MAT 102/121

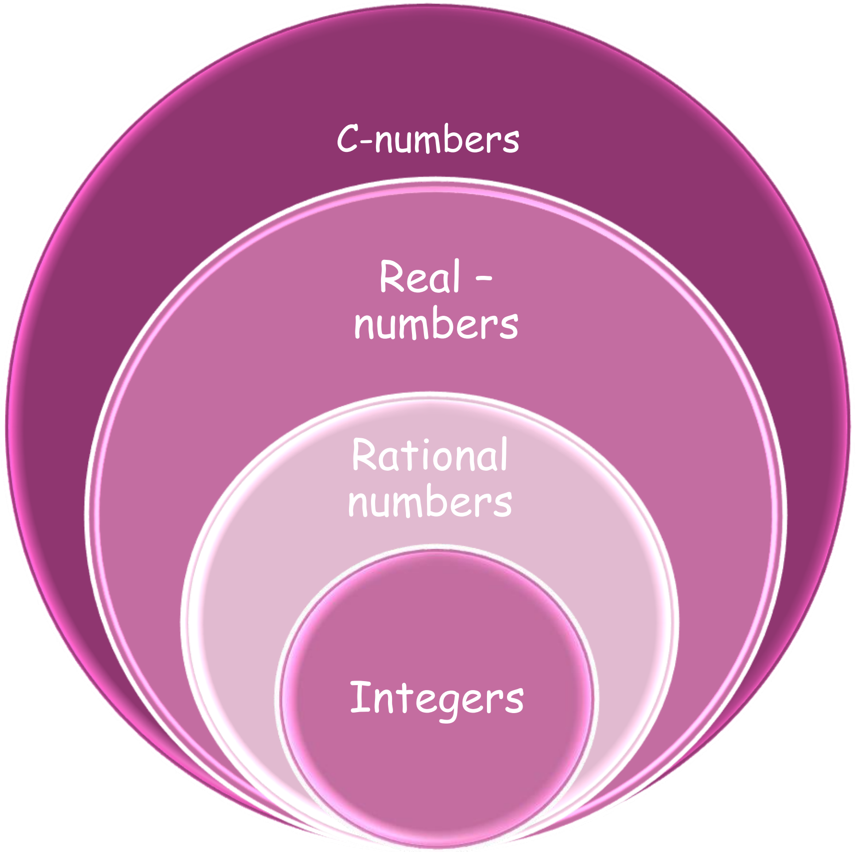
COMPLEX NUMBERS

COMPLEX NUMBER

- Definition
- Different forms
- Transformation from one form to another
- Euler's identity
- Simplification

