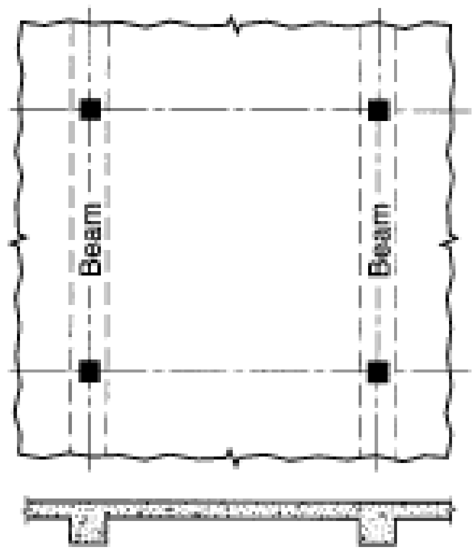
Analysis and Design of Slabs

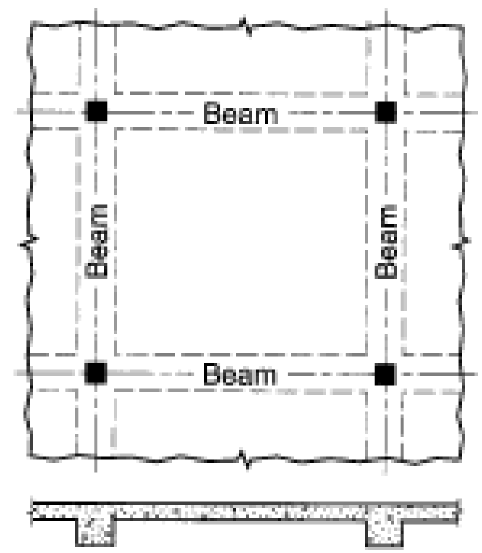
Chapter 13

Types of Slabs

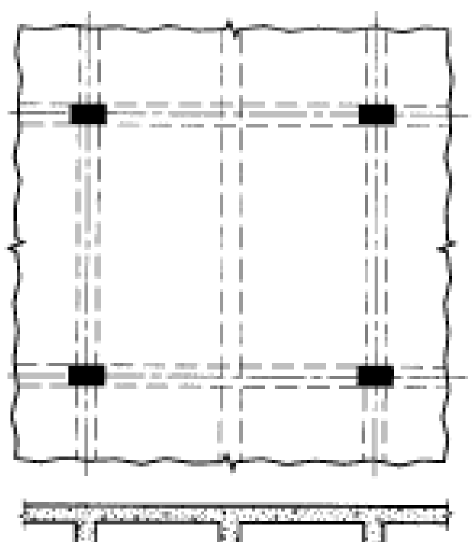
- Useful surface
- Supported on monolithic beams, steel joist, masonry or RC wall, directly on column, continuously on ground
-

