Lecture 2: Ray theory transmission in optical fibers

- Nature of light
- Ray optics
- Refractive indices
- Snell's Law
- Total internal reflection
- Acceptance angle
- Numerical Aperture
- Optical fiber structures

Reading: Senior 2.1 – 2.2
Keiser 2.1 – 2.2

The nature of light

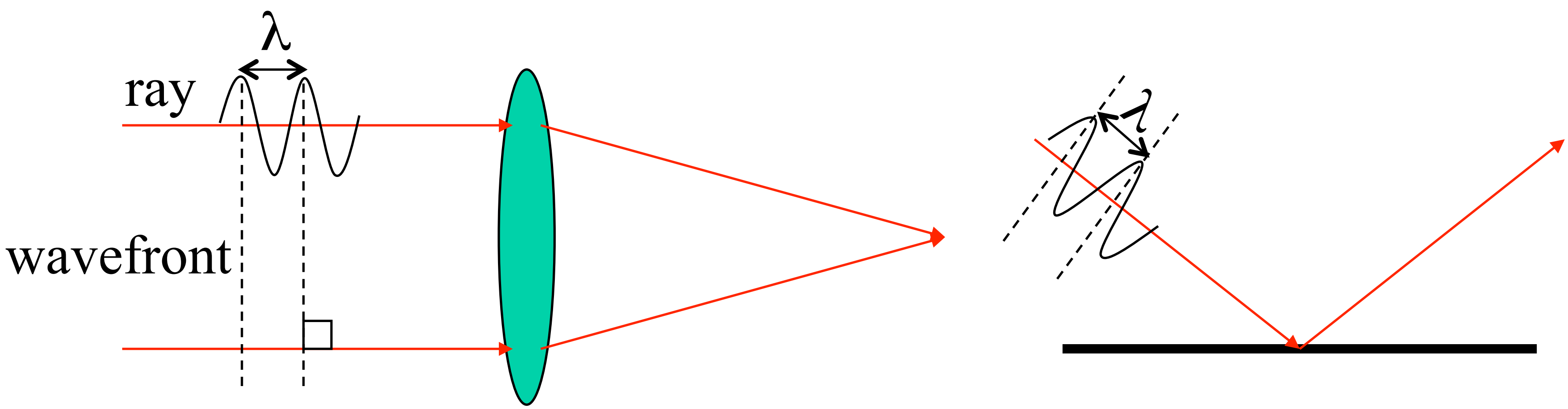
– Modeling light:
  - ***Ray optics:*** propagation of **light rays** through simple optical components and systems.
  - ***Wave optics:*** propagations of **light waves** through optical components and systems.
  - ***Electromagnetic optics***: description of light waves in terms of **electric and magnetic fields**.
  - ***Quantum optics***: emission/absorption of **photons**, which are characteristically quantum mechanical in nature and cannot be explained by classical optics (e.g. lasers, light-emitting diodes, photodiode detectors, solar cells)

Light as waves, rays and photons

- Light is an ***electromagnetic wave***.
- While light is a wave, it nevertheless travels along straight lines or ***rays***, enabling us to analyze simple optical components (e.g. *lenses* and *mirrors*) and instruments in terms of <u>geometrical optics</u>.
- Light is also a stream of ***photons***, discrete particles carrying packets of *energy* and *momentum*.

Ray Optics or Geometrical Optics

Wavelength λ << size of the optical component



- In many applications of interest the *wavelength* λ of light is *short* compared with the relevant length scales of the optical components or system (e.g. mirrors, prisms, lenses).
- This branch of optics is referred to as **Ray optics** or **Geometrical Optics**, where energy of light is propagated along rays.
- The rays are perpendicular to the wavefronts.

Ray Optics: basic laws

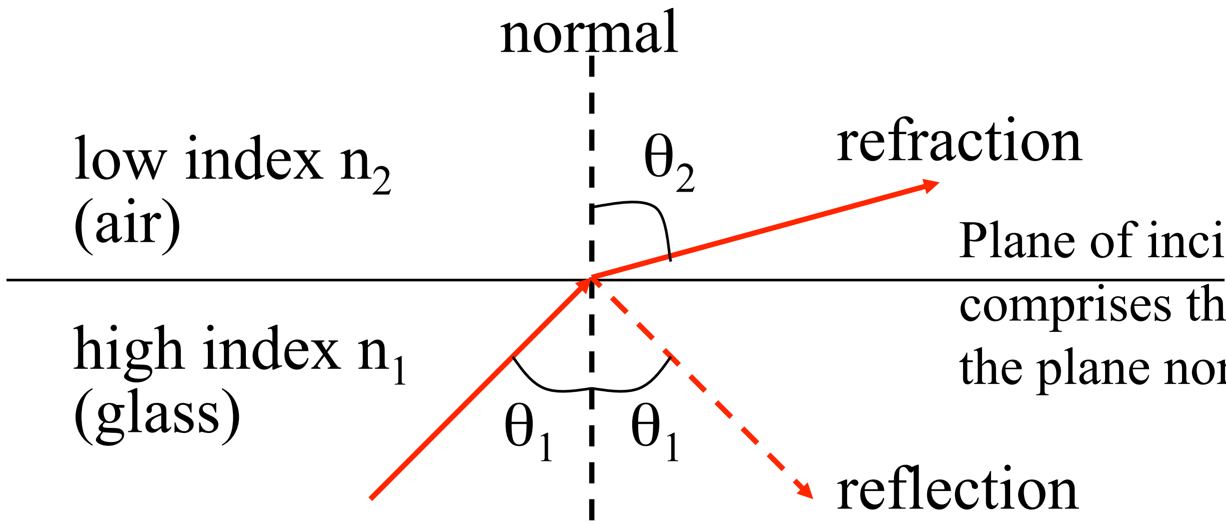
- Ray optics is based on <u>three</u> laws which describe the propagation of rays:

