Course Code: NFE-131
Course Title: Unit Operation in Food Industries
Course Teacher: Professor Dr. Md. Bellal Hossain

Fluid Flow

Lesson 5: Simple Manometer, Piezometer, U-Tube Manometer

5.1 Introduction

There are many techniques for the measurement of pressure and vacuum. Instruments used to measure pressure are called pressure gauges. Manometers are used for the measurement of very low pressures as well as vacuum especially in hydraulic laboratories.

5.2 Manometers

Manometers are used for measuring pressures by balancing the fluid column of fluid against another column of fluid of known specific gravity. Manometers can be classified as:

1. Simple manometers
   a. Piezometer
   b. U-tube manometer
2. Micro manometers (or single column manometers)
   a. Vertical column micro manometer
   b. Inclined column micro manometer
3. Differential manometers
   a. Upright U-tube differential manometer
   b. Inverted U-tube differential manometer

1. Simple manometers

It consists of a glass tube with one end open to the atmosphere and other end connected to a point at which pressure is to be measured.

a. Piezometer

It consists of glass tube connected to a vessel or pipe at which static pressure is to be measured. It is the simplest of all the manometers (Fig. 5.1). It is used to measure very low pressures.

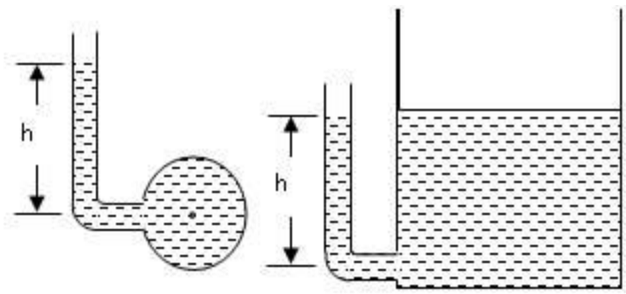


Fig. 5.1 Piezometer

The pressure in piezometer is given by the following equation.

$$p = \rho g h$$

Where,

ρ = density of liquid

h = height of liquid in the piezometer from the centre of the pipe.

g = acceleration due to gravity.

b. U-tube manometer

The manometer is named so because it consists of a glass tube having the shape of alphabet 'U�. One end is open to the atmosphere and other end connected to a point at which pressure is to be measured.

Let ρ_1 = density of liquid for which pressure has to be determined

ρ_2 = density of manometer liquid (assume mercury)

ω_1 = weight density of liquid for which pressure has to be determined

ω_2 = weight density of manometer liquid

S_1 = Specific gravity of liquid for which pressure has to be determined

S_2 = Specific gravity of manometer liquid

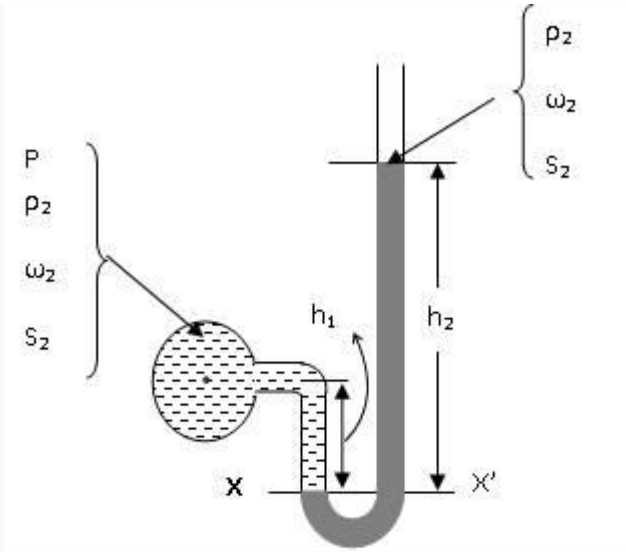


Fig. 5.2 U-tube manometer

Let h^{ρ} be the pressure in terms of height of fluid in the pipe.

h_1^{ρ} 1 is the distance from the datum line XX_1 to the centre of pipe

delta $h_2^{\rho}{}_2$ is the height of manometer liquid from the datum line XX_2 in the right limb

