Contents

In today's lecture we'll have a look at:

– Bresenham's line drawing algorithm

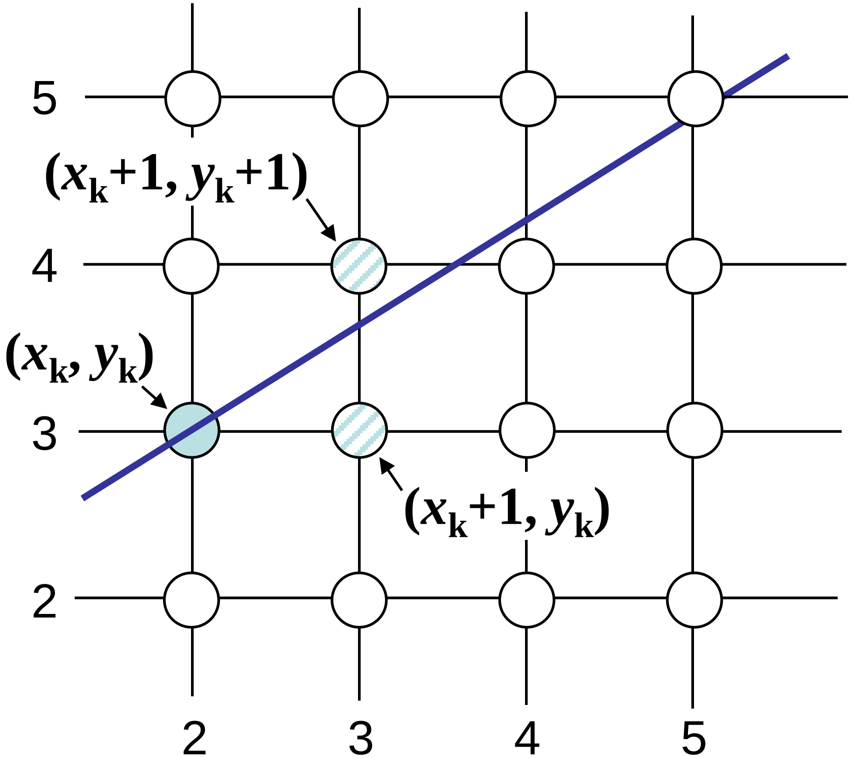
The Bresenham Line Algorithm

- The Bresenham algorithm is another **incremental scan conversion algorithm**.
- The big advantage of this algorithm is that it uses only **integer calculations**.

- **Jack Bresenham** worked for **27 years** at **IBM** before entering **Academia**.
- **Bresenham** developed his famous algorithms at IBM in the early **1960s.**

The Big Idea

Move across the x axis in unit intervals and at each step choose between two different y coordinates

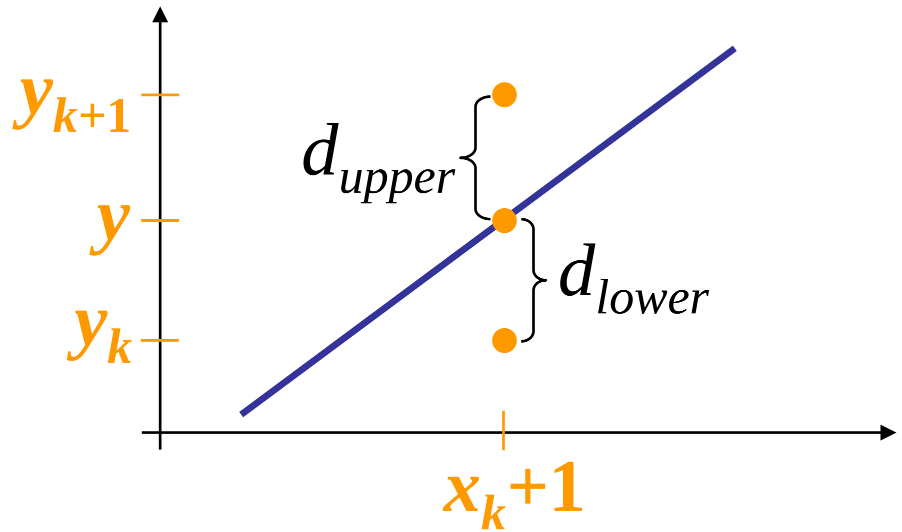


For example,

- from position **(2, 3)** we have to choose between **(3, 3)** and **(3, 4)**
- We would like the point that is closer to the original line

