rather than glass, fibers are available for short-haul communications (e.g. within an automobile). These fibers are restricted to short lengths because of the relatively high attenuation in plastic materials.

Plastic optical fibers (POFs) are designed to have high numerical apertures (typically, 0.4 – 0.5) to improve coupling efficiency, and so partially offset the high propagation losses and also enable alignment tolerance.

Types of ray propagation in OF



Skew ray follows helical path in optical fiber

Limitation of ray optics

- For smaller fiber diameters that are only few times of the wavelength, *geometrical optics approach becomes inadequate*. This is because ray optics only describes the direction a plane wave component takes in the fiber, but does not take into account *interference* among such components.

- When interference phenomena are considered it is found that only rays with certain discrete characteristics propagate in the fiber core.

- Thus the fiber will only support a discrete number of guided modes.

- This becomes critical in small core diameter fibers which only support one (singlemode) or a few modes (multimode). Electromagnetic theory must be applied in this case.

Numerical Aperture Example

Example 2.4 Consider a multimode silica fiber that has a core refractive index $n_1 = 1.480$ and a cladding index $n_2 = 1.460$. Find (a) the critical angle, (b) the numerical aperture, and (c) the acceptance angle.

Solution: (a) From Eq. (2.21), the critical angle is given by

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.460}{1.480} = 80.5^\circ$$

(b) From Eq. (2.23) the numerical aperture is

$$\text{NA} = \left(n_1^2 - n_2^2\right)^{1/2} = 0.242$$

(c) From Eq. (2.22) the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1} 0.242 = 14^\circ$$

Example 2.5 Consider a multimode fiber that has a core refractive index of 1.480 and a core-cladding index difference 2.0 percent ($\Delta = 0.020$). Find the (a) numerical aperture, (b) the acceptance angle, and (c) the critical angle.

Solution: From Eq. (2.20), the cladding index is $n_2 = n_1(1-\Delta) = 1.480(0.980) = 1.450$.

(a) From Eq. (2.23) we find that the numerical aperture is

$$\text{NA} = n_1\sqrt{2\Delta} = 1.480(0.04)^{1/2} = 0.296$$

(b) Using Eq. (2.22) the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1} 0.296 = 17.2^\circ$$

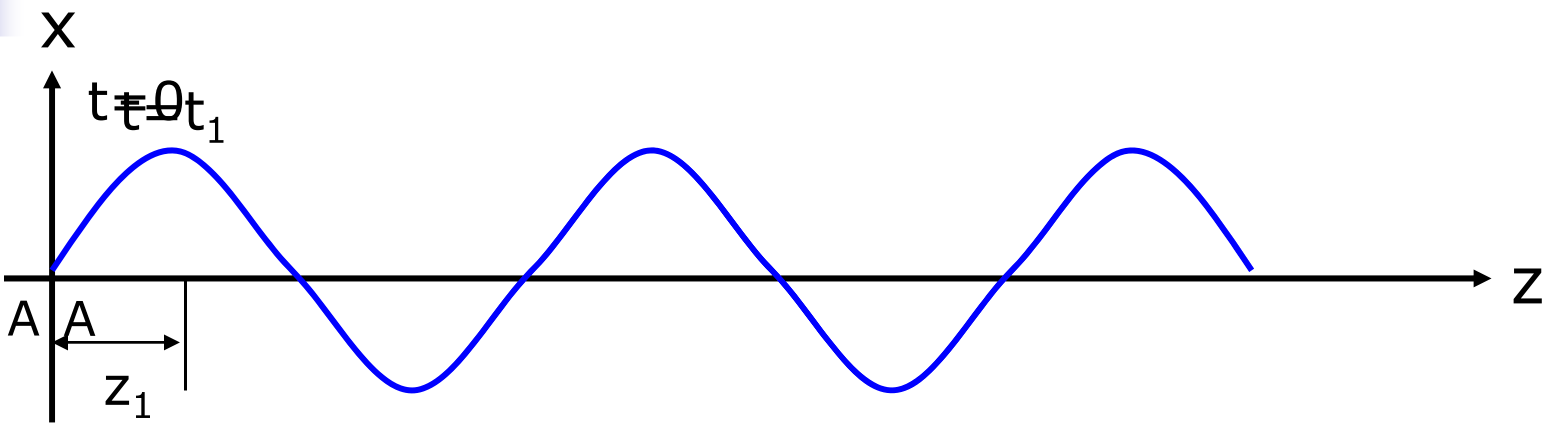
(c) From Eq. (2.21) the critical angle at the core–cladding interface is