Pressure in the left limb at XX_1 $= P + \rho_1 g h_1 = P + \omega_1 h_1$

Pressure in the right limb at XX_2 $= \rho_2 g h_2 = \omega_2 h_2$

According to Pascal's law, at datum line pressure will be equal

$P + \omega_1 h_1 = \omega_2 h_2$

$P = \omega_2 h_2 - \omega_1 h_1$ (i)

$P = \rho_2 g h_2 - \rho_1 g h_1$ (ii)

On dividing equation (ii) by ρg where ρ is the density of the water:

$$\frac{P}{\rho g} = \frac{\rho_2 g h_2}{\rho g} - \frac{\rho_1 g h_1}{\rho g} \quad \text{(iii)}$$

$$\text{Sp. Gravity (s)} = \frac{\text{Density of substance}}{\text{Density of Water}}$$

(iv)

Since $P = \rho g h$ where delta h_1 is head of water

$h = S_2 h_2 / S_1 h_1$ (v)

5.3 Rules for Writing Equations for Manometers

Step 1: Draw a neat diagram of a manometer

Step 2: Consider a suitable datum line XX1. It should be in such a manner so that manometer liquid touching the datum line in the two limbs is the same. At figure 5.3, point A and B are on the datum line, liquids is same in both the columns i.e. left and right limbs.

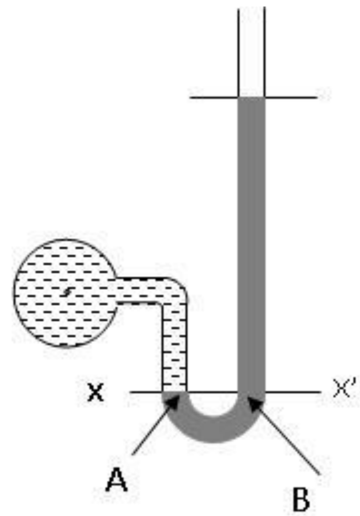


Fig. 5.3 Selection of datum line XX′

Step 3: Mark the distances of centre of pipe and the liquid level in the vertical column from the datum line X-X′

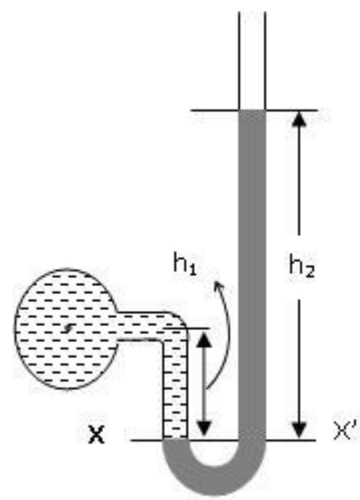


Fig. 5.4 Marking liquid levels

Step 4: Let h in meters is the pressure head in the centre of pipe.

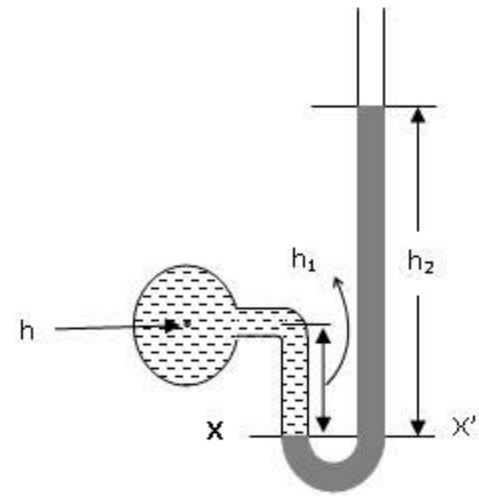


Fig. 5.5 Pressure head h at the centre of pipe

Step 5: Write the equation for pressure head in the left limb starting from center of pipe.

Pressure head in left limb at X-X′ = $h + h_1S_1$

Where,
S_1 = Specific gravity of liquid for which pressure has to be determined
S_2 = Specific gravity of manometer liquid (assume mercury)
Let delta h_1 be the pressure in terms of height of fluid in the pipe.
Delta h_1s1 is the distance from the datum line XX′ to the centre of pipe
Delta h_2 s_2 is the height of heavy liquid from the datum line XX′ in the right limb

Step 6: Write the equation for pressure head in the right limb starting from center of pipe.

Pressure head in right limb at X-X′ = h_2S_2

Step 7: Equate the pressure heads in the two limbs (left and right) to get the value of h.