1. Light rays in homogeneous media are straight lines.
2. **Law of reflection**: Reflection from a mirror or at the boundary between two media of different *refractive indices*: the reflected ray lies in the plane of incidence, the angle of reflection equals the angle of incidence (i.e. $\theta_r = \theta_i$)
5. **Snell's law of refraction:** At the boundary between two media of different refractive index n, the refracted ray lies in the plane of incidence; the angle of refraction θ_t is related to the angle of incidence θ_i by

$$n_i \sin \theta_i = n_t \sin \theta_t$$

Snell's Law

When a ray is incident on the interface between two dielectrics of different refractive indices (e.g. glass-air), reflection and refraction occur.



Plane of incidence ≡ plane that comprises the incident ray and the plane normal

The *angle of incidence* θ_1 and the *angle of refraction* θ_2 are related to each other, and to the refractive indices of the dielectrics by Snell's law of refraction:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Refractive index

- In any dielectric medium, the speed of light becomes

$$v = c/n$$

The factor n is the *index of refraction* (or *refractive index*) of the medium.

e.g. For air and gases, v ~ c, so that n ~ 1. At optic frequencies, the refractive index of water is 1.33.

e.g. Glass has many compositions, each with a slightly different n. An approximate refractive index of 1.5 is representative for the silica glasses used in fibers; more precise values for these glasses lie between ~1.45 and ~1.48.

Index of refraction for some materials

Air	1.0
Water	1.33
Magnesium fluoride	1.38
Fused silica (SiO_2)	1.46
Sapphire (Al_2O_3)	1.8
Lithium niobate ($LiNbO_3$)	2.25
Indium phosphide (InP)	3.21
Gallium arsenide (GaAs)	3.35
Silicon (Si)	3.48
Indium gallium arsenide phosphide (InGaAsP)	3.51
Aluminum gallium arsenide (AlGaAs)	3.6
Germanium (Ge)	4.0

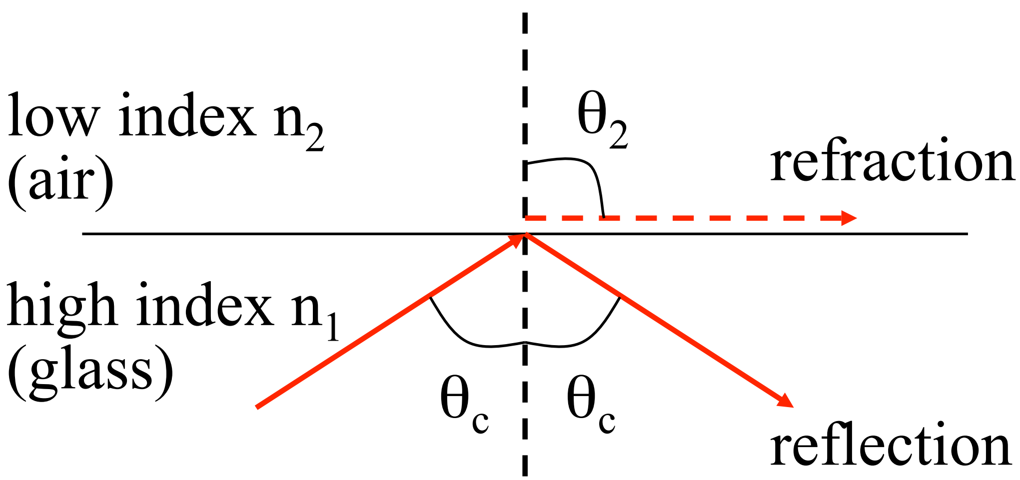
*The index varies with a number of parameters, such as wavelength and temperature.

Critical angle

For $n_1 > n_2$, the angle of refraction θ_2 is always *greater* than the angle of incidence θ_1.

- When the angle of refraction θ_2 is 90^o, the refracted ray emerges *parallel* to the interface between the media.

