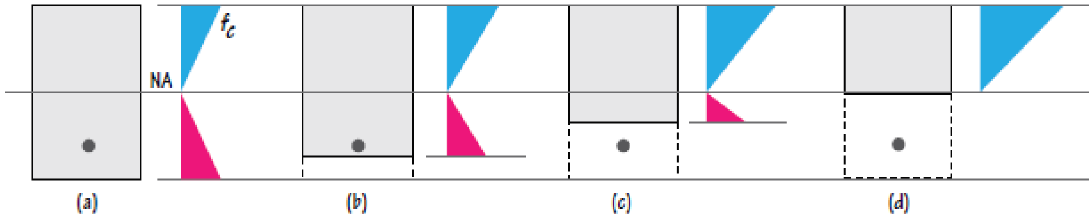
Elastic Analysis of RC Beam (Section Transformation)

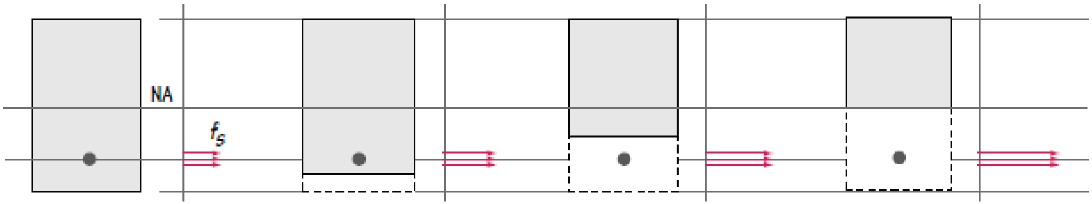
Bending Stresses in RC Beam

Stress in Concrete



- *Case (a):* If concrete is uncracked, the flexural stress diagram exists in both compressive zone (shaded blue) and tensile zone (shaded red).
- *Case (b):* As load is increased, the crack propagates upward, and the depth of effective concrete section (shaded grey) reduces. Since there in no concrete in the cracked zone (dotted line), the stress diagram shrinks in size in tensile zone. The stress diagram in compressive zone increases.
- *Case (c):* The crack continues to propagate upward upon addition of more loads. The stress diagram in tensile zone shrinks further and the stress diagram in compressive zone increases even more.
- *Case (d):* When concrete is fully cracked, i.e., cracked has reached neutral axis, the stress diagram in tensile zone completely vanishes and only the stress in compressive zone remains.

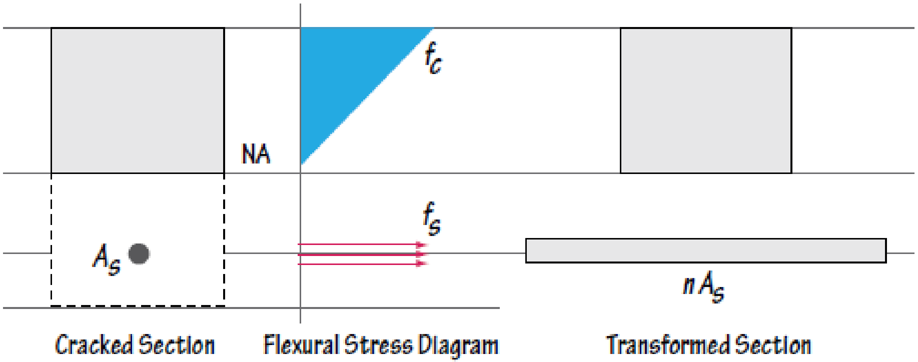
Stress in Steel



For all cases, from (a) to (d), the stress in steel exists. But it's magnitude increases as load is increased.

Transformed Section

- Since concrete beam is made of composite material, i.e., it consists of both concrete and steel, it is therefore necessary to convert it into one homogeneous material before any analysis.
- Usually the steel is converted into its equivalent concrete but the other-way is also possible and the resulting section is known as *transformed section*.
- To convert the steel area (A_s) into its equivalent area of fictitious concrete, it should be multiplied by modular ratio (n). The resulting area (nA_s) is known as *transformed area* and it must be at the same depth of actual steel.



- Though a real concrete cannot resist any tension, it is assumed that the fictitious concrete of the transformed area can resist tension.
- The modular ratio is defined as, $n = E_s/E_c$, where E_s is the elastic modulus of steel and E_c is the elastic modulus of concrete.

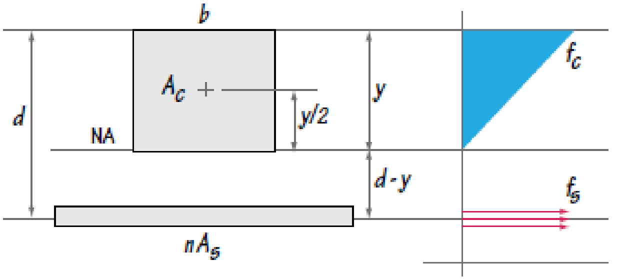
Determination of Stress in Concrete and Steel

Location of Neutral Axis

The depth of neutral axis (y) is unknown before analysis. But the *moment of compression area* (A_C) about neutral axis always equals the *moment of tensile area* (nA_s) about neutral axis. This property is used to determine y.

