Newton's Divided Difference Interpolation Formula

Interpolation is an estimation of a value within two known values in a sequence of values.
Newton's divided difference interpolation formula is an interpolation technique used when the interval difference is not same for all sequence of values.
Suppose $f(x_0)$, $f(x_1)$, $f(x_2)$……$f(x_n)$ be the (n+1) values of the function y=f(x) corresponding to the arguments $x=x_0$, x_1, $x_2 \ldots x_n$, where interval differences are not same
Then the first divided difference is given by

$$f[x_0, x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

The second divided difference is given by

$$f[x_0, x_1, x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$$

and so on…
Divided differences are symmetric with respect to the arguments i.e **independent of the order of arguments.**
so,
$f[x_0, x_1]=f[x_1, x_0]$
$f[x_0, x_1, x_2]=f[x_2, x_1, x_0]=f[x_1, x_2, x_0]$
By using first divided difference, second divided difference as so on .A table is formed which is called the divided difference table.

Divided difference table:

xi	fi	F(xi,xj)	F(xi,xj,xk)
x1	f1		
		$f[x1,x2]=\frac{f2-f1}{x2-x1}$	
x2	f2		$f[x1,x2,x3]=\frac{f[x3,x2]-f[x2-x1]}{x3-x1}$
		$f[x2,x3]=\frac{f3-f2}{x3-x2}$	
x3	f3		

NEWTON'S DIVIDED DIFFERENCE INTERPOLATION FORMULA

$$f(x) = f(x_0)+(x-x_0)f[x_0,x_1]+(x-x_0)(x-x_1)f[x_0,x_1,x_2]+\ldots\ldots\ldots\ldots\ldots\ldots+(x-x_0)(x-x_1)\ldots(x-x_{k-1})f[x_0,x_1,x_2\ldots x_k]$$

Examples:

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Input : Value at 7
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x	5	6	9	11
Y=f(x)	12	13	14	16

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Output :
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x	y=f(x)	f[xi,xj]	f[xi,xj,xk]	f[xi,xj,xk,xl]
5	12			
		1		
6	13		$\frac{-1}{6}$	
		$\frac{1}{3}$		$\frac{1}{20}$
9	14		$\frac{2}{15}$	
		1		
11	16			

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Value at 7 is 13.47
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