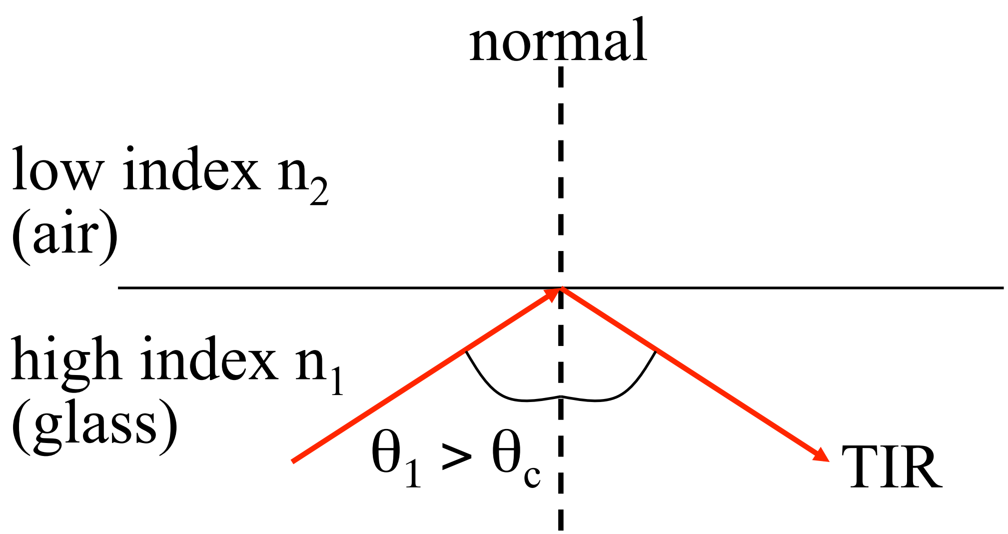
This is the *limiting* case of refraction and the angle of incidence is known as the critical angle θ_c.



$$\sin \theta_c = n_2 / n_1$$

Total internal reflection

- At angles of incidence $\theta > \theta_c$, the light is totally reflected back into the incidence higher refractive index medium. This is known as total internal reflection.

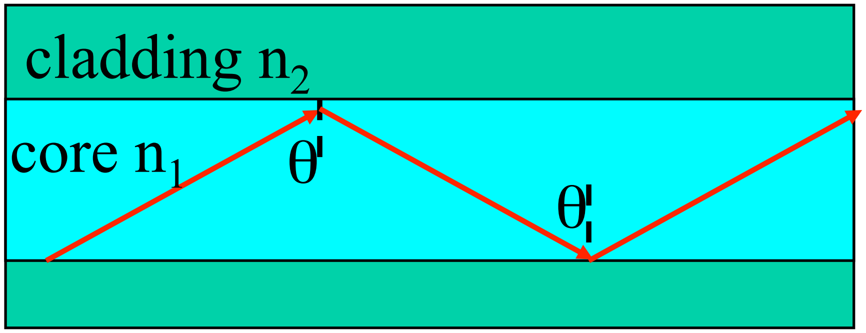


e.g. $n_1 = 1.44$, $n_2 = 1$, then $\theta_c = \sin^{-1}(1/1.44) = 44^o$

Total internal reflection: $\theta_1 > \theta_c$

Light ray guiding condition

• Light ray that satisfies *total internal reflection* at the interface of the higher refractive index core and the lower refractive index cladding can be guided along an optical fiber.



e.g. Under what condition will light be trapped inside the fiber core?

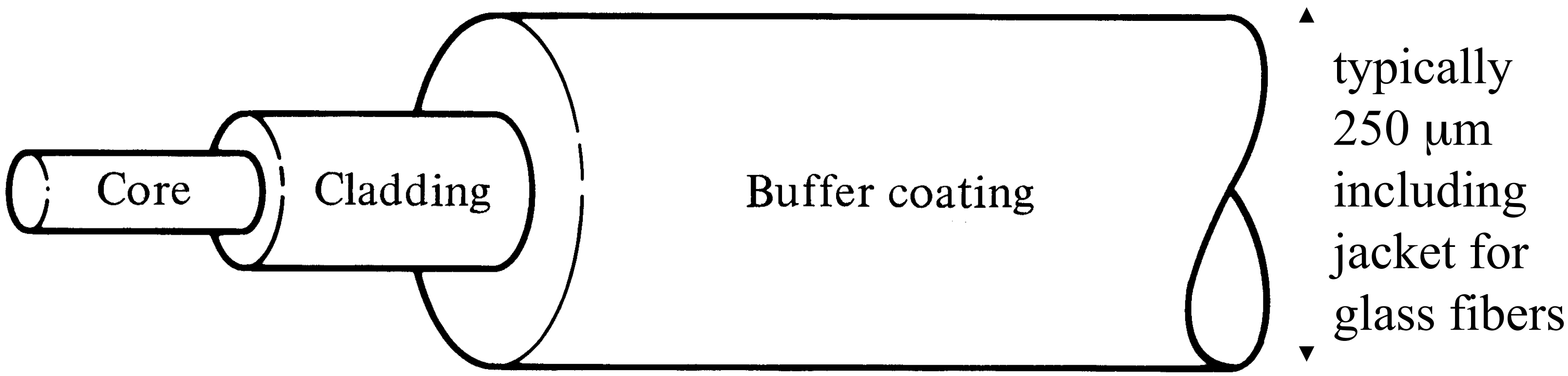
$n_1 = 1.46$; $n_2 = 1.44$

$$\theta > \theta_c$$

$$\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1.44/1.46) = 80.5^o$$

Optical fiber structures

- A typical bare fiber consists of a core, a cladding and a polymer jacket (buffer coating).