Moment of Compressive Area = Moment of Tensile Area

$$A_C\left(\frac{y}{2}\right) = (nA_s)(d-y)$$



Moment of Inertia

Following is the general formula for moment of inertia of the transformed section,

$$I = \left[\frac{bh^3}{3}\right]_{\text{conc.}} + \left[\frac{bh^3}{12} + Ad^2\right]_{\text{steel}}$$

According to the figure, $h = y$, for concrete. Since the height of steel is very small, $bh^3/12$ is very close to zero, thus could be ignored. For steel, $A = nA_s$ and $d = d - y$. Therefore, the general formula becomes,

$$I = \left[\frac{by^3}{3}\right]_{\text{conc.}} + \left[(nA_s)(d-y)^2\right]_{\text{steel}}$$

Stress in Concrete and Steel

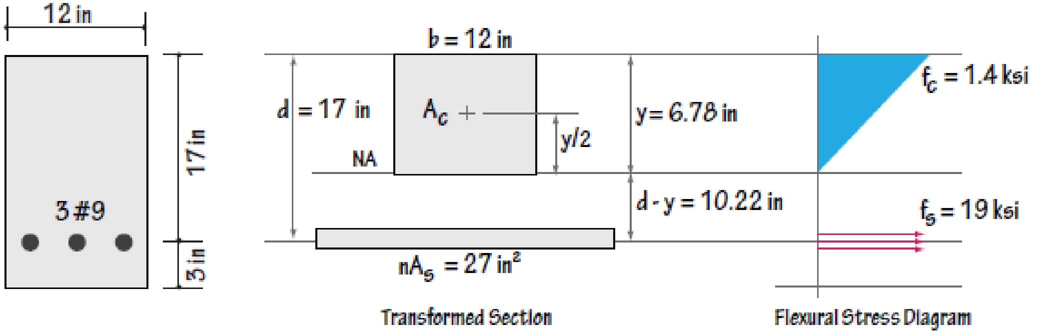
$$f_c = \frac{My}{I}, \qquad f_s = n\frac{M(d-y)}{I}$$

Notice the presence of n in the equation of f_s. Since the steel was transformed to its equivalent concrete, the concrete stress $M(d-y)/I$ must be multiplied by n to determine the steel stress.

Symbol	Description
y $y/2$	Depth of neutral axis Distance between center of effective concrete area and neutral axis
A_c A_s nA_s	Area of concrete, $A_c = by$ Area of steel Transformed area of steel
d $d-y$	Effective depth of beam Distance between steel and n.a.
f_c f_s	Maximum stress in concrete Stress in steel

Section Cracked, Example 1

Ques. Calculate the maximum bending stresses in concrete and stress in steel of the following beam section by using the transformed area method. Given that, $n = 9$, and $M = 70$ k-ft.



Solution.

Determine Steel Area

Each number #9 steel bar has an area of 1.00 in^2. Therefore, three #9 has three times of that amount.

$$A_s = 3\times1.00 = 3.00 \text{ in}^2$$

Location of Neutral Axis

$$A_c\left(\frac{y}{2}\right) = (nA_s)(d-y)$$

$$(12\cdot y)\left(\frac{y}{2}\right) = (9\times3.00)(17-y)$$

$$6y^2 + 27y - 459 = 0$$

Solving we find, $y = 6.78$ in, -11.28 in. Since only positive value is accepted for length, $y = 6.78$ in.

Moment of Inertia of Transformed Section

$$I = \left[\frac{by^3}{3}\right]_{\text{conc.}} + \left[(nA_s)(d-y)^2\right]_{\text{steel}}$$

$$= \frac{12\times6.78^3}{3} + (9\times3.00)(17-6.78)^2$$

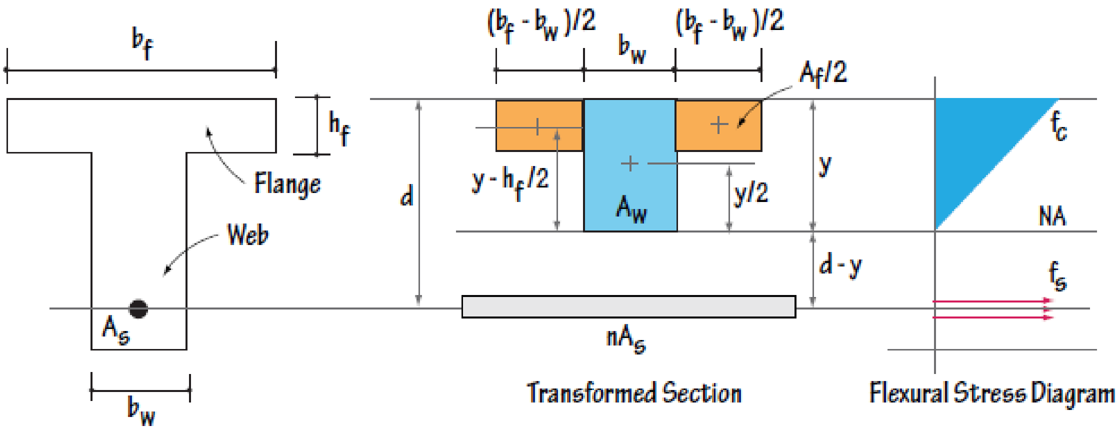
$$= 4067 \text{ in}^4$$

Bending Stresses

$$f_c = \frac{My}{I} = \frac{(70\times12)6.78}{4067} = 1.40 \text{ ksi}$$

$$f_s = n\frac{M(d-y)}{I} = 9\times\frac{(70\times12)10.22}{4067} = 19.0 \text{ ksi}$$