Deriving The Bresenham Line Algorithm

At sample position $\boldsymbol{x_k+1}$ the vertical separations from the mathematical line are labelled $\boldsymbol{d_{upper}}$ and $\boldsymbol{d_{lower}}$



The $\boldsymbol{y}$ coordinate on the mathematical line at $\boldsymbol{x_k+1}$ is:

$$y = m(x_k + 1) + b$$

Deriving The Bresenham Line Algorithm (cont…)

So, d_{upper} and d_{lower} are given as follows:

$$d_{lower} = y - y_k$$

and:

$$= m(x_k + 1) + b - y_k$$

$$d_{upper} = (y_k + 1) - y$$

$$= y_k + 1 - m(x_k + 1) - b$$

We can use these to make a simple decision about which pixel is closer to the mathematical line.

Deriving The Bresenham Line Algorithm (cont…)

This simple decision is based on the difference between the two pixel positions:

$$d_{lower} - d_{upper} = 2m(x_k + 1) - 2y_k + 2b - 1$$

Let's substitute ***m*** with Δ***y***/Δ***x*** where Δ***x*** and Δ***y*** are the differences between the end-points:

$$\Delta x(d_{lower} - d_{upper}) = \Delta x(2\frac{\Delta y}{\Delta x}(x_k + 1) - 2y_k + 2b - 1)$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + 2\Delta y + \Delta x(2b - 1)$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

Deriving The Bresenham Line Algorithm (cont…)

So, a decision parameter p_k for the kth step along a line is given by:

$$p_k = \Delta x(d_{lower} - d_{upper})$$
$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

- The sign of the **decision parameter** $\boldsymbol{p_k}$ is the same as that of $d_{lower} - d_{upper}$
- If $\boldsymbol{p_k}$ **is negative**, then we choose the **lower pixel**, otherwise we choose the **upper pixel**.

Deriving The Bresenham Line Algorithm (cont…)

Remember that, coordinate changes occur along the ***x* axis** in unit steps so we can do everything with integer calculations.

At step k+1 the decision parameter is given as:

$$p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$$

Subtracting $\boldsymbol{p_k}$ from this we get:

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

Deriving The Bresenham Line Algorithm (cont…)

But, $\boldsymbol{x_{k+1}}$ is the same as $\boldsymbol{x_k}\mathbf{+1}$ so:

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

where $\boldsymbol{y_{k+1} - y_k}$ is either **0** or **1** depending on the sign of $\boldsymbol{p_k}$

The first decision parameter $\mathbf{p_0}$ is evaluated at $\mathbf{(x_0, y_0)}$ is given as:

$$p_0 = 2\Delta y - \Delta x$$

The Bresenham Line Algorithm

Bresenham's Line Drawing Algorithm
(for $|m| < 1.0$)

Step 1: Input the two line end-points, storing the left end-point in (x_0, y_0)

Step 2: Plot the point (x_0, y_0)

Step 3: Calculate the constants Δx, Δy, $2\Delta y$, and $(2\Delta y - 2\Delta x)$ and get the first value for the decision parameter as:

$$p_0 = 2\Delta y - \Delta x$$

Step 4: At each x_k along the line, starting at $k = 0$, perform the following test. If $p_k < 0$, the next point to plot is (x_k+1, y_k) and:

$$p_{k+1} = p_k + 2\Delta y$$

The Bresenham Line Algorithm (cont…)

Otherwise, the next point to plot is (x_k+1, y_k+1) and:

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

Step 5: Repeat **Step 4** $(\Delta x - 1)$ times

Attention!

- The algorithm and derivation above assumes slopes are less than 1.
- For other slopes we need to adjust the algorithm slightly.