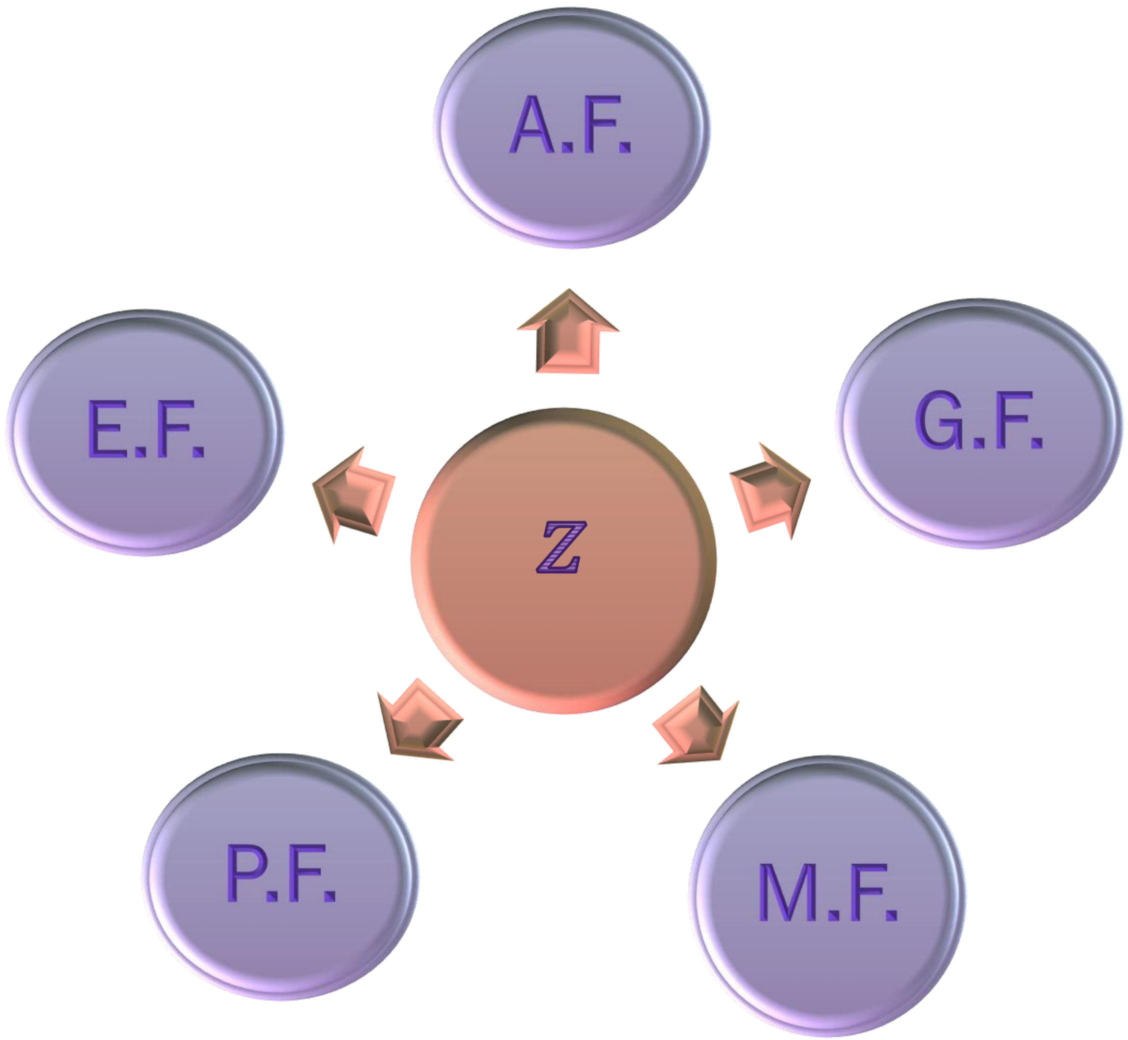
DEFINITION

- Each number is a complex number.
- We shall use z to denote a c-number.



DIFFERENT FORMS OF A C-NUMBER z

- Algebraic form (A.F.)
- Point/Geometric form (G.F.)
- Matrix form (M.F.)
- Polar/vector form (P.F.)
- Exponential form (E.F.)



ALGEBRAIC FORM OF z

$$z = x + iy$$

where: x and y real number

$$(\pm i)^2 = -1$$

Examples :

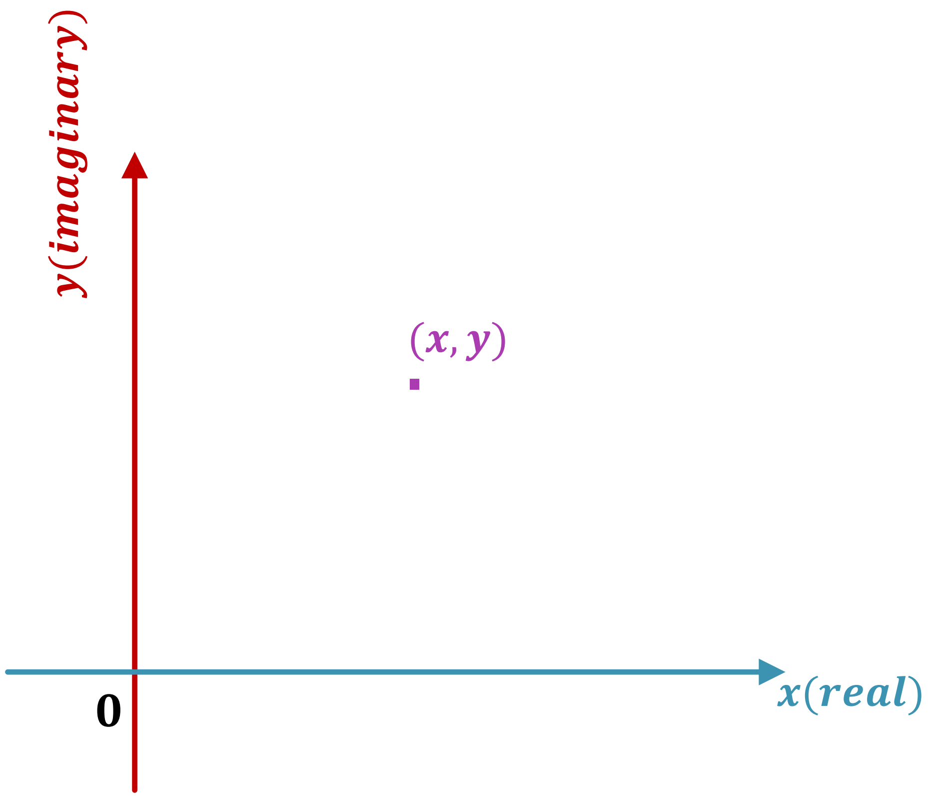
- $z = 4 + 5i$
- $z = 4 - 15i = 4 + (-15)i$
- $z = -44 - 35i = -44 + (-35)i$
- $z = 4 = 4 + (0)i$
- $z = 6i = 0 + 6i$

POINT FORM OF Z

The point/geometric form of a c-number $x + iy$ is (x, y)

The point (a, b) can be written as the c-number: $a + ib$

Conclusion: A point in a plane is a complex number and conversely.