- The polymer coating is the first line of mechanical protection.

- The coating also reduces the internal reflection at the cladding, so light is only guided by the core.

***In Lab1**, we shall learn how to strip off the jacket to expose the cladding. This is necessary in order to "cleave" the fiber for a smooth end face for light coupling.

Silica optical fibers

• Both the core and the cladding are made from a type of glass known as silica (SiO_2) which is *almost transparent in the visible and near-IR*.

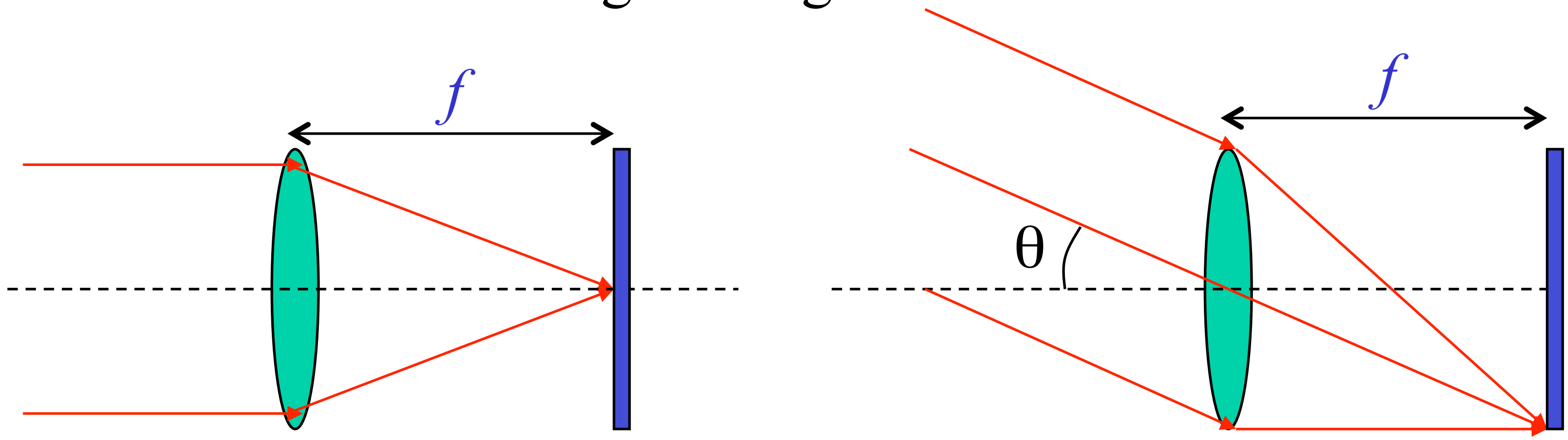
• In the case that the refractive index changes in a "step" between the core and the cladding. This fiber structure is known as step-index fiber.

• The higher core refractive index (~ 0.3% higher) is typically obtained by doping the silica core with germanium dioxide (GeO_2).

***In Lab 1**, we should be able to see the step boundary between the core and the cladding, by end-illuminating the fiber and imaging the output-end cross-section using a microscope.

Numerical aperture

- An important characteristic of an optic system is its ability to collect light incident over a wide range of angles.



The numerical aperture (NA) is defined as:

$$NA = n_o \sin\theta$$

where n_o is the refractive index of the medium between the lens and the image plane (e.g. a photodetector) and θ is the maximum acceptance angle.

- The definition of numerical aperture applies to *all light-collecting systems*, including optical fibers.

e.g. Light rays incident at angles *outside* the collection cone for a fiber will *not* propagate along the fiber (*instead will attenuate rapidly*).