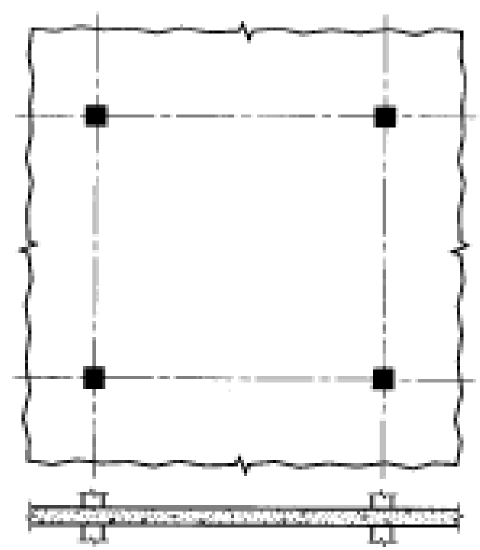


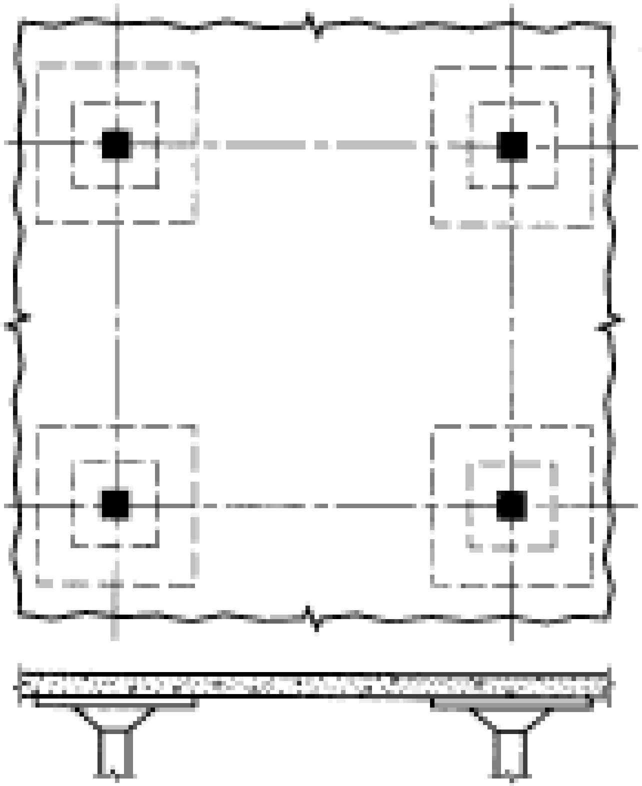
(*a*) One-way slab



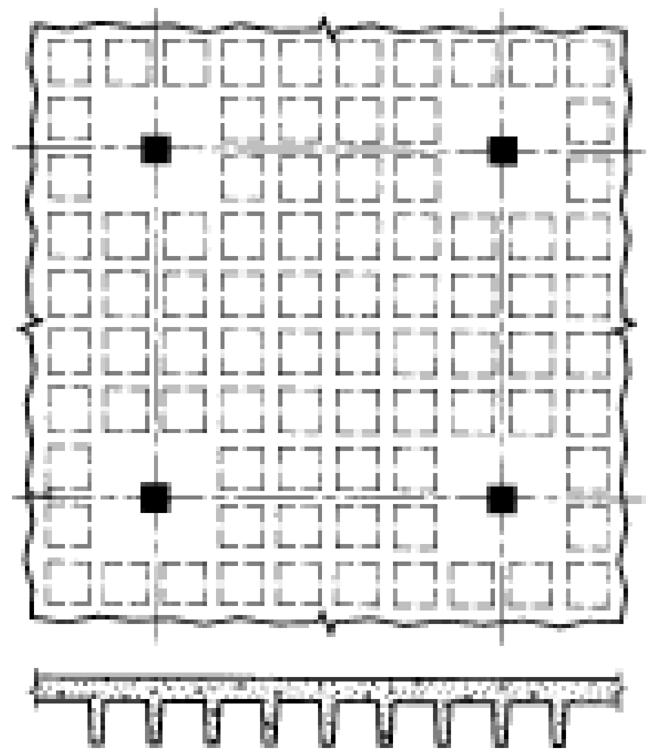
(*b*) Two-way slab


(*c*) One-way slab

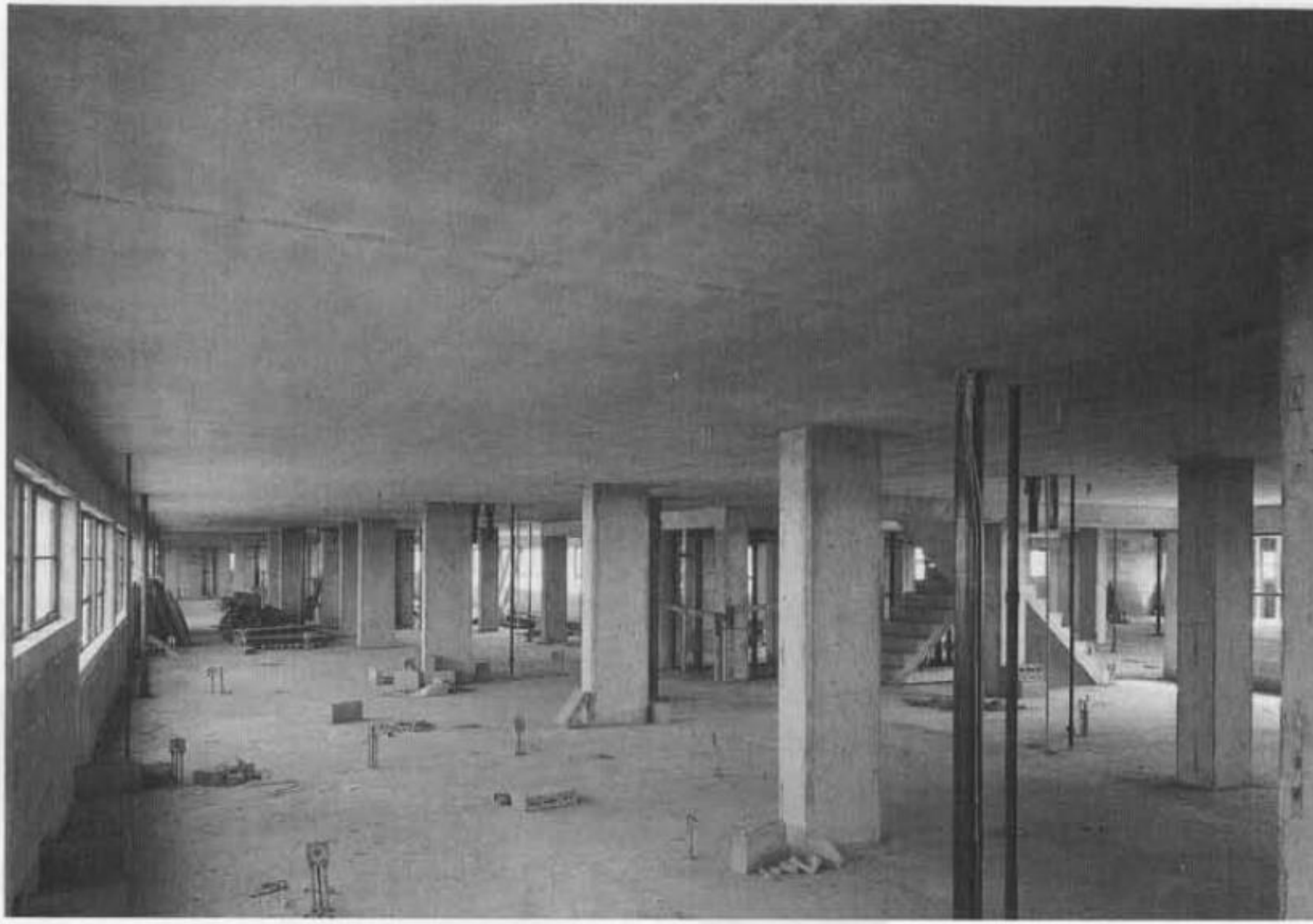

(*d*) Flat plate



(*e*) Flat slab



(*f*) Grid or waffle slab

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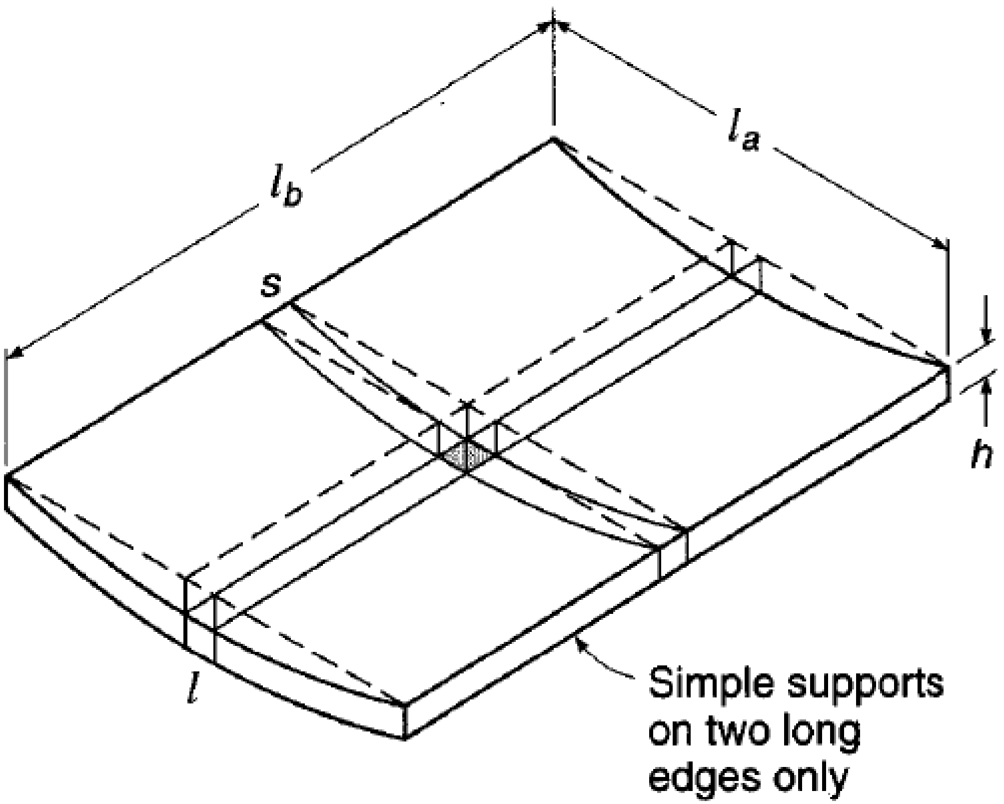
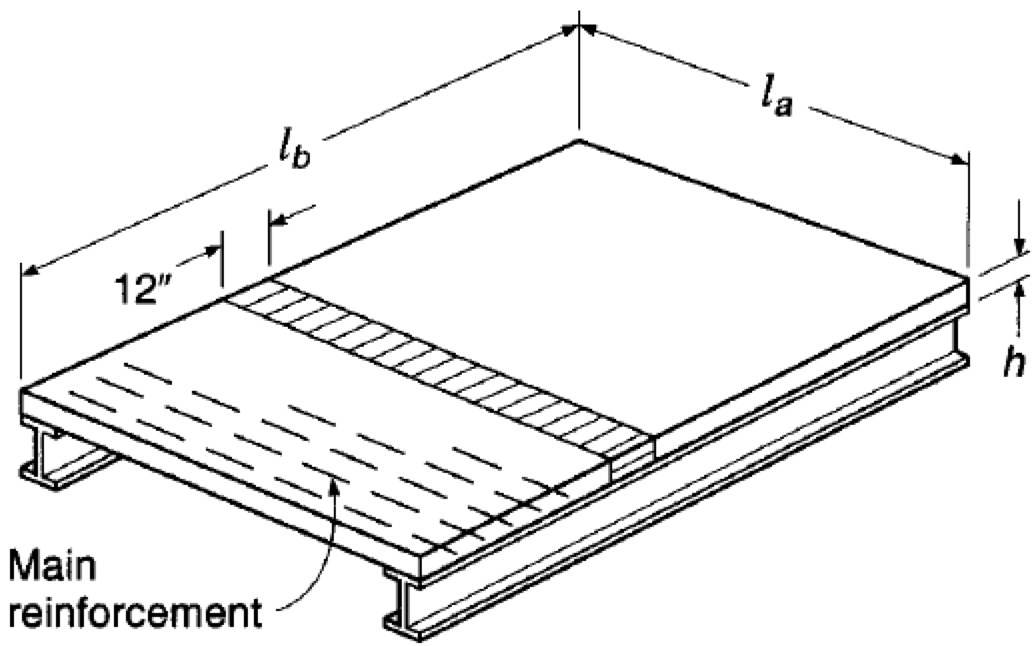
Design of One-way slab





l_b
l_a
12″
h
Main reinforcement

Thickness of one-way slab

TABLE 13.1
Minimum thickness *h* of nonprestressed one-way slabs

Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

ACI Code 9.5.2 specifies the minimum thickness in Table 13.1 for nonprestressed slabs of normalweight concrete (w_c = 145 pcf) using Grade 60 reinforcement, provided that the slab is not supporting or attached to construction that is likely to be damaged by large deflections. Lesser thicknesses may be used if calculation of deflections indicates no adverse effects. For concretes having unit weight w_c in the range from 90 to 115 pcf, the tabulated values should be multiplied by $1.65 - 0.005w_c$, but not less than 1.09. For reinforcement having a yield stress f_y other than 60,000 psi, the tabulated values should be multiplied by $0.4 + f_y/100{,}000$. Slab deflections may be calculated, if required, by the same methods as for beams (see Section 6.7).

Design as a beam

- Same as beam of unit width

One-way slabs are normally designed with tensile reinforcement ratios well below the maximum practical value of $\rho_{0.005}$. Typical reinforcement ratios range from about 0.004 to 0.008. This is partially for reasons of economy, because the saving in steel associated with increasing the effective depth more than compensates for the cost of the additional concrete, and partially because very thin slabs with high reinforcement ratios would be likely to permit large deflections. Thus, flexural design may start with selecting a relatively low reinforcement ratio, say about $0.3\rho_{0.005}$, setting $M_u = \phi M_n$ in Eq. (3.38), and solving for the required effective depth d, given that $b = 12$ in. for the unit strip. Alternatively, Table A.5 or Graph A.1 of Appendix A may be used. Table A.9 is also useful. The required steel area per 12 in. strip $A_s = \rho bd$ is then easily found.