EXAMPLE

Arithmetic Form	Geometric Form
$3 + 4i$	$(3, 4)$
$-13 - 43i$	$(-13, -14)$
9	$(9, 0)$
$4i$	$(0, 4)$

Geometric Form	Arithmetic Form
$(-1, 4)$	$-1 + 4i$
$(-13, -4)$	$-13 - 4i$
$(-13, 0)$	-13
$(0, -14)$	$-14i$

MATRIX FORM OF Z

- The matrix form of a complex number $x + iy$ is

$$\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$$

- Every matrix of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ can be written as the c-number

$$a + ib$$

EXAMPLE

Arithmetic Form	Geometric Form
$3 - 4i$	$\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$
9	$\begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$

Geometric Form	Arithmetic Form
$\begin{pmatrix} -5 & -2 \\ 2 & -5 \end{pmatrix}$	$-5 + 2i$
$\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$	$-3i$

2 IMPORTANT CONCEPTS

Modulus of z :

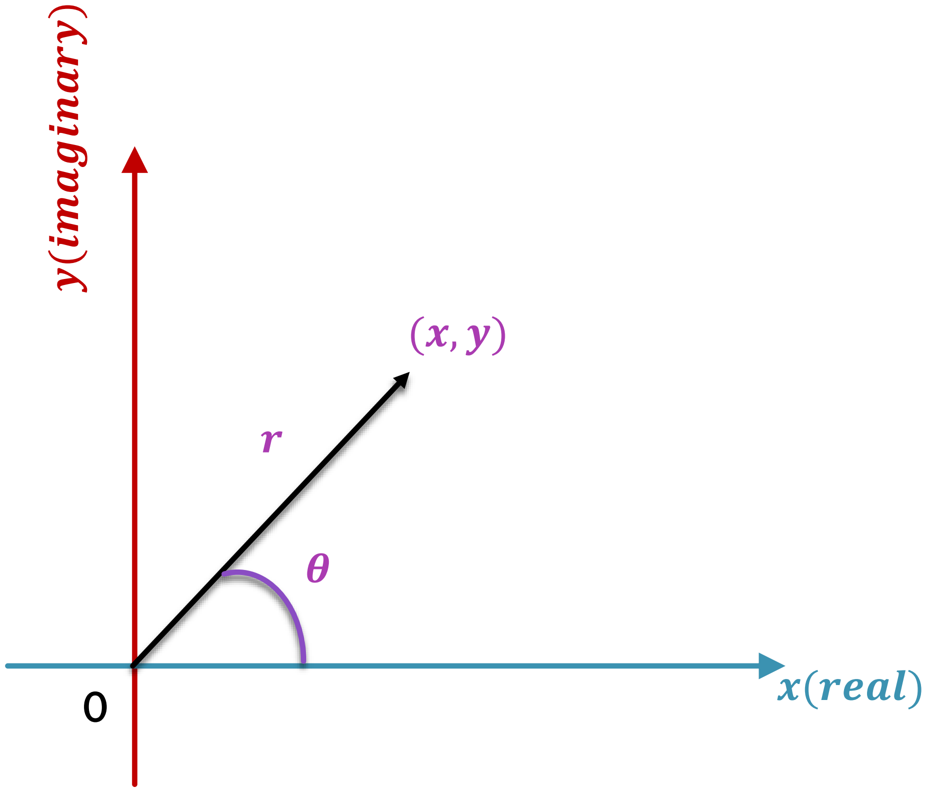
Notation : $|z| = r$

Rule : $|z| = \sqrt{(x^2 + y^2)}$

Argument of z :

Notation :

$\arg(z)$ or θ



FINDING MODULUS OF Z

Modulus of z :

- Notation : $|\,z\,|$
- Rule : $|\,z\,| = \sqrt{x^2 + y^2}$

Examples :

$z = 5 + 5\,i \qquad \therefore |\,z\,| = \sqrt{5^2 + 5^2} = \sqrt{50}$

$z = -5 + 5\,i \qquad \therefore |\,z\,| = \sqrt{(-5)^2 + 5^2} = \sqrt{50}$

$z = -5 - 5\,i \qquad \therefore |\,z\,| = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50}$

$z = 5 - 5\,i \qquad \therefore |\,z\,| = \sqrt{5^2 + (-5)^2} = \sqrt{50}$