Equating pressure head at X-X′ as the pressure at datum line would be equal.
$h + h_1S_1 = h_2S_2$
or,
$h = h_2S_2 - h_1S_1$
Note

1. Moving downward from a point in a manometer, all the pressure heads will be added.
2. Moving upward from a point in a manometer, all the pressure heads will be subtracted.

Lesson 6: Classification, Steady, Uniform And Non Uniform Flow, Laminar And Turbulent

11.1 Introduction

In Fluid Mechanics, the knowledge of flow behavior is important as the analysis and calculations depends on the flow conditions.

11.2 Types of flow

1. *Steady flow*

In steady flow fluid parameters such as velocity, density, pressure, acceleration etc. at a point do not change with time.

$$\frac{\partial v}{\partial t} = 0; \quad \frac{\partial \rho}{\partial t} = 0; \quad \frac{\partial P}{\partial t} = 0; \quad \frac{\partial a}{\partial t} = 0;$$

Where v: velocity; P: Pressure; ρ: Density; a: acceleration; t: time

2. ***Unsteady flow***

In unsteady flow fluid parameters such as velocity, density, pressure, acceleration etc. at a point changes with time.

$$\frac{\partial v}{\partial t} \neq 0; \quad \frac{\partial \rho}{\partial t} \neq 0; \quad \frac{\partial P}{\partial t} \neq 0; \quad \frac{\partial a}{\partial t} \neq 0;$$

3. ***Uniform flow***

In uniform flow if the velocity at a given instant of time is same in both magnitude and direction at all points in the flow, the flow is said to be uniform flow.

4. ***Non-uniform flow***

When the velocity changes from point to point in a flow at any given instant of time, the flow is described as non-uniform flow.

5. ***Compressible flow***

The flow in which density of the fluid varies during the flow is called compressible fluid flow. (i. e. $\rho \neq constant$). This is applicable in gas flow.

6. ***Incompressible flow***

In case of in compressible fluid flow, the density of the fluid remains constant during the flow. (i. e. $\rho = constant$). Practically, all liquids are treated as incompressible.

7. *Pressurized flow*

Flow under pressure. e..g. liquid flowing in pipes with pressure.

8. *Gravity flow*

Flow of fluid due to gravity.

9. *One, two and three dimensional flow*

a. One Dimensional: When the flow properties (e.g. velocity, density pressure etc) vary only in one direction.
b. Two Dimensional flows: When the flow properties (e.g. velocity, density pressure etc) vary in only two directions.
c. Three Dimensional flows: When the flow properties (e.g. velocity, density pressure etc) vary in all the three directions.

10. Rotational and irrotational flows:

Rotational flow: The fluid particles while flowing also rotate about their own axis.
Irrotational flow: The fluid particles while flowing do not rotate about their own axis.

11. *Laminar flow*

In this type of fluid flow, particles move along well defied paths or steam lines. The fluid layers moves smoothly over the adjacent layer. The fluid particles move in a definite path and their paths do not cross each other (Fig. 11.1).

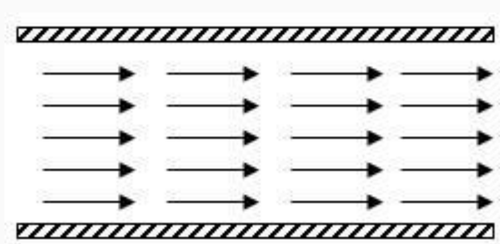


Fig. 11.1 Laminar flow

12. *Turbulent Flow*

In turbulent fluid flow, fluid particles move in a random and zigzag way (Fig. 11.2). Turbulence is characterized by the formation of eddies.

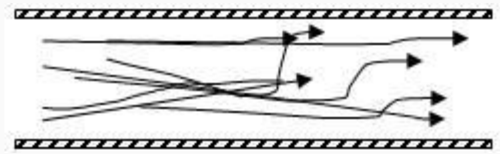


Fig. 11.2 Turbulent flow

The type of flow is determined by Reynold�s Number.