Clear Cover

The total slab thickness h is usually rounded to the next higher $\frac{1}{4}$ in. for slabs up to 6 in. thickness, and to the next higher $\frac{1}{2}$ in. for thicker slabs. Best economy is often achieved when the slab thickness is selected to match nominal lumber dimensions. The concrete protection below the reinforcement should follow the requirements of ACI Code 7.7.1, calling for $\frac{3}{4}$ in. below the bottom of the steel (see Fig. 3.13*b*). In a typical slab, 1 in. below the center of the steel may be assumed. The lateral spacing of the bars, except those used only to control shrinkage and temperature cracks (see Section 13.3), should not exceed 3 times the thickness h or 18 in., whichever is less, according to ACI Code 7.6.5. Generally, bar size should be selected so that the actual spacing is not less than about 1.5 times the slab thickness, to avoid excessive cost for bar fabrication and handling. Also, to reduce cost, straight bars are usually used for slab reinforcement, cut off where permitted as described for beams in Section 5.10.

Temperature and Shrinkage Reinforcement

- Why?

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement should be provided in a structural slab in which the principal reinforcement extends in one direction only. ACI Code 7.12.2 specifies the minimum ratios of reinforcement area to *gross concrete area* (i.e., based on the total depth of the slab) shown in Table 13.2, but in no case may such reinforcing bars be placed farther apart than 5 times the slab thickness or more than 18 in. In no case is the reinforcement ratio to be less than 0.0014.

TABLE 13.2
Minimum ratios of temperature and shrinkage reinforcement in slabs based on gross concrete area

Slabs where Grade 40 or 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars or welded wire fabric (smooth or deformed) is used	0.0018
Slabs where reinforcement with yield strength exceeding 60,000 psi measured at yield strain of 0.35 percent is used	$\frac{0.0018 \times 60{,}000}{f_y}$

EXAMPLE 13.1

One-way slab design. A reinforced concrete slab is built integrally with its supports and consists of two equal spans, each with a clear span of 15 ft. The service live load is 100 psf, and 4000 psi concrete is specified for use with steel with a yield stress equal to 60,000 psi. Design the slab, following the provisions of the ACI Code.

SOLUTION. The thickness of the slab is first estimated, based on the minimum thickness from Table 13.1: $l/28 = 15 \times 12/28 = 6.43$ in. A trial thickness of 6.50 in. will be used, for which the weight is $150 \times 6.50/12 = 81$ psf. The specified live load and computed dead load are multiplied by the ACI load factors:

$$\begin{aligned} \text{Dead load} &= 81 \times 1.2 = 97 \text{ psf} \\ \text{Live load} &= 100 \times 1.6 = \underline{160 \text{ psf}} \\ \text{Total} &= 257 \text{ psf} \end{aligned}$$

For this case, factored moments at critical sections may be found using the ACI moment coefficients (see Table 12.1):

At interior support: $-M = \frac{1}{9} \times 0.257 \times 15^2 = 6.43$ ft-kips

At midspan: $+M = \frac{1}{14} \times 0.257 \times 15^2 = 4.13$ ft-kips

At exterior support: $-M = \frac{1}{24} \times 0.257 \times 15^2 = 2.41$ ft-kips

The maximum practical reinforcement ratio is, according to Eq. (3.30*d*),

$$\rho_{0.005} = (0.85^2)\frac{4}{60}\frac{0.003}{0.003 + 0.005} = 0.0181$$

TABLE 12.1
Moment and shear values using ACI coefficients†

Positive moment	
End spans	
If discontinuous end is unrestrained	$\frac{1}{11} w_u l_n^2$
If discontinuous end is integral with the support	$\frac{1}{14} w_u l_n^2$
Interior spans	$\frac{1}{16} w_u l_n^2$
Negative moment at exterior face of first interior support	
Two spans	$\frac{1}{9} w_u l_n^2$
More than two spans	$\frac{1}{10} w_u l_n^2$
Negative moment at other faces of interior supports	$\frac{1}{11} w_u l_n^2$
Negative moment at face of all supports for (1) slabs with spans not exceeding 10 ft and (2) beams and girders where ratio of sum of column stiffness to beam stiffness exceeds 8 at each end of the span	$\frac{1}{12} w_u l_n^2$
Negative moment at interior faces of exterior supports for members built integrally with their supports	
Where the support is a spandrel beam or girder	$\frac{1}{24} w_u l_n^2$
Where the support is a column	$\frac{1}{16} w_u l_n^2$
Shear in end members at first interior support	$1.15 \frac{w_u l_n}{2}$
Shear at all other supports	$\frac{w_u l_n}{2}$

† w_u = total factored load per unit length of beam or per unit area of slab.
l_n = clear span for positive moment and shear and the average of the two adjacent clear spans for negative moment.

If this value of ρ were actually used, the minimum required effective depth, controlled by negative moment at the interior support, would be found from Eq. (3.38) to be

$$d^2 = \frac{M_u}{\phi \rho f_y b(1 - 0.59\rho f_y/f'_c)}$$

$$= \frac{6.43 \times 12}{0.90 \times 0.0181 \times 60 \times 12[1 - 0.59 \times 0.0181 \times (60/4)]} = 7.83 \text{ in}^2$$

$$d = 2.80 \text{ in.}^\dagger$$

This is less than the effective depth of 6.50 − 1.00 = 5.50 in. resulting from application of Code restrictions, and the latter figure will be adopted. At the interior support, if the stress-block depth a = 1.00 in., the area of steel required per foot of width in the top of the slab is [Eq. (3.37)]

$$A_s = \frac{M_u}{\phi f_y(d - a/2)} = \frac{6.43 \times 12}{0.90 \times 60 \times (5.50 - 1.00/2)} = 0.29 \text{ in}^2$$

Checking the assumed depth a by Eq. (3.32), one gets

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.29 \times 60}{0.85 \times 4 \times 12} = 0.43 \text{ in.}$$

A second trial will be made with a = 0.43 in. Then

$$A_s = \frac{6.43 \times 12}{0.90 \times 60 \times (5.50 - 0.43/2)} = 0.27 \text{ in}^2$$

for which $a = 0.43 \times 0.27/0.29 = 0.40$ in. No further revision is necessary. At other critical-moment sections, it will be satisfactory to use the same lever arm to determine steel areas, and

At midspan: $$A_s = \frac{4.13 \times 12}{0.90 \times 60 \times (5.50 - 0.40/2)} = 0.17 \text{ in}^2$$

At exterior support: $$A_s = \frac{2.41 \times 12}{0.90 \times 60 \times (5.50 - 0.40/2)} = 0.10 \text{ in}^2$$

The minimum reinforcement is that required for control of shrinkage and temperature cracking. This is

$$A_s = 0.0018 \times 12 \times 6.50 = 0.14 \text{ in}^2$$

per 12 in. strip. This requires a small increase in the amount of steel used at the exterior support.

The factored shear force at a distance d from the face of the interior support is

$$V_u = 1.15 \times \frac{257 \times 15}{2} - 257 \times \frac{5.50}{12} = 2100 \text{ lb}$$

By Eq. (4.12b), the nominal shear strength of the concrete slab is

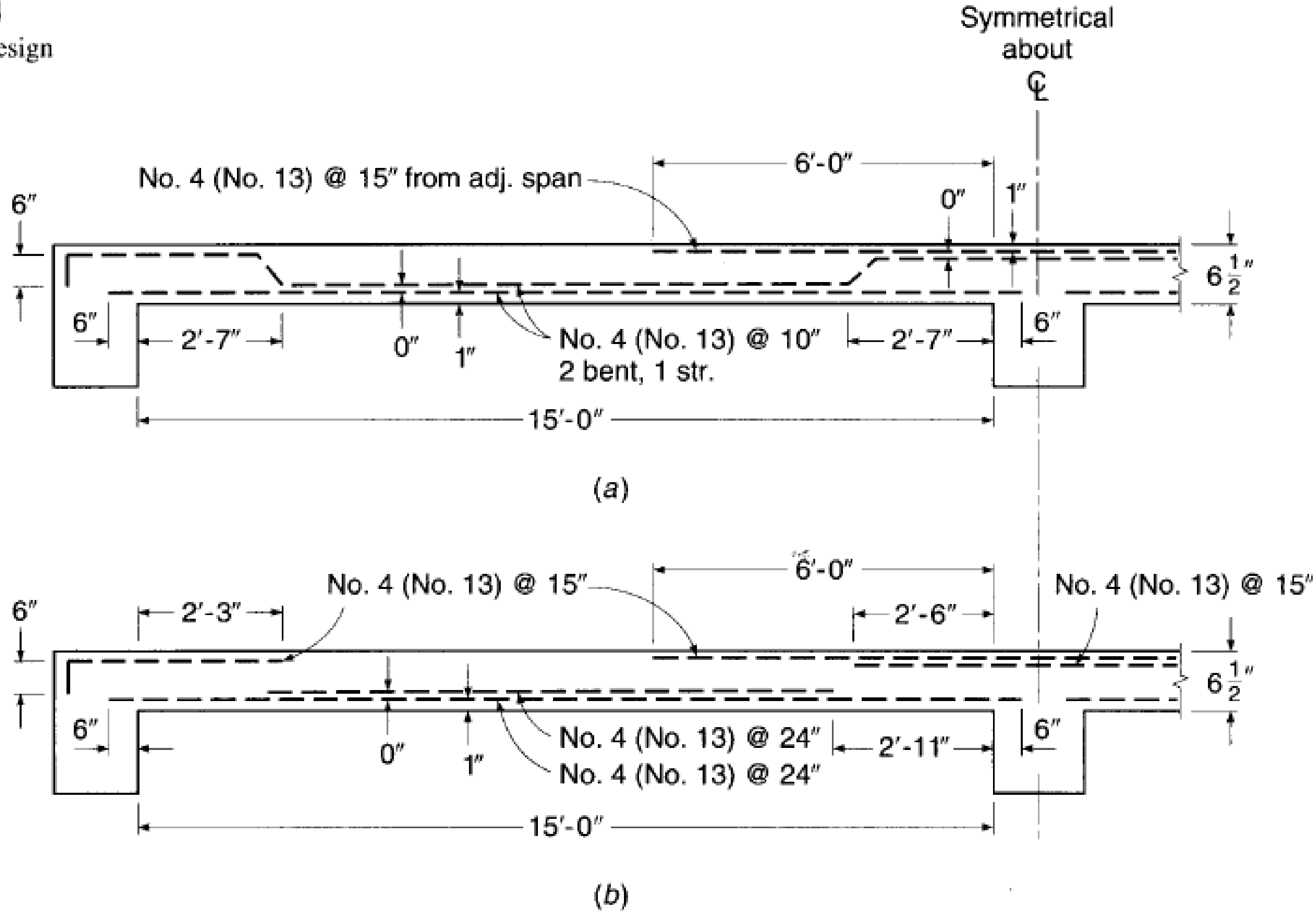
$$V_n = V_c = 2\lambda\sqrt{f_c'}\, bd = 2 \times 1\sqrt{4000} \times 12 \times 5.50 = 8350 \text{ lb}$$