- The numerical aperture is often measured in air, $n_o = 1$

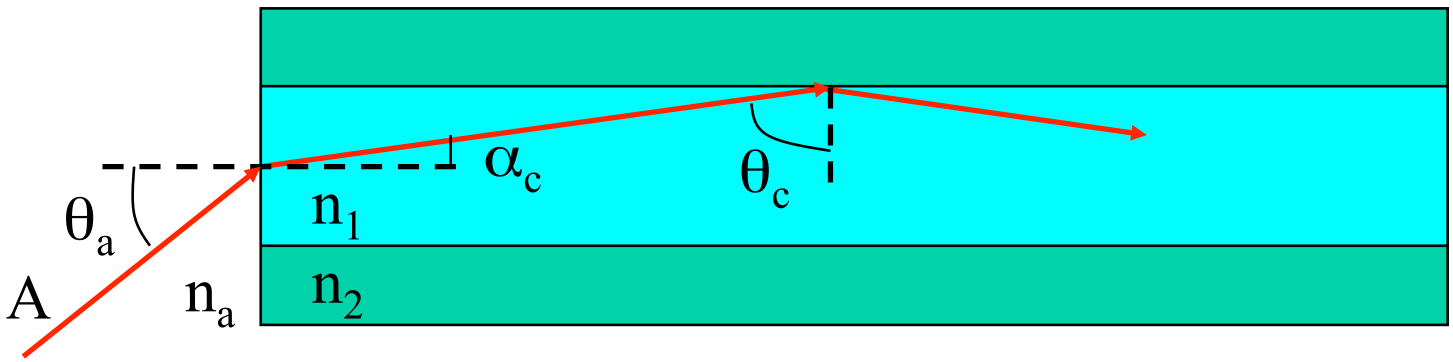
$$NA = \sin\theta$$

- A *low* NA indicates a *small* acceptance angle.

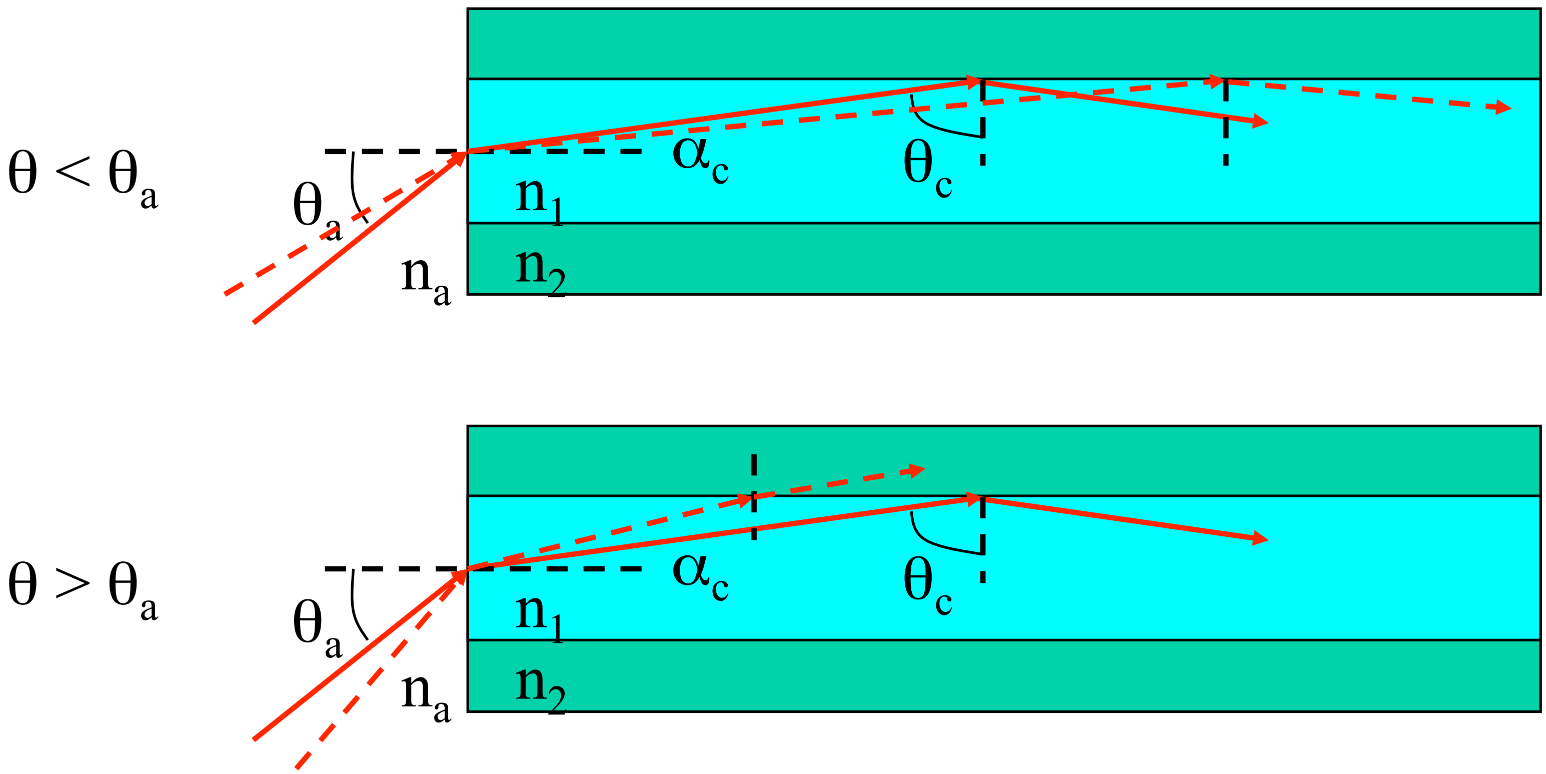
⇒Light coupling to a low-NA optical system (e.g. fiber) is more difficult (*alignment is more sensitive*) and less efficient (*some of the rays are outside the acceptance angle*) than is coupling to a high-NA optical system.