11.3 Reynold�s Number

It is defined as the ratio of inertia force of the flowing fluid to the viscosity force of the fluid. In case of pipe flow, it is determined by using the following equation.

$$Re = \frac{\rho V D}{\mu}$$

Where, Re=Reynold′s Number
ρ= Density of fluid
V= Velocity of fluid
D= Diameter of pipe
μ= Viscosity of fluid

Reynold′s Number (Re)	Flow type
Re < 2100	Laminar flow
2100 < Re < 4000	Transitional (flow can be laminar or turbulent)
Re > 4000	Turbulent

11.4 Streamline, Path Line and Streakline

Streamline: Is an imaginary line and velocity vector at any point on a stream line is tangent to the streamline (Fig. 11.3).

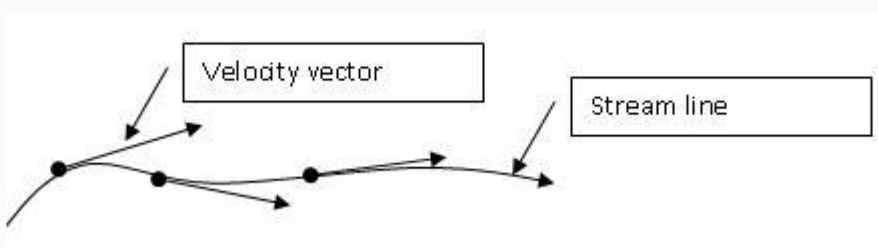


Fig. 11.3 Streamline

Path line is the path traced by a fluid particles.

Streaklines are obtained by joining the locus of points of all the fluid particles that have passed continuously through a fixed point during time t. Dye steadily injected into the fluid at a fixed point extends along a streakline.

11.5 Numericals

Q.1 Predict whether the flow would be laminar or turbulent in a pipe of diameter 5 cm. Consider density of liquid to be 950 kg/m^3, viscosity 0.2 Ns/m^2 and flow velocity 20 m/s.

Q.2 Calculate the Reynlod′s number if pipe diameter is 4 cm, liquid density 900 kg/cm^3, viscosity 0.5 Ns/m^2 and flow velocity 10 m/s.

Lesson 7: Continuity Equation

12.1 Introduction

Continuity equation is one of the widely used formulae in Fluid Mechanics. The equation based on the conservation of mass is called continuity equation. When fluid flows in any pipeline, the rate of fluid flowing at every section remains constant.

12.2 Continuity Equation

It is based on principle of conservation of mass and is expressed by the following relation.

Consider a pipe of varying diameter as shown in (Fig 12.1). The fluid is compressible and the density at section XX′ is ρ_1 and at YY′ is ρ_2.

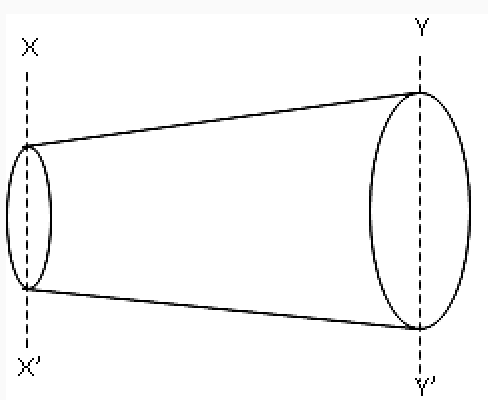


Fig. 12.1 Pipe of increasing diameter

At section XX′:

Area of pipe = A_1

Velocity of fluid = V_1

Density of fluid = ρ_1

At YY′:

Area = A_2

Velocity=V_2

Density = ρ_2

Volume flowing = A_1V_1 at section XX′

Mass of fluid flowing per second at section XX′ = $\rho_1A_1V_1$. (i)
Mass of flowing/sec at section YY′ = $\rho_2A_2V_2$..(ii)

Mass flow rate into the system = Mass flow rate out of the system

For compressible fluid applying conservation of mass,

$\rho_1A_1V_1 = \rho_2A_2V_2$ (Mass flow rate remains constant)

For incompressible fluid

ρ = constant i.e. $\rho_1=\rho_2= \rho$

$\therefore A_1V_1 = A_2V_2$

Discharge, Q = AV

At XX, Discharge (m^3/s) $Q_1 = A_1V_1$

At YY, Discharge (m^3/s) $Q_2 = A_2V_2$

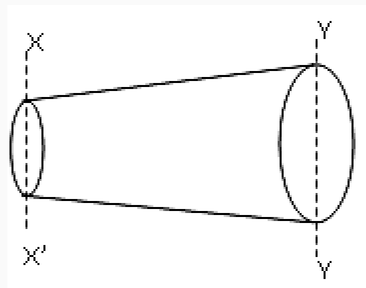
According to continuity equation,

$Q_1 = Q_2$

Thus, continuity equation relates flow velocity with area of the section, if area of flow at any section is decreased there is an increase in flow velocity.