Thus, the design strength of the concrete slab $\phi V_c = 0.75 \times 8350 = 6260$ lb is well above the required strength in shear of $V_u = 2100$.

The required tensile steel areas may be provided in a variety of ways, but whatever the selection, due consideration must be given to the actual placing of the steel during construction. The arrangement should be such that the steel can be placed rapidly with the minimum of labor costs even though some excess steel is necessary to achieve this end.

Two possible arrangements are shown in Fig. 13.4. In Fig. 13.4a, bent bars are used, while in Fig. 13.4b all bars are straight.

FIGURE 13.4
One-way slab design example.



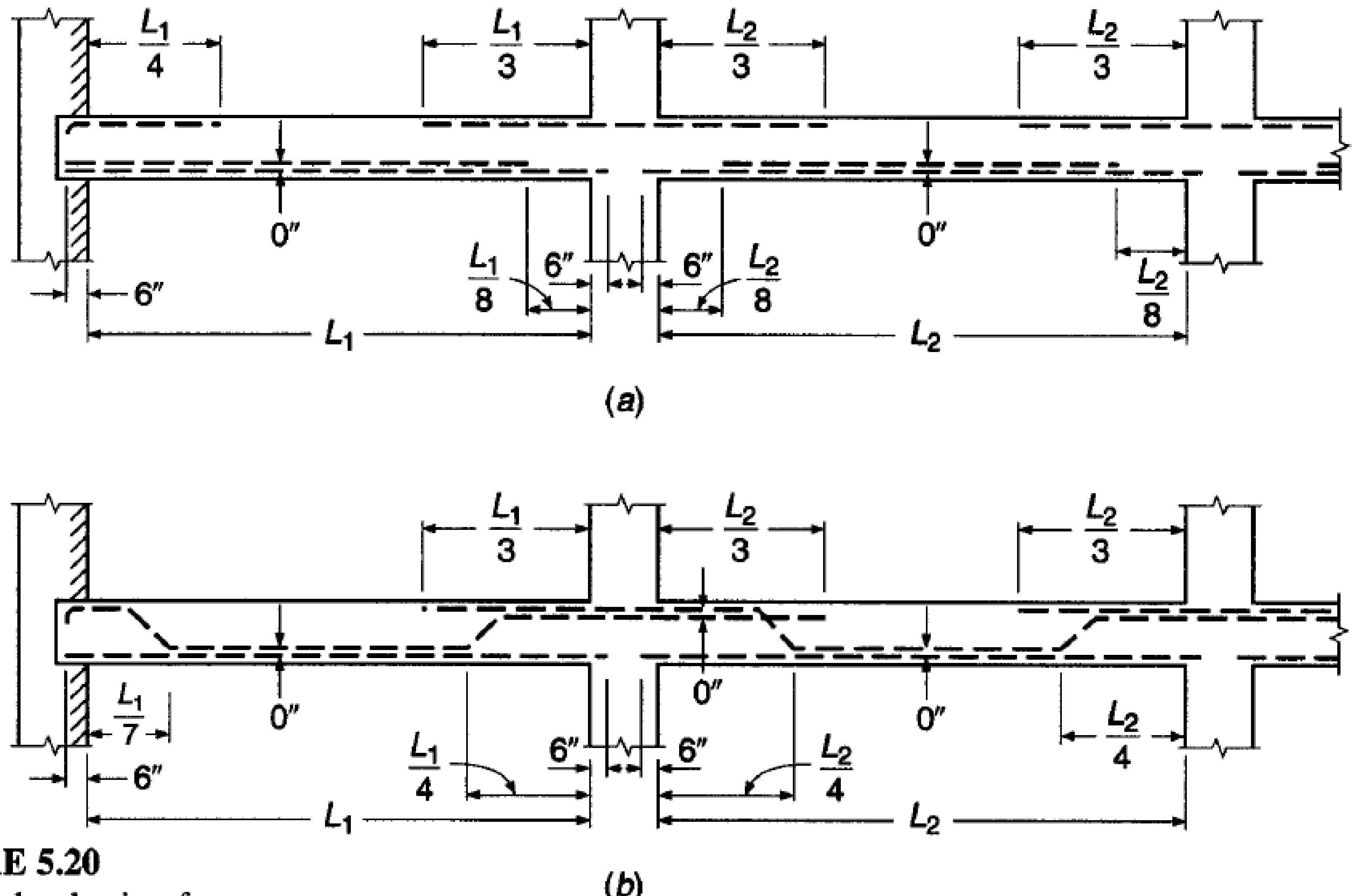


FIGURE 5.20
Cutoff or bend points for bars in approximately equal spans with uniformly distributed loads.

Two-way edge supported slab

Two-way slab

- Load transmits in two direction
- Dished shape
- Types
  - Supported on walls or stiff beams on all sides
  - Flat plate
  - Flat slab
  - Waffle slab

Slab supported on edges

- By wall
- By stiff RC beam
- By steel joist

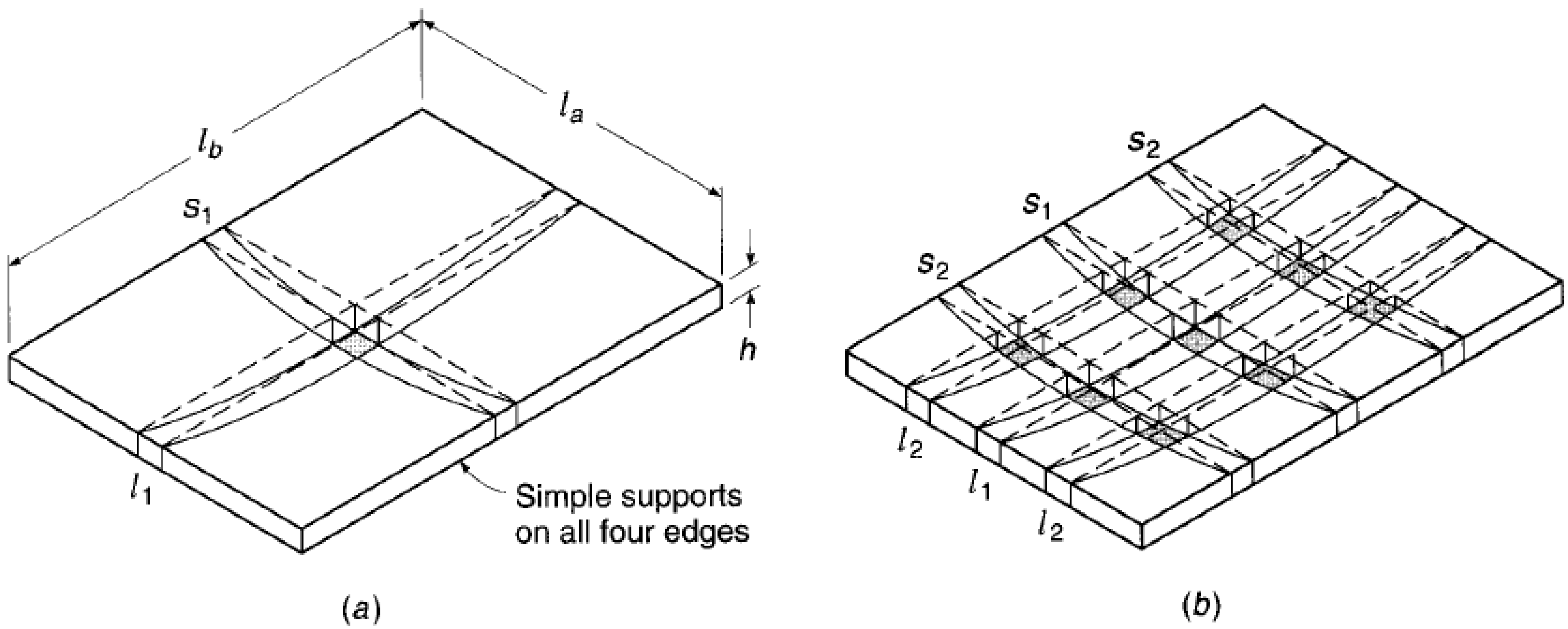


FIGURE 13.5
Two-way slab on simple edge supports: (*a*) bending of center strips of slab; (*b*) grid model of slab.