Bresenham Line Algorithm: Example

Let's have a go at this

Let's plot the line from (20, 10) to (30, 18)

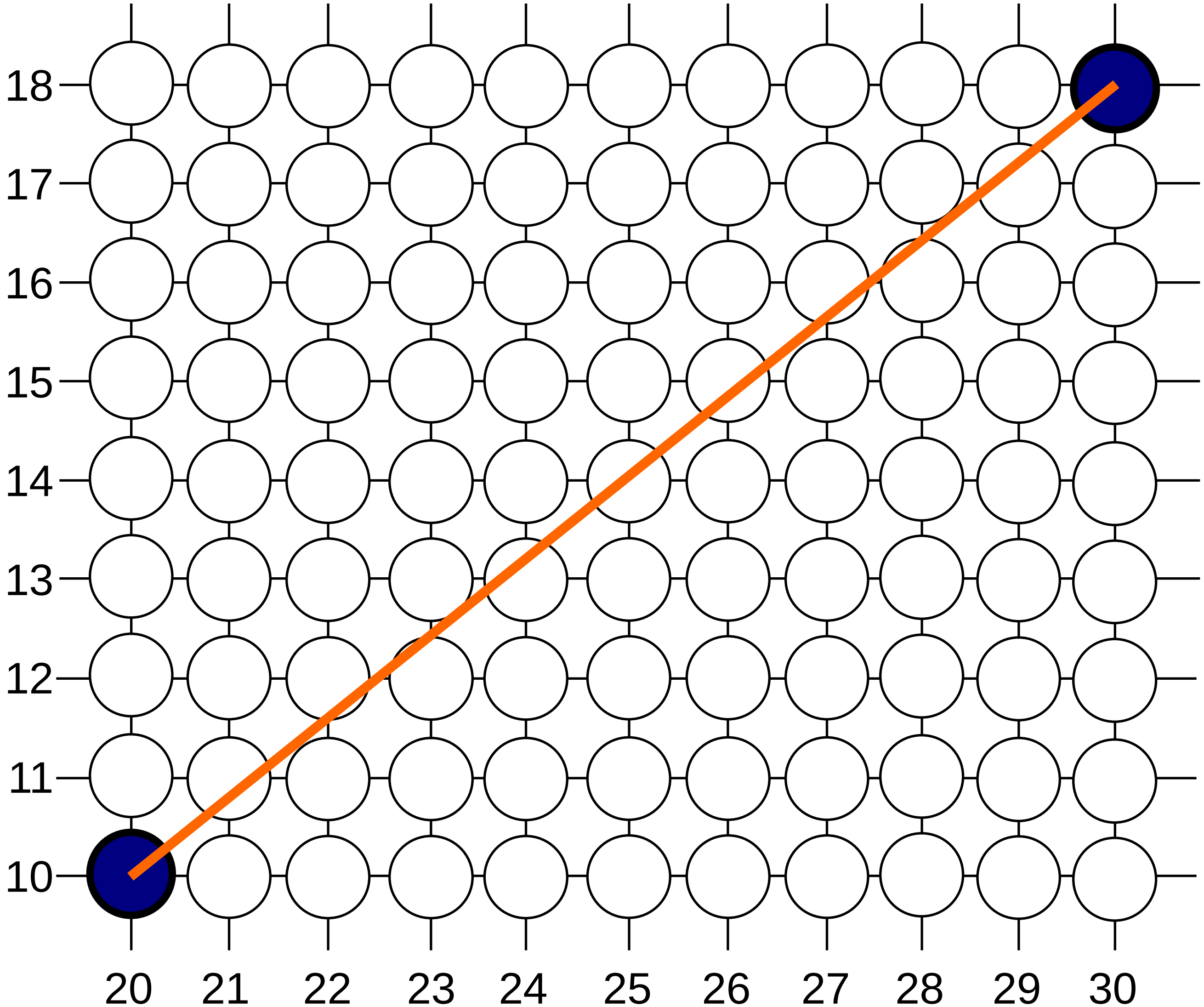
First off calculate all of the constants:

- Δx: 10
- Δy: 8
- $2\Delta y$: 16
- $2\Delta y$ - $2\Delta x$: -4

Calculate the initial decision parameter p_0:

- $p0 = 2\Delta y - \Delta x = 6$

Bresenham Line Algorithm: Example (cont…)