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 0.980 = 78.5^\circ$$

Phase Velocity and Group Velocity

Phase Velocity

Sinusoidal variation of electric field with time and distance



At t=0 and z=0, the amplitude of the wave vector is zero at point A

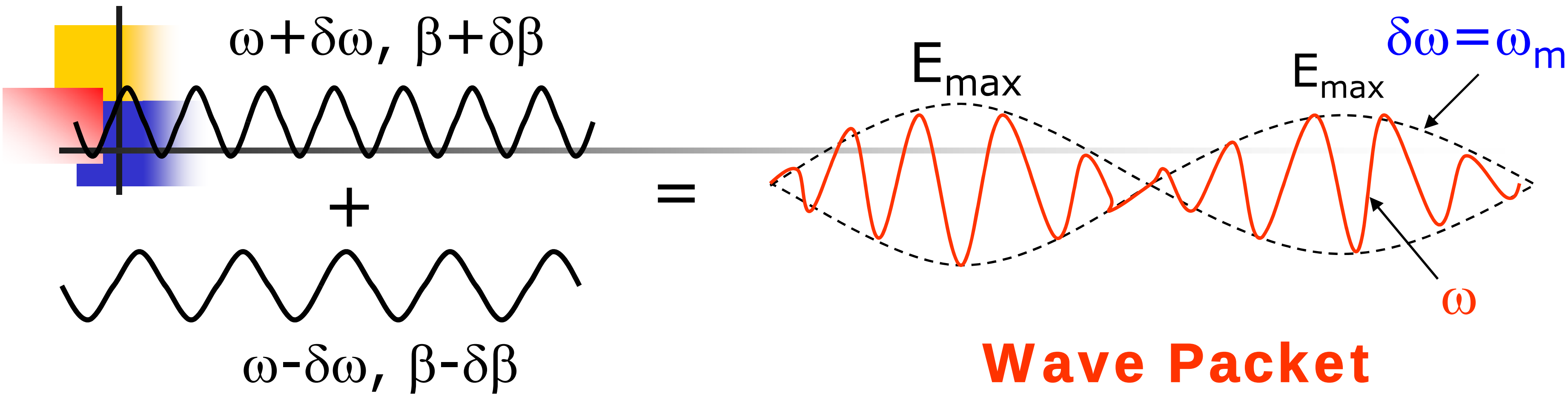
At $t=t_1$ the point A has moved z_1, the amplitude of the wave is still zero

$$\sin(\omega t_1 - \beta z_1) = 0 \qquad \omega t_1 - \beta z_1 = 0$$

Phase velocity

$$v_p = \frac{z_1}{t_1} = \frac{\omega}{\beta}$$

Group Velocity



$$e_{AM} = E_0(1 + m\cos\omega_m t)\cos\omega_c t$$

$$= E_0\left\{\cos\omega_c t + \frac{m}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]\right\}$$

$$= E_0\cos(\omega_c t - \beta z) + E_0\frac{m}{2}\left[\cos(\omega_c + \delta\omega)t - (\beta + \delta\beta)z + \cos(\omega_c - \delta\omega)t - (\beta - \delta\beta)z\right]$$

$$= E_0\cos(\omega_c t - \beta z) + E_0 m\cos(\omega_c t - \beta z)\cos(\delta\omega t - \delta\beta z)$$

Carrier wave travels with phase velocity $v_p = \dfrac{\omega}{\beta}$

Modulated envelop travels with group velocity $v_g = \dfrac{d\omega}{d\beta}$