Elastic Analysis of T-Beam



Location of Neutral Axis

To ease the analysis procedure, T-beam is considered into two parts, the *web part* (shaded in blue) and the *flange part* (shaded in orange).

Moment of Compressive Area = Moment of Tensile Area

$$\left[A_w\left(\frac{y}{2}\right)\right]_{web} + 2\left[\left(\frac{A_f}{2}\right)\left(y-\frac{h_f}{2}\right)\right]_{flange} = (nA_s)(d-y)$$

Notice the presence of 2 in the flange part. Since only one side of flange is considered in the expression, it must be multiplied by 2 to account the other side of flange.

Symbol	Description
b_w	Width of web
b_f	Width of flange
A_w	Area of web $A_w = b_w y$
A_f	Area of flange $A_f = (b_f - b_w)h_f$

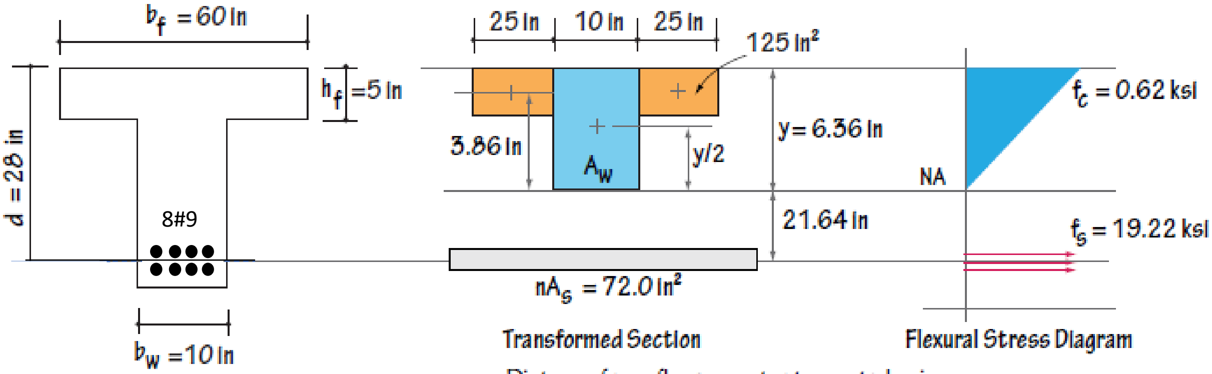
Moment of Inertia

$$I = \left[\frac{b_w y^3}{3}\right]_{web} + 2\left[\frac{(b_f - b_w)/2 \cdot h_f^3}{12} + \left(\frac{A_f}{2}\right)\left(y - \frac{h_f}{2}\right)^2\right]_{flange} + \left[(nA_s)(d-y)^2\right]_{steel}$$

Though expressions of T-beam looks scary, in numeric problems, they turn out to be quite simple.

Section Cracked, Example 2

Ques. Calculate the bending stresses in the T-beam by using the transformed area method, $n = 9$, and $M = 250$ k-ft.



Solution.

Transformed steel area,

$$nA_s = 9\times(8\times1.00) = 72.00 \text{ in}^2$$

Width of each flange block,

$$\frac{b_f - b_w}{2} = \frac{60-10}{2} = 25 \text{ in}$$

Therefore, area of each flange block (shaded orange),

$$A_f/2 = 25\times5 = 125 \text{ in}^2$$

Location of Neutral Axis

$$(10\cdot y)\left(\frac{y}{2}\right) + 2(125)(y-2.5) = 9(6.00)(28-y)$$

$$5y^2 + 304y - 2137 = 0, \qquad \boxed{y = 6.36 \text{ in}}$$

Distance from flange center to neutral axis

$$y - \frac{h_f}{2} = 6.36 - \frac{5}{2} = 3.86 \text{ in}$$

Moment of Inertia of Transformed Section

$$I = \frac{10\times6.36^3}{3} + 2\left[\frac{25\times5^3}{12} + 125\times3.86^2\right] + 72.00\times21.64^2$$

$$= 30,391 \text{ in}^4$$

Bending Stresses

$$f_c = \frac{My}{I} = \frac{(250\times12)6.36}{30,391} = 0.62 \text{ ksi}$$

$$f_s = n\frac{My}{I} = 9\times\frac{(250\times12)21.64}{30,391} = 19.22 \text{ ksi}$$