Acceptance angle

- Only rays with a sufficiently shallow grazing angle (i.e. with an angle to the normal greater than θ_c) at the core-cladding interface are transmitted by total internal reflection.



- Ray A incident at the critical angle θ_c at the core-cladding interface enters the fiber core at an angle θ_a to the fiber axis, and is refracted at the air-core interface.



• Any rays which are incident into the fiber core at an angle > θ_a have an incident angle less than θ_c at the core-cladding interface.

These rays will NOT be totally internal reflected, thus eventually loss to radiation (at the cladding-jacket interface).

• Light rays will be confined inside the fiber core if it is input-coupled at the fiber core end-face within the acceptance angle θ_a.

e.g. What is the fiber acceptance angle when $n_1 = 1.46$ and $n_2 = 1.44$?

$\theta_c = \sin^{-1}(n_2/n_1) = 80.5^o \Rightarrow \alpha_c = 90^o - \theta_c = 9.5^o$

using $\sin\theta_a = n_1 \sin\alpha_c$ (taking $n_a = 1$)

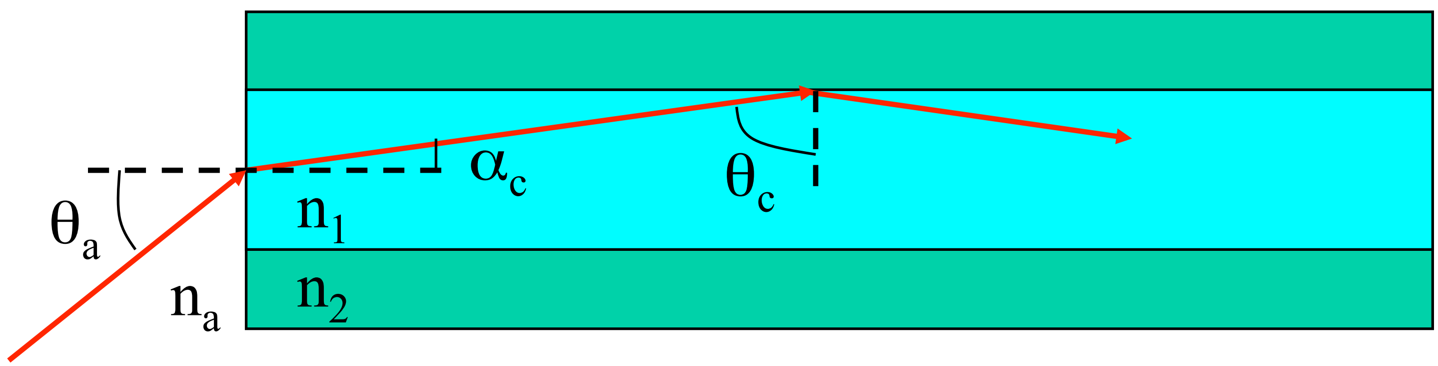
$\theta_a = \sin^{-1}(n_1 \sin\alpha_c) = \sin^{-1}(1.46 \sin 9.5^o) \sim 14^o$

=> the acceptance angle $\theta_a \sim 14^o$

Fiber numerical aperture

In fiber optics, we describe the fiber acceptance angle using **Numerical Aperture (NA):**

$$NA = n_a \sin \theta_a = \sin \theta_a = (n_1^2 - n_2^2)^{1/2}$$



- We can relate the <u>acceptance angle θ_a</u> and the <u>refractive indices of the core n_1, cladding n_2 and air n_a</u>.

• Assuming the end face at the fiber core is *flat* and *normal* to the fiber axis (when the fiber has a "nice" cleave), we consider the refraction at the air-core interface using Snell's law:

$$\text{At } \theta_a\text{:} \quad n_a \sin \theta_a = n_1 \sin \alpha_c$$

launching the light from air: $\sin \theta_a = n_1 \sin \alpha_c$

$(n_a \sim 1)$

$$= n_1 \cos \theta_c$$

$$= n_1 (1 - \sin^2\theta_c)^{1/2}$$

$$= n_1 (1 - n_2^2/n_1^2)^{1/2}$$

$$= (n_1^2 - n_2^2)^{1/2}$$

• Fiber NA therefore characterizes the fiber's ability to gather light from a source and guide the light.

e.g. What is the fiber numerical aperture when $n_1 = 1.46$ and $n_2 = 1.44$?