$$\frac{5q_a l_a^4}{384EI} = \frac{5q_b l_b^4}{384EI} \Rightarrow \frac{q_a}{q_b} = \frac{l_b^4}{l_a^4}$$

$$\frac{(q/2)l^2}{8} = 0.0625ql^2 \Rightarrow 0.048ql^2$$

$$\Downarrow$$

$$0.036ql^2$$

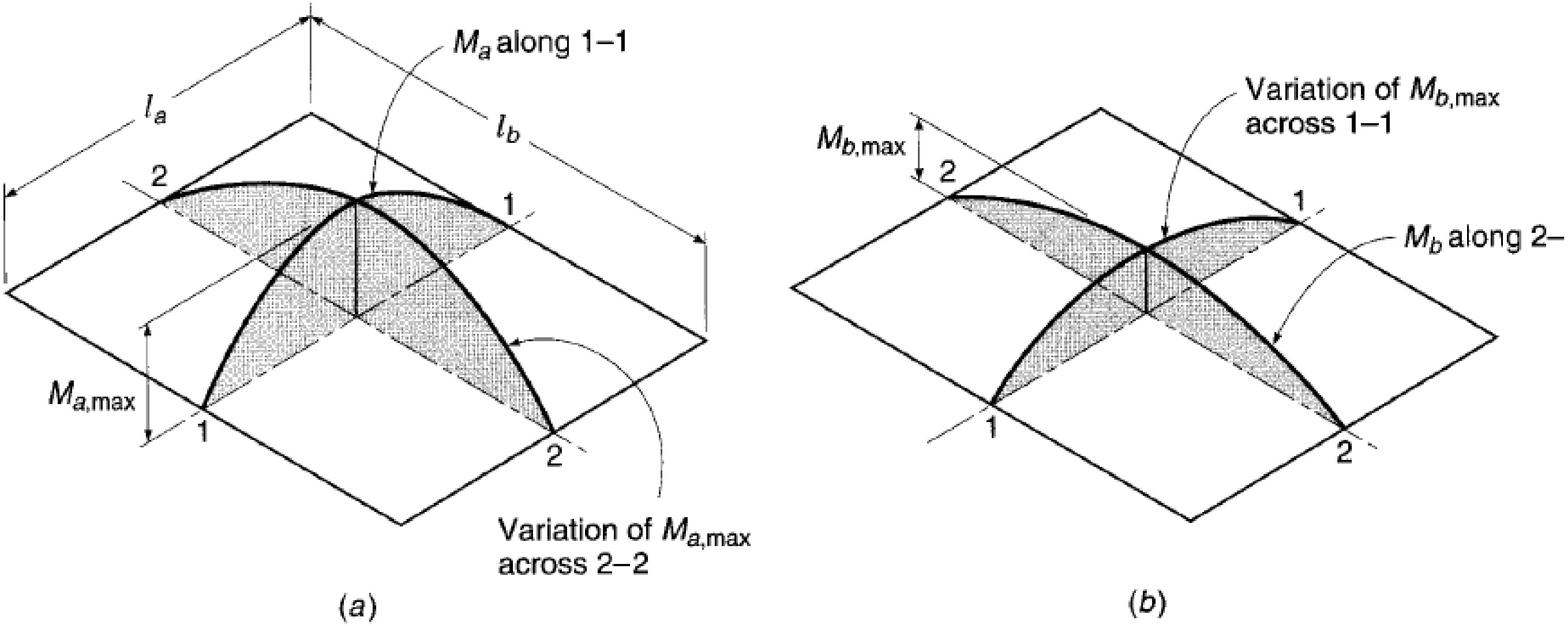


FIGURE 13.6
Moments and moment variations in a uniformly loaded slab with simple supports on four sides.

Moment Coefficient method

- Simplified method of calculating moments and shear
- 1963 ACI Code- later discontinued from 1977
- Still part of BNBC
- For two-way slabs supported on all sides by stiff beams (not less than 3 time slab thickness)

Moment Coefficient method

- Use tables of moment coefficient for various end conditions
- Based on elastic plate analysis and considers stress redistribution
- Still valid

tion. The moments in the middle strips in the two directions are computed from

$$M_a = C_a w l_a^2 \tag{12.1}$$

and

$$M_b = C_b w l_b^2 \tag{12.2}$$

where C_a, C_b = tabulated moment coefficients

w = uniform load, psf

l_a, l_b = length of clear span in short and long directions respectively

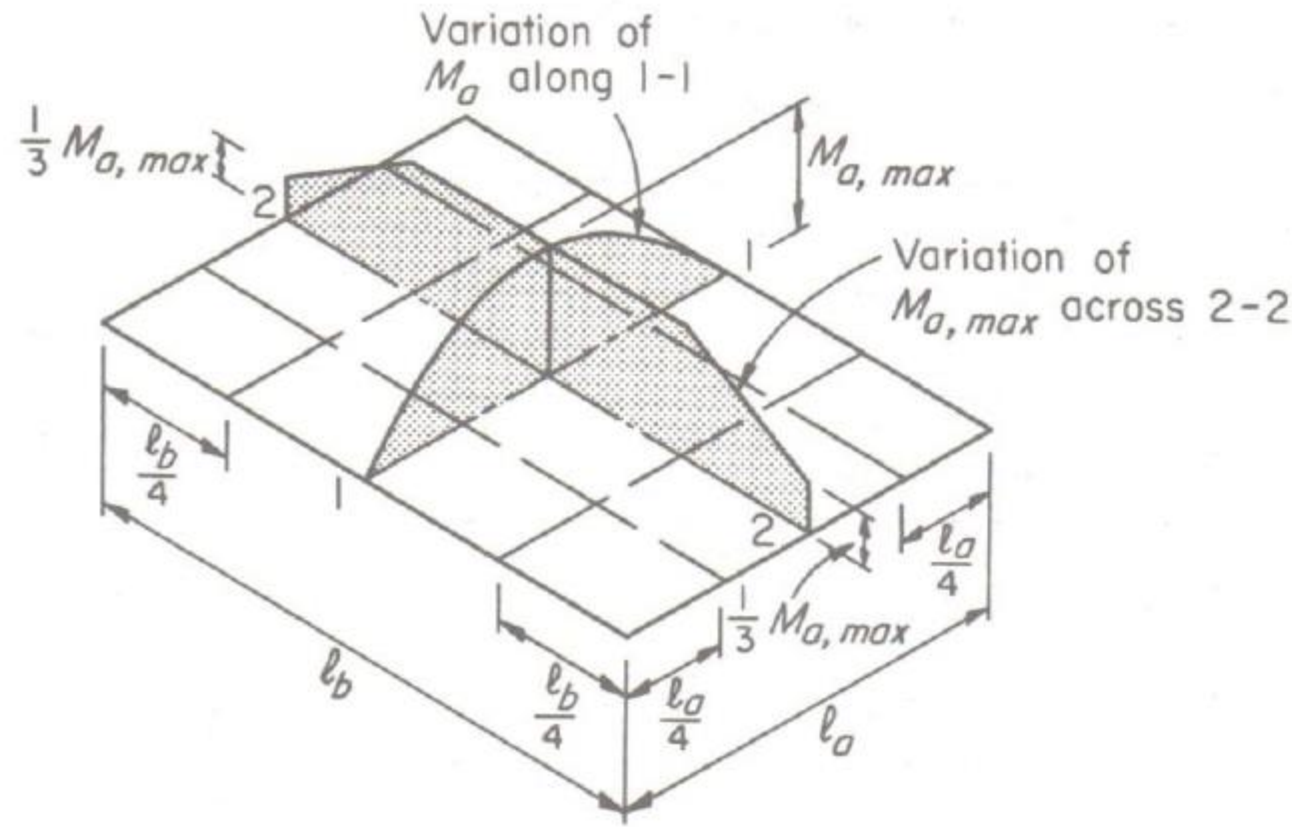


FIGURE 12.7

Variation of moments across the width of critical sections assumed for design.