12.3 Numericals

Q 1. The area of a pipe at section XX′ is 315 cm^2. The area of pipe at section YY′ is twice the area of section XX′. The velocity at section XX′ is 4.5 m/s. Find the velocity at section YY′. Also determine the flow rate through the pipe.



Solution:

$A_1V_1 = A_2V_2$

$315 \times 10^{-4} \times 4.5 = 630 \times V \times 10^{-4}$

$V = \frac{315 \times 4.5}{630} = 2.25 m/s$

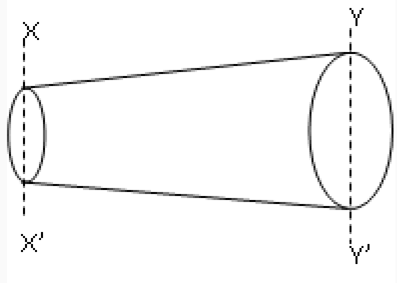
Discharge = A_1V_1

$= 315 \times 10^{-4} \times 4.5$

$= 1417.5 \times 10^{-4}$

$= 0.14175$ m^3/s

Q 2. A compressible liquid is flowing in a pipe. The diameter of section XX′ and YY′ is 150 mm and 250 mm respectively. The density at XX′ is 998 kg/m^3 and at YY′ is 994 kg/m^3. If the velocity of liquid at section XX′ is 3.5 m/s, find the velocity at section YY′.



Solution:

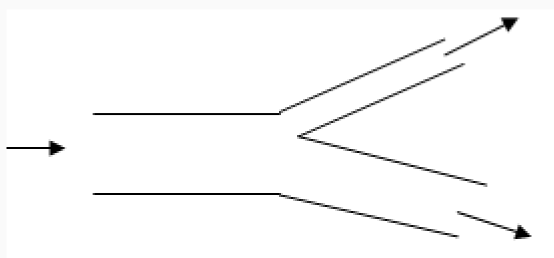
$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$998 \times \pi/4 \times 150^2 \times 10^{-6} \times 3.5 = 994 \times \pi/4 \times 250^2 \times 10^{-6} \times V_2$

$$V_2 = \frac{998X150X150X3.5}{994X250X250} = \frac{7859250}{621250} = 12.65 m/s$$

Q 3. A pipe 500 mm in diameter bifurcates into two pipes of diameter 200 mm and 350 mm respectively. If the flow is incompressible and the velocity of flow in 500 mm and 200 mm pipe is 4.0 m/s and 3.0 m/s respectively. Find the (a) Discharge through 500 mm & 350 mm diameter pipe.

Solution:



Discharge through 500 mm = $(\pi/4) \times 500^2 \times 10^{-6} \times 4$ m^3/s

$Q_1 = Q_2 + Q_3$

Or,

$$A_1V_1 = A_2V_2 + A_3V_3$$

$$\frac{\pi}{4}\times 500^2 \times 10^{-6} \times 4 = \frac{\pi}{4}\times 200^2 \times 10^{-6} \times 3 + \frac{\pi}{4}\times 350^2 \times 10^{-6} \times V_3$$

$$25000 \times 4 = 40000 \times 3 + 350 \times 350 \times V_3$$

$$1000000 - 120000 = 350x \times 350 \times V_3$$

$$880000 = 350 \times 350 \times V_3$$

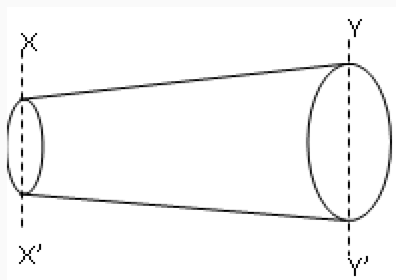
$$V_3 = \frac{880000}{350 \times 350} = 7.183 m/s \quad Ans$$

$$\text{Discharge through 35mm pipe} = \frac{\pi}{4}(350)^2 \times 10^{-6} \times V_3$$