$$NA = \sin\theta_a = (1.46^2 - 1.44^2)^{1/2} = 0.24$$

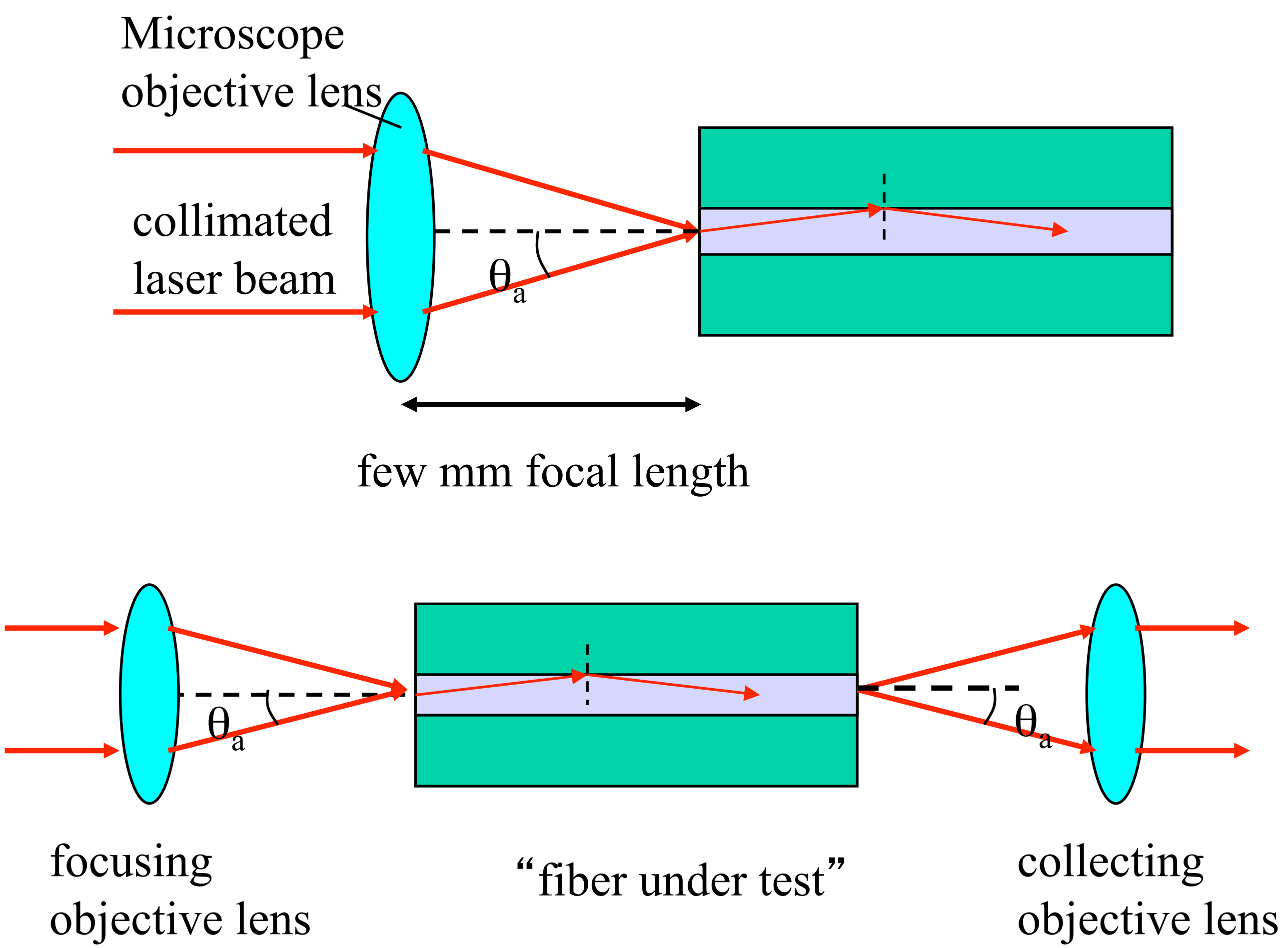
• It is a common practice to define a relative refractive index Δ as:

$$\Delta = (n_1 - n_2) / n_1$$

$$(n_1 \sim n_2) \quad \Rightarrow \quad NA = n_1 (2\Delta)^{1/2}$$

i.e. Fiber NA only depends on n_1 and Δ.

Lens coupling to fiber end faces



- By measuring the output couple ray cone angle, we can measure the fiber acceptance angle. (This is like part of Lab 1 but without using lenses.)

Large-NA fibers?

- Developing ways for fiber to collect light efficiently was an important early step in developing practical fiber optic communications (particularly in the 1970s)

- It seems logical to have optical fibers with NA as large as possible … with as large Δ as possible … in order to couple maximum amount of light into the fiber.

- Soon, we will find out that such large-NA fibers tend to be "multimode" and are *unsuitable* for high-speed communications because of a limitation known as modal dispersion.

- Relatively small-NA fibers are therefore used for high-speed optical communication systems.

Typical fiber NA

- Silica fibers for long-haul transmission are designed to have numerical apertures from about 0.1 to 0.3.

The low NA makes coupling efficiency tend to be poor, but turns out to improve the fiber's bandwidth! (details later)

- Plastic, rather than glass, fibers are available for short-haul communications (e.g. within an automobile). These fibers are restricted to short lengths because of the relatively high attenuation in plastic materials.