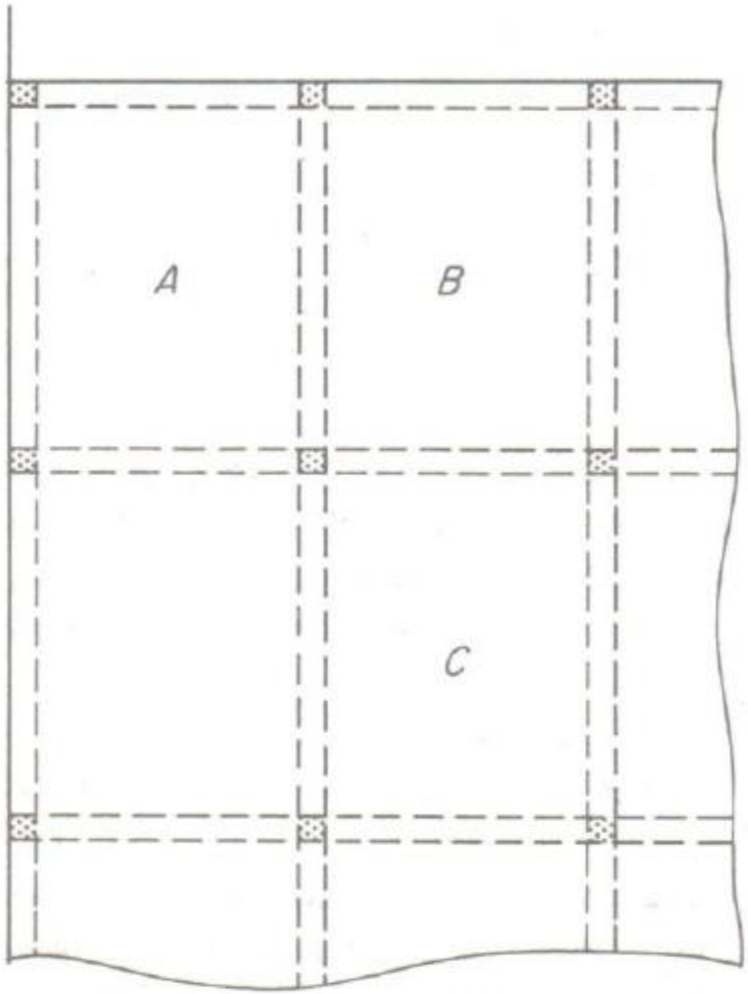


FIGURE 12.8
Plan of a typical two-way slab floor with beams on column lines.

Table 12.3 Coefficients for negative moments in slabs[a]

$M_{a,\text{neg}} = C_{a,\text{neg}} w l_a^2$
$M_{b,\text{neg}} = C_{b,\text{neg}} w l_b^2$ where w = total uniform dead plus live load

1

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,\text{neg}}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{b,\text{neg}}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{a,\text{neg}}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{b,\text{neg}}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{a,\text{neg}}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{b,\text{neg}}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{a,\text{neg}}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{b,\text{neg}}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{a,\text{neg}}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{b,\text{neg}}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{a,\text{neg}}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{b,\text{neg}}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{a,\text{neg}}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{b,\text{neg}}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{a,\text{neg}}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{b,\text{neg}}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{a,\text{neg}}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{b,\text{neg}}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{a,\text{neg}}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{b,\text{neg}}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{a,\text{neg}}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{b,\text{neg}}$		0.006	0.022	0.006			0.014	0.010	0.003

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

2

Table 12.4 Coefficients for dead load positive moments in slabs[a]

$M_{a,pos,dl} = C_{a,dl} w l_a^2$
$M_{b,pos,dl} = C_{b,dl} w l_b^2$ where w = total uniform dead load

Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,dl}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{b,dl}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{a,dl}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{b,dl}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{a,dl}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{b,dl}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{a,dl}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{b,dl}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{a,dl}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{b,dl}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{a,dl}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{b,dl}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{a,dl}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{b,dl}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{a,dl}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{b,dl}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{a,dl}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{b,dl}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{a,dl}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{b,dl}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{a,dl}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{b,dl}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

Table 12.5 Coefficients for live load positive moments in slabs[a]

$M_{a,\text{pos},ll} = C_{a,ll} w l_a^2$
$M_{b,\text{pos},ll} = C_{b,ll} w l_b^2$
where w = total uniform live load

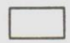
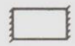
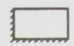
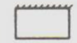
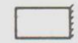
Ratio $m = \frac{l_a}{l_b}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{a,ll}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{b,ll}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{a,ll}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{b,ll}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{a,ll}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{b,ll}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{a,ll}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{b,ll}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{a,ll}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{b,ll}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{a,ll}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{b,ll}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{a,ll}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{b,ll}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{a,ll}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{b,ll}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{a,ll}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{b,ll}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{a,ll}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{b,ll}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{a,ll}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{b,ll}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

3

4

Table 12.6 Ratio of load W in l_a and l_b directions for shear in slab and load on supports[a]

Ratio $m = \frac{l_a}{l_b}$		Case 1 	Case 2 	Case 3 	Case 4 	Case 5 	Case 6 	Case 7 	Case 8 	Case 9 
1.00	W_a	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_b	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_a	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_b	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_a	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_b	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_a	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_b	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_a	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_b	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_a	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_b	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_a	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_b	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_a	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_b	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_a	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_b	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_a	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_b	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_a	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_b	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

[a] A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unmarked edge indicates a support at which torsional resistance is negligible.

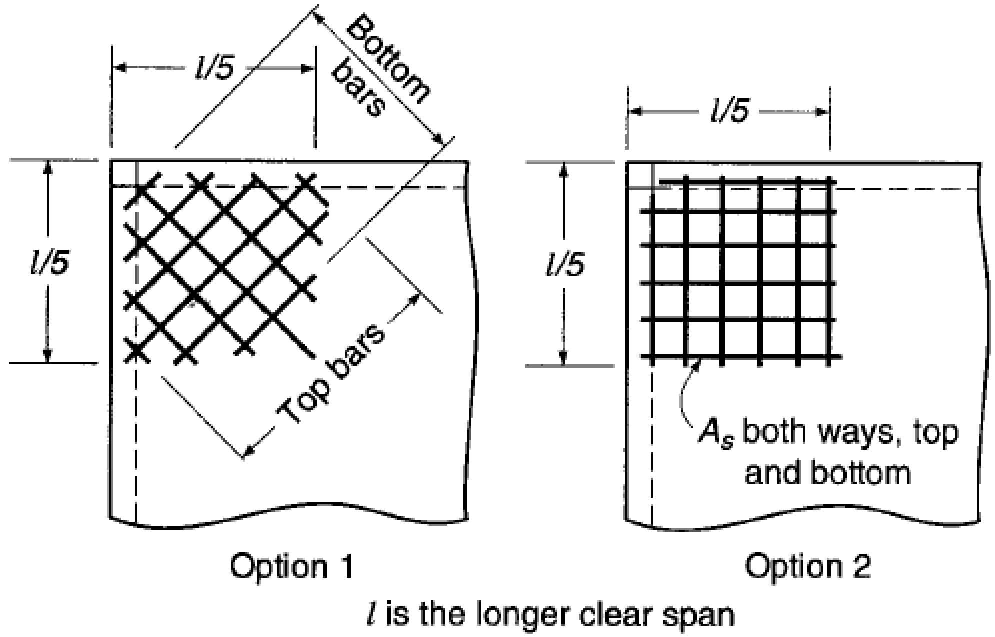
- Negative moment at discontinous edge =1/3 positive moment in the span
- d smaller by 1d in midspan
- Short direction reinf placed below
- Minimum reinforcement
- Spacing at critical section less than 2h

Corner reinforcement

The twisting moments discussed in Sec. 12.4 are usually of consequence only at exterior corners of a two-way slab system, where they tend to crack the slab at the bottom along the panel diagonal, and at the top perpendicular to the panel diagonal. Special reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the longer span of the corner panel, as shown in Fig. 12.9. The reinforcement at the top of the slab should be parallel to the diagonal from the corner, while that at the bottom should be perpendicular to the diagonal. Alternatively, either layer of steel may be placed in two bands parallel to the sides of the slab. The positive and negative reinforcement, in any case, should be of a size and spacing equivalent to that required for the maximum positive moment in the panel, according to ACI Code 13.4.6.

Corner reinforcement

FIGURE 13.7
Special reinforcement at exterior corners of a beam-supported two-way slab.



Example 12.2 Design of two-way edge-supported slab. A monolithic reinforced concrete floor is to be composed of rectangular bays measuring 21 × 26 ft, as shown in Fig. 12.10. Beams of width 12 in. and depth 24 in. are provided on all column lines; thus the clear-span dimensions for the two-way slab panels are 20 × 25 ft. The floor is to be designed to carry a service live load of 137 psf uniformly distributed over its surface, in addition to its own weight, using concrete of strength $f'_c = 3000$ psi and reinforcement having $f_y = 60{,}000$ psi. Find the required slab thickness and reinforcement for the corner panel shown.