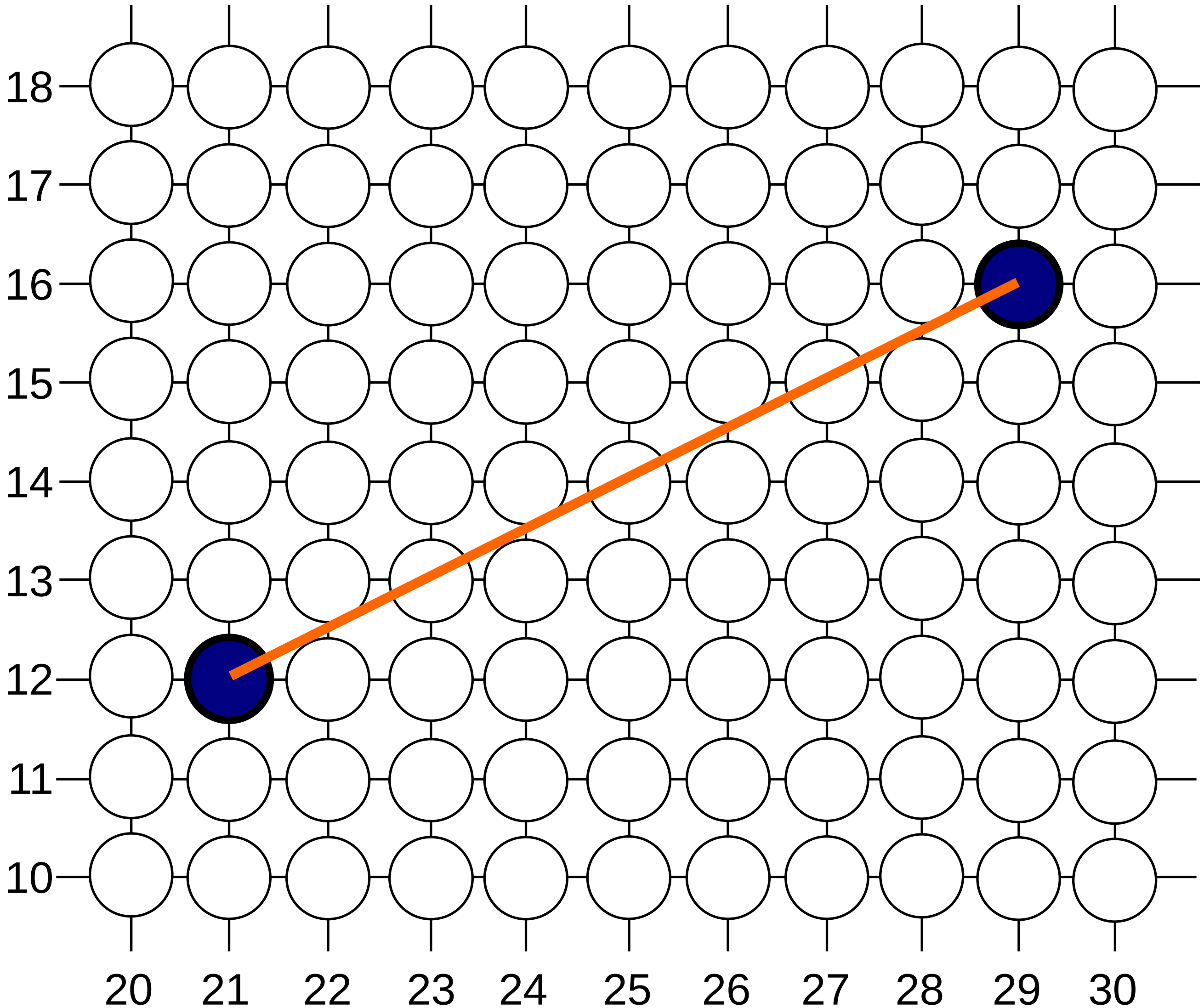


k	p_k	(x_{k+1},y_{k+1})
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

Bresenham Line Algorithm: Exercise

Go through the **Step 1** to **Step 5** of the Bresenham line drawing algorithm for a line going from (21,12) to (29,16)

Bresenham Exercise (cont…)



k	p_k	(x_{k+1},y_{k+1})
0		
1		
2		
3		
4		
5		
6		
7		
8		

Bresenham Line Algorithm: Summary
Advantages and Problems

The Bresenham line algorithm has the following **advantages**:

- A fast incremental algorithm
- Uses only integer calculations

Comparing this to the DDA algorithm, DDA has the following **problems**:

- Accumulation of round-off errors can make the pixelated line drift away from what was intended
- The rounding operations and floating point arithmetic involved are time consuming