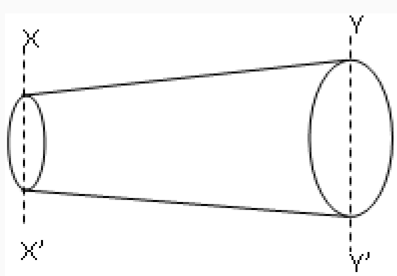
$$= 0.785 \times (350)^2 \times 7.18 \times 10^{-6} = 690446.75 \times 10^{-6} = 0.69\ m^3/s \text{ Ans}$$

12.4 Numericals

Q.1. A incompressible liquid is flowing in a pipe. The diameter of section XX′ and YY′ is 170 mm and 200 mm respectively. If the velocity of liquid at section XX′ is 5.5 m/s, find the velocity at section YY′.



Q. 2. A compressible liquid is flowing in a pipe. The diameter of section XX′ and YY′ is 50 mm and 100 mm respectively. The density at XX′ is 995 kg/m^3 and at YY′ is 990 kg/m^3. If the velocity of liquid at section XX′ is 5.5 m/s, find the velocity at section YY′.



Lesson 8: Bernoulli′s Theorem

13.1 Introduction

Bernoulli's Theorem is based on the conservation of energy in fluid flow. There are three types of energy namely potential energy, kinetic energy and pressure energy possessed by the liquid. The theorem explains how these energies change from one form to another form. Many instruments such as Pitot tube, Venturimeter etc. are working on the principle of Bernouilli's theorem for the measurement of fluid flow.

13.2 Bernoulli's Theorem

Bernoulli's Theorem states that in a steady ideal flow of in compressible fluid flow, the sum of pressure energy, kinetic energy and potential energy remains constant at every section provided no energy is added or taken out by an external sources

It is based on principle of conservation of energy and is expressed by the following relation:

Pressure energy + Kinetic energy + Potential energy = constant

Assumption for Bernoulli's equation

Bernoulli's theorem holds well under the following assumptions:

(1) The flow is along stream line.

(2) The flow is steady and continuous.

(3) The fluid is non-viscous (ideal fluid) and incompressible (ρ = constant).

(4) Flow is irrotational.

Pressure head = $\frac{P}{\rho g}$

Velocity head = $\frac{V^2}{2g}$

Potential head = Z

According to Bernoulli's theorem,

$$\frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{constant}$$

Consider a pipe of varying diameter as shown in (Fig 13.1).

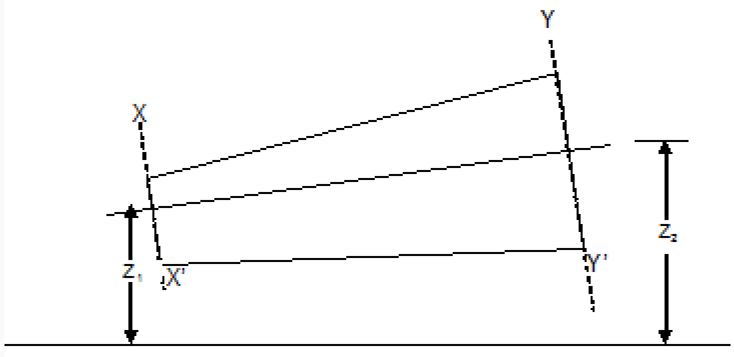


Fig. 13.1 Pipe of varying diameter

At section XX′

Cross sectional area of pipe = A_1
Velocity of fluid = V_1
Pressure of fluid = P_1

At YY′

Cross sectional area of pipe = A_2
Velocity of fluid = V_2
Pressure of fluid = P_2

Applying Bernoulli′s theorem at section XX′ and YY′

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 = \text{constant}$$

The above equation holds true for an ideal fluid (i.e. when there is no loss of head between two points). However, in practice some energy is lost due to friction and it is denoted by H_L then