Plastic optical fibers (POFs) are designed to have high numerical apertures (typically, 0.4 – 0.5) to improve coupling efficiency, and so partially offset the high propagation losses and also enable alignment tolerance.

Limitation of ray optics

- For smaller fiber diameters that are only few times of the wavelength, *geometrical optics approach becomes inadequate*. This is because ray optics only describes the direction a plane wave component takes in the fiber, but does not take into account *interference* among such components.

- When interference phenomena are considered it is found that only rays with certain discrete characteristics propagate in the fiber core.

- Thus the fiber will only support a discrete number of guided modes.

- This becomes critical in small core diameter fibers which only support one (singlemode) or a few modes (multimode). Electromagnetic theory must be applied in this case.

Optical Fiber Type Comparisons

- The indices are uniform in a **step-index** fiber
- The index varies with the core radius in a **graded-index** fiber

Typical diameters

SM core: 8-10 μm

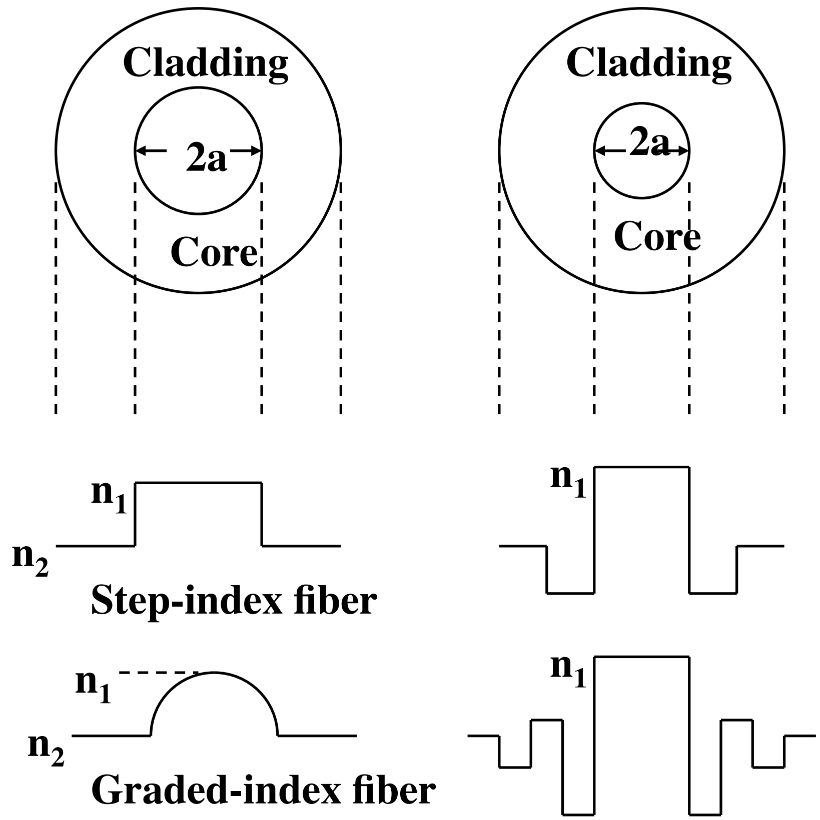
SM cladding: 125 μm

MM core: 50 or 62.5 μm

MM cladding: 125 μm

(SM = single mode)

(MM = multimode)



(a) Basic fiber types (b) Sample tailored profiles

Numerical Aperture Example

Example 2.4 Consider a multimode silica fiber that has a core refractive index $n_1 = 1.480$ and a cladding index $n_2 = 1.460$. Find (a) the critical angle, (b) the numerical aperture, and (c) the acceptance angle.

Solution: (a) From Eq. (2.21), the critical angle is given by

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.460}{1.480} = 80.5^\circ$$

(b) From Eq. (2.23) the numerical aperture is

$$\text{NA} = \left(n_1^2 - n_2^2\right)^{1/2} = 0.242$$

(c) From Eq. (2.22) the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1} 0.242 = 14^\circ$$

Example 2.5 Consider a multimode fiber that has a core refractive index of 1.480 and a core-cladding index difference 2.0 percent ($\Delta = 0.020$). Find the (a) numerical aperture, (b) the acceptance angle, and (c) the critical angle.

Solution: From Eq. (2.20), the cladding index is $n_2 = n_1(1 - \Delta) = 1.480(0.980) = 1.450$.

(a) From Eq. (2.23) we find that the numerical aperture is

$$\text{NA} = n_1\sqrt{2\Delta} = 1.480(0.04)^{1/2} = 0.296$$

(b) Using Eq. (2.22) the acceptance angle in air ($n = 1.00$) is

$$\theta_A = \sin^{-1}\text{NA} = \sin^{-1} 0.296 = 17.2^\circ$$

(c) From Eq. (2.21) the critical angle at the core–cladding interface is

$$\varphi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 0.980 = 78.5^\circ$$