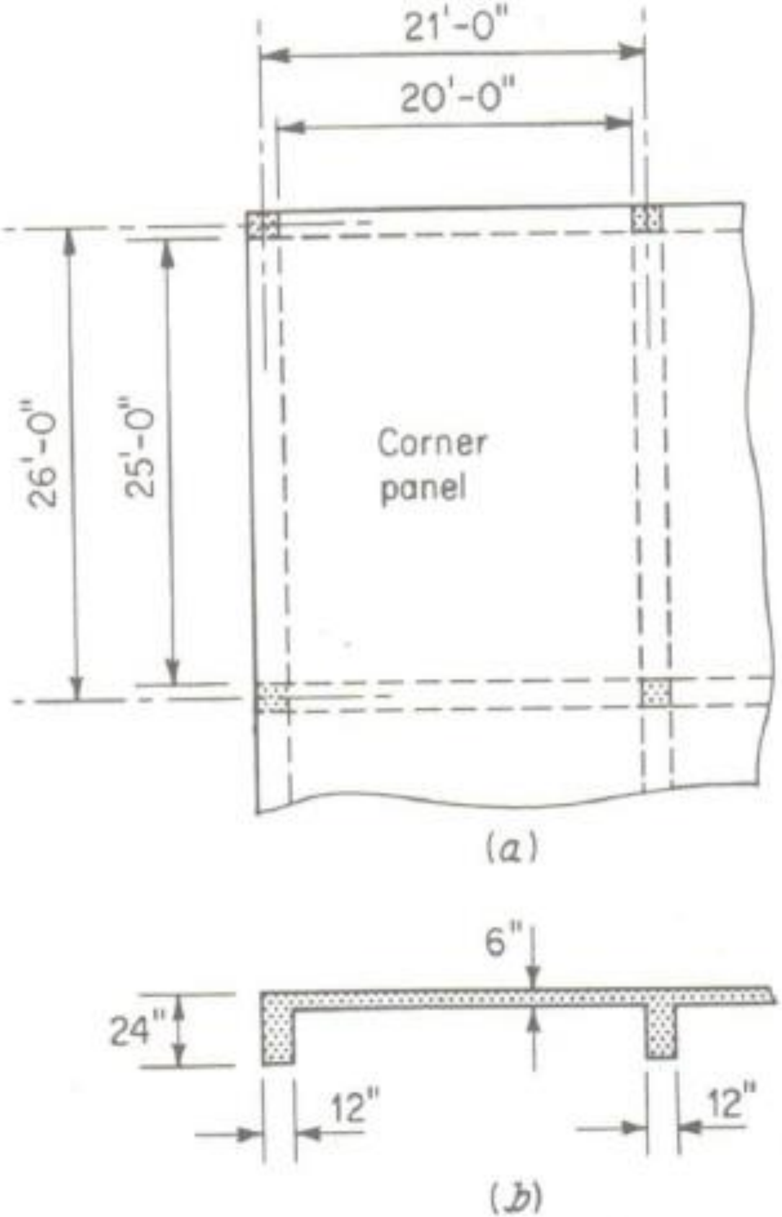


FIGURE 12.10
Two-way edge-supported slab example: (*a*) partial floor plan; (*b*) typical cross section.

This will be selected for a trial depth. The corresponding dead load is $\frac{1}{2} \times 150 = 75$ psf. Thus, the factored loads on which the design is to be based are

$$\begin{aligned} \text{Live load} &= 1.7 \times 137 = 233 \text{ psf} \\ \text{Dead load} &= 1.4 \times \ \ 75 = \underline{105} \text{ psf} \\ \text{Total load} &= 338 \text{ psf} \end{aligned}$$

With the ratio of panel sides $m = l_a/l_b = 20/25 = 0.8$, the moment calculations for the slab *middle strips* are as follows.

Negative moments at continuous edges (Table 12.3)

$$M_{a,\text{neg}} = 0.071 \times 338 \times 20^2 = 9600 \text{ ft-lb} = 115{,}000 \text{ in-lb}$$

$$M_{b,\text{neg}} = 0.029 \times 338 \times 25^2 = 6130 \text{ ft-lb} = 73{,}400 \text{ in-lb}$$

Positive moments (Tables 12.4 and 12.5)

$$\begin{aligned} M_{a,\text{pos},dl} &= 0.039 \times 105 \times 20^2 = 1638 \text{ ft-lb} = 19{,}700 \text{ in-lb} \\ M_{a,\text{pos},ll} &= 0.048 \times 233 \times 20^2 = 4470 \text{ ft-lb} = \underline{53{,}700} \text{ in-lb} \\ M_{a,\text{pos,tot}} & \qquad\qquad\qquad\qquad\qquad\qquad\qquad = 73{,}400 \text{ in-lb} \\ M_{b,\text{pos},dl} &= 0.016 \times 105 \times 25^2 = 1050 \text{ ft-lb} = 12{,}600 \text{ in-lb} \\ M_{b,\text{pos},ll} &= 0.020 \times 233 \times 25^2 = 2910 \text{ ft-lb} = \underline{35{,}000} \text{ in-lb} \\ M_{b,\text{pos,tot}} & \qquad\qquad\qquad\qquad\qquad\qquad\qquad = 47{,}600 \text{ in-lb} \end{aligned}$$

Negative moments at discontinuous edges ($\frac{1}{3}$ × positive moments)

$$M_{a,\text{neg}} = \tfrac{1}{3}(73{,}400) = 24{,}500 \text{ in-lb}$$

$$M_{b,\text{neg}} = \tfrac{1}{3}(47{,}600) = 15{,}900 \text{ in-lb}$$

The required reinforcement in the *middle strips* will be selected with the help of Graph A.1 of App. A.

Short direction

1. Midspan

$$\frac{M_u}{\phi b d^2} = \frac{73{,}400}{0.90 \times 12 \times 5^2} = 272 \qquad \rho = 0.0048$$

$A_s = 0.0048 \times 12 \times 5 = 0.288$ in^2/ft. From Table A.4 of App. A, No. 4 bars at 7 in. spacing are selected, giving $A_s = 0.34$ in^2/ft.

2. Continuous edge

$$\frac{M_u}{\phi b d^2} = \frac{115{,}000}{0.90 \times 12 \times 5^2} = 426 \qquad \rho = 0.0078\dagger$$

$A_s = 0.0078 \times 12 \times 5 = 0.468$ in^2/ft. If two of every three positive bars are bent up, and likewise for the adjacent panel, the negative-moment steel area furnished at the continuous edge will be $\frac{4}{3}$ times the positive-moment steel in the span, or $A_s = \frac{4}{3} \times 0.34 = 0.453$ in^2/ft. It is seen that this is 3 percent less than the required amount of 0.468. On the other hand, the positive-moment steel furnished, 0.34 in^2/ft, represents about 15 percent more than the required amount. As discussed in Sec. 11.9*e*, the ACI Code permits a certain amount of inelastic redistribution, within strictly specified limits. In the case at hand, the negative steel furnished suffices for only 97 percent of the calculated moment, but the positive steel permits about 115 percent of the calculated moment to be resisted. This more than satisfies the conditions for inelastic moment redistribution set by the ACI Code. This situation illustrates how such moment redistribution can be utilized to obtain a simpler and more economical distribution of steel.

3. Discontinuous edge. The negative moment at the discontinuous edge is one-third the positive moment in the span; it would be adequate to bend up every third bar from the bottom to provide negative-moment steel at the discontinuous edge. However, this would result in a 21 in. spacing, which is larger than the maximum spacing of $2h = 12$ in. permitted by the ACI Code. Hence, for the discontinuous edge, two of every three bars will be bent up from the bottom steel.

Long direction

1. Midspan

$$\frac{M_u}{\phi b d^2} = \frac{47{,}600}{0.90 \times 12 \times 4.5^2} = 218 \qquad \rho = 0.0038$$

---

†Note that this value of ρ, which is the maximum required anywhere in the slab, is about half the permitted maximum value of $0.75\rho_b = 0.0160$, indicating that a thinner slab might be used. However, use of the minimum possible thickness would require an increase in the tensile steel area and would be less economical for this reason. In addition, a thinner slab may produce undesirably large deflections. The trial depth of 6 in. will be retained for the final design.

(The positive-moment steel in the long direction is placed on top of that for the short direction. This is the reason for using $d = 4.5$ in. for the positive-moment steel in the long direction and $d = 5$ in. in all other locations.) $A_s = 0.0038 \times 12 \times 4.5 = 0.205$ in^2/ft. From Table A.4 of App. A, No. 3 bars at 6 in. spacing are selected, giving $A_s = 0.22$ in^2/ft.

2. Continuous edge

$$\frac{M_u}{\phi b d^2} = \frac{73,400}{0.90 \times 12 \times 5^2} = 272 \qquad \rho = 0.0048$$

$A_s = 0.0048 \times 12 \times 5 = 0.288$ in^2/ft. Again bending up two of every three bottom bars from both panels adjacent to the continuous edge, one has, at that edge, $A_s = \frac{4}{3} \times 0.22 = 0.29$ in^2/ft.