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + H_L$$

13.3 Total Energy Line (TEL) and Hydraulic Grade Line (HGL)

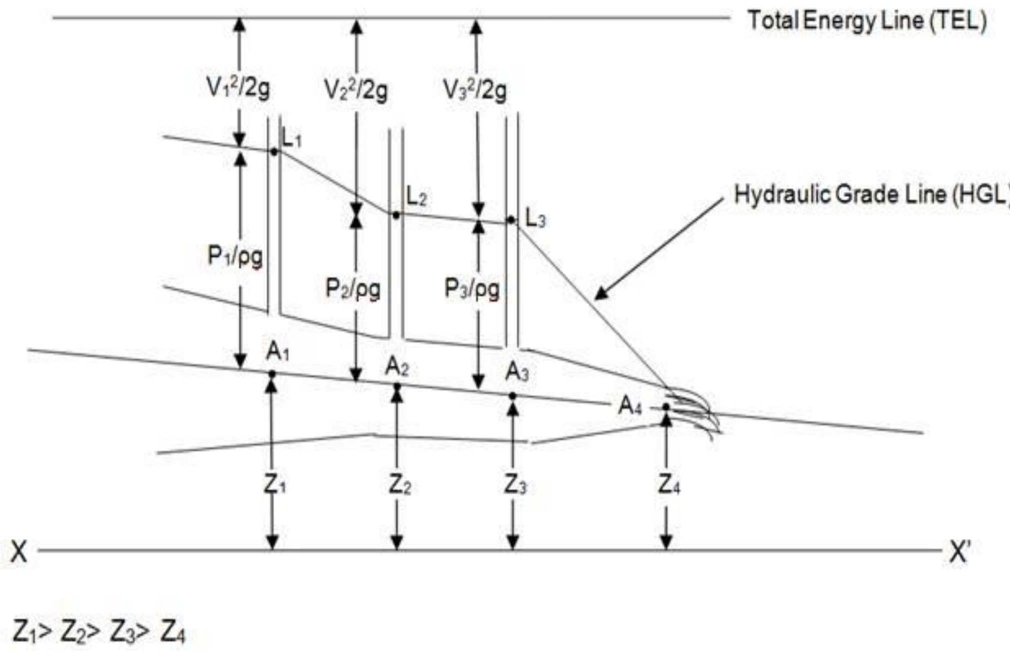


Fig. 13.2 Total Energy Line (TEL) and Hydraulic Grade Line (HGL)

(1) At points A_1, A_2 and A_3, piezometer tube measures the pressure head ($P/\rho g$) by rise of liquid in the peizometer tube h_1, h_2 and h_3 respectively.

(2) Points A_1, A_2, A_3 and A_4 are located at a height of Z_1, Z_2, Z_3 and Z_4 above datum (reference line XX′). Z_1, Z_2, Z_3 and Z_4 are known as potential heads.

(3) Points L_1, $L_{2,}$ and L_3 represent the water level in the peizometer and are located at a height of (Z_1+h_1), (Z_2+h_2) and (Z_3+h_3).

(4) Point L_4 is located at tip of nozzle is at height Z_4 above datum line.

(5) The line joining points L_1, L_2, L_3 and L_4 is known as HGL and ($P/\rho g$ + Z) is known as hydraulic gradient.

(6) Total energy line is line joining points denoting total head is represented as

$$\frac{P}{\rho g}+\frac{v^2}{2g}+Z$$

(7) At tip of nozzle (L_4) the liquid is exposed to atmosphere as a result pressure head becomes zero.

(8) For flow in converging pipe there is a decrease in the pressure head ($P/\rho g$) and consecutively there is a rise in velocity energy.

(9) At tip of nozzle pressure head and entire pressure energy gets converted into K.E. causing the flow velocity to increase.

13.4 Numericals

Q 1. Water is flowing through an inclined pipeline of diameter 20 cm & 40 cm at section A & B respectively. Section A & B are located at height of 2 m & 2.5 m respectively from ground level. The discharge through pipe is 30 l/s. If the pressure at A is 20 kPa, find the pressure at point B.

Solution:

$$1\ P_a = 1\ N/m^2$$

$$P_A = 20\ kPa;\ \mathrm{P_B} = ?$$

$$1000\,l = 1\,\mathrm{m}^3$$

$$1\,l = 10^{-3}\,m^3$$

$$Q = 30\ l/s = 0.03\ m^3/s$$

$$A_1V_1 = A_2V_2 = Q$$

$$A_1 = \frac{\pi}{4}(20)^2 \times 10^{-4} = 0.0314\ m^2$$

$$A_2 = \frac{\pi}{4}(40)^2 \times 10^{-4} = 0.1256\ m^2$$

$$V_1 = \frac{0.03}{A_1} = 0.955414012\ m/s$$

$$V_2 = \frac{0.03}{A_2} = 0.2388\ m/s$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20000}{1000 \times 9.81} + \frac{0.912}{2 \times 9.81} + 2 = \frac{P_2}{1000 \times 9.81} + \frac{0.057}{2 \times 9.81} + 2$$

$$2.038 + 0.046 + 2 = \frac{P_2}{9810} + 0.0002 + 2.5 = 1.582 = \frac{P_2}{9810}$$

$$P_2 = 15519.42\ \mathrm{Pa} \qquad P_2 = 15.519\ \mathrm{kPa}$$

Q 2. A horizontal pipe of diameter 250 mm carries oil (sp. gravity = 0.89) at the rate of 125 l/s and a pressure of 32 kPa. The pipe converges to a 100 mm diameter at a section located in down stream. Determine the pressure at down stream.

Solution:

$P_1 = 32\,\text{kPa}\,;\, \text{P}_2 = ?$

$Q = 125\,\text{l/s} = 0.125\,\text{m}^3/s$

$$0.049\,m^2 = A_1 = \frac{\pi}{4}\times(250)^2\times10^{-6}$$

$$0.0078\,m^2 = A_2 = \frac{\pi}{4}\times(100)^2\times10^{-6}$$

$$A_1V_1 = A_2V_2 = Q$$

$$V_1 = \frac{Q}{A_1} = \frac{0.125}{A_1} = 2.55\,m/s$$

$$V_2 = \frac{0.125}{A_2} = 16.02\,m/s$$

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{32000}{890\times9.81}+\frac{(2.55)^2}{2\times9.81} = \frac{P_2}{890\times9.81}+\frac{(16.02)^2}{2\times9.81}$$

$$3.66+0.33 = \frac{P_2}{8730.9}+13.08 = -79363.88 = -79.36\,kPa\text{ Ans}$$

Q 3. The area of cross-section at point A in a converging pipe reduces from 0.45 m^2 to 0.22 m^2 at point B. The velocity & pressure at point A are 2.5 m/s and 200 kN/m^2 respectively. Neglecting the frictional head loss in pipe, calculate the pressure at point B which is 6 m above the level of A.

Solution: $P_1 = P_2$

$$A_1 = \frac{\pi}{4}\times\frac{(200)^2}{(1000)^2}$$

$$A_2 = \frac{\pi}{4}\times\frac{(40)^2}{(1000)^2}$$

$$P_1 = \frac{F_1}{\frac{\pi}{4}\times\frac{(200)^2}{(1000)^2}};\quad P_2 = \frac{500}{\frac{\pi}{4}\times\frac{(40)^2}{(1000)^2}}$$

$$\frac{F_1}{\frac{\pi}{4}\times\frac{(200)^2}{(1000)^2}} = \frac{500}{\frac{\pi}{4}\times\frac{(40)^2}{(1000)^2}}$$

$$F_1 = \frac{200\times200\times500}{40\times40} = 62500N$$

$P_A = 200\,\text{kN/m}^2\,;\,\text{P}_\text{B} = ?$

$V_A = 2.5\, m/s;\; V_B = ?$

$A_1V_1 = A_2V_2 = 0.45 \times 2.5 = 0.22 \times V_B$

$V_B = 5.11\, m/s$

$$\frac{P_A}{\rho g} + \frac{V_A^{\ 2}}{2g} + Z_1 = \frac{P_B}{\rho g} + \frac{V_B^{\ 2}}{2g} + Z_2$$

$$\frac{200 \times 10^3}{1000 \times 9.81} + \frac{(2.5)^2}{2 \times 9.81} + 0 = \frac{P_B}{1000 \times 9.81} + \frac{(5.11)^2}{2 \times 9.81} + = 6$$

$$20.38 + 0.31 = \frac{P_B}{9810} + 1.33 + 6$$

$$13.36 = \frac{P_B}{9810}$$

$P_B = 131061.6 = 131.06\, kN/m^2$ Ans