3. Discontinuous edge. For the reasons discussed in connection with the short direction, two out of every three bottom bars will, likewise, be bent up at this edge.

The preceding steel selections refer to the *middle strips* in both directions. For the *column strips*, the moments are assumed to decrease linearly from the full calculated value at the inner edge of the column strip to one-third of this value at the edge of the supporting beam. To simplify steel placement, a uniform spacing will be used in the column strips. The average moments in the column strips being two-thirds of the corresponding moments in the middle strips, adequate column steel will be furnished if the spacing of this steel is $\frac{3}{2}$ times that in the middle strip. Maximum spacing limitations should be checked.

Bend points for rebars will be located as suggested in Fig. 5.15, i.e., $l/4$ from should be checked.

Bend points for rebars will be located as suggested in Fig. 5.15, i.e., $l/4$ from the face of the supporting beam at the continuous ends and $l/7$ from the beam face at the discontinuous ends. The corresponding distances from the beam face to bend point are 5 ft 0 in. and 2 ft 10 in. for the short-direction positive bars, and 6 ft 3 in. and 3 ft 7 in. for the long-direction positive bars, at the continuous end and discontinuous end, respectively. Negative bars carried over from the adjacent panels will be cut off at $l/3$ from the support face, at 6 ft 8 in. for the short-direction negative bars and 8 ft 4 in. for the long-direction negative bars. At the exterior edges, negative bars will be extended as far as possible into the supporting beams, then bent downward in a 90° hook to provide anchorage.

At the exterior corner of the panel, No. 4 bars at 8 in. spacing will be used, parallel to the slab diagonal at the top, and perpendicular to the diagonal at the bottom, according to Fig. 12.9. This will provide an area of 0.29 in^2/ft each way, equal to that required for the maximum positive bending moment in the panel. This reinforcement will be carried to a point $25/5 = 5$ ft from the corner, with lengths varying as indicated in Fig. 12.9.

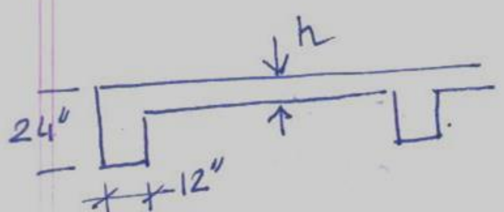
The reactions of the slab are calculated from Table 12.6, which indicates that 71 percent of the load is transmitted in the short direction and 29 percent in the long direction. The total load on the panel being $20 \times 25 \times 338 = 169{,}000$ lb, the load per foot on the long beam is $(0.71 \times 169{,}000)/(2 \times 25) = 2400$ lb/ft and on the short beam is $(0.29 \times 169{,}000)/(2 \times 20) = 1220$ lb/ft. The shear to be transmitted by the slab to these beams is numerically equal to these beam loads, reduced to a critical section a distance d from the beam face. The shear strength of the slab is

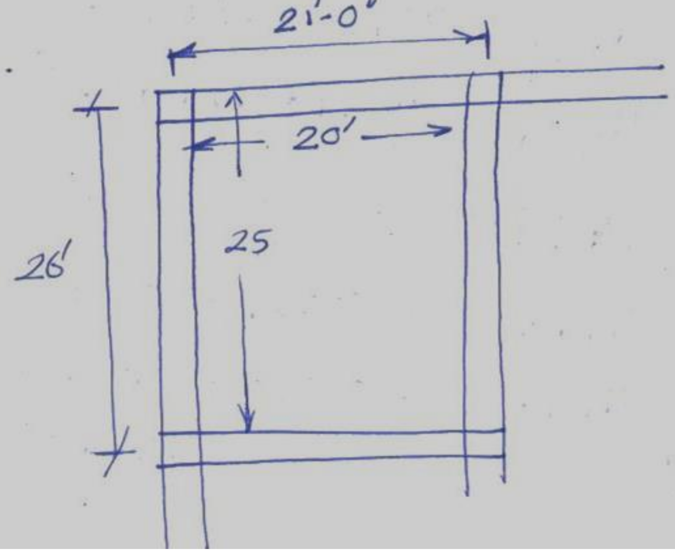
$$\phi V_c = 0.85 \times 2\sqrt{3000} \times 12 \times 5 = 5590 \text{ lb}$$

well above the required shear strength at factored loads.

Edge supported slab · Problem

A monolithic RC floor is to be composed of rectangular bays measuring 21 × 26 ft, as shown in fig. Beams of width 12 in. and depth 24 in. are provided on all column lines; thus the clear span dimensions for the two-way slab panels are 20×25 ft. The floor is to be designed to carry LL = 60 psf, FF = 25 psf in addition to radom PW = 40 psf and its own weight, using $f_c' = 3$ ksi and $f_y = 60$ ksi. Find the reqd slab thickness and reinforcement for the slab system shown.





Old method

$$h = \frac{\text{perimeter}}{180}$$

$$= \frac{2(20+25)\times 12}{180} \doteq 6 \text{ in.}$$

New formula $\alpha_m > 2.0$

$$h = \frac{l_n(0.8 + f_y/200{,}000)}{36+9\beta}$$

$$= \frac{25\times 12\left(0.8 + \frac{60{,}000}{200{,}000}\right)}{36 + 9\frac{25}{20}} = 6.98 \text{ in} \simeq 7 \text{ in.}$$

$$\text{self wt} = \frac{7}{12} \times 150 = 87.5 \text{ psf.}$$

$$DL = 1.2\,(87.5 + 40 + 25) = 1.2 \times 152.5 = 183 \text{ psf}$$

$$LL = 1.6 * 60 \qquad\qquad = 96$$

$$w_u = 279 \text{ psf.}$$

$$m = \frac{l_a}{l_b} = \frac{20}{25} = 0.8 \qquad \boxed{\text{Case 4}}$$

Moments

Negative moments at cont edge

$$M_{a,neg} = 0.071 * 279 * 20^2 = 7923.6 \text{ lb-ft.}$$

$$M_{b,neg} = 0.029 * 279 * 25^2 = 5057 \text{ lb-ft.}$$

Positive moments

Positive moments

$M_{a,pos,dL} = 0.039 * 183 * 20^2 = 2854.8$

$M_{a,pos,LL} = 0.048 * 96 * 20^2 = 1843.2$

$M_{a,pos,tot} = 4698$ lb-ft.

$M_{b,pos,dL} = 0.016 * 183 * 25^2 = 1830$ lb-ft

$M_{b,pos,LL} = 0.020 * 96 * 25^2 = 1200$

$M_{b,pos,tot} = 3030$ lb-ft

Negative moment at discontinuous end

$= \frac{1}{3}$ positive mom

$M_{a,neg} = \frac{1}{3} \times 4698 = 1566$ lb-ft.

$M_{b,neg} = \frac{1}{3} \times 3030 = 1010$ "

d check $\varepsilon_t = 0.005$

$$\rho = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000}{1000} \qquad 0.65 \leq \beta_1 \leq 0.85$$

$$\Rightarrow \rho = 0.85 * 0.85 * \frac{3}{60} \frac{.003}{.003 + .005}$$

$$= 0.0135$$

$$M_u = \phi M_n = \phi \times \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$\Rightarrow 7923.6 * 12 = 0.9 * 0.0135 * 60{,}000 * 12 * d^2 \left(1 - 0.59 * .0135 * \frac{60}{3}\right)$$

$d = 3.59$ in $\qquad d_{prov} = 7 - 1 = 6$ in.

A_s

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

$$\Rightarrow \quad 79236 * 12 = 0.9 * A_s * 60{,}000\left(6 - \frac{1}{2}\right)$$

$$A_s = 0.32 \text{ in}^2/\text{ft}.$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = 0.628$$

$$A_s = 0.31$$

$$a = 0.61$$

$$A_s = \underline{0.31 \text{ in}^2/\text{ft}}$$

$$a = 0.61$$

$$c = \frac{0.61}{0.85} = 0.7176$$

$$\varepsilon_t = \varepsilon_u \frac{d-c}{c}$$

$$= 0.02.$$

7924 #4 $\Rightarrow$ 0.31 in^2

4698 lb-ft/ft

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

$$\Rightarrow 4698 * 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right)$$

$A_s = 0.1898$

$a = \frac{A_s f_y}{0.85 f'_c b} = 0.372$

$A_s = 0.1796$

$a = 0.352$

$A_s = \underline{0.179}$ in^2/ft.

$60{,}000\left(6 - \frac{1}{2}\right)$

$A_s = \underline{\underline{0.1??}}$

<u>1566</u>

$1566 * 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right.$

$A_s = 0.063$ ✓

$a = 0.124$ ✓

$A_s =$ ~~[illegible]~~ 0.059

$a =$ ~~[illegible]~~ 0.1149

$A_s = \underline{\underline{0.059}}$

Long span

5057 lb-ft/ft $\qquad d = 6$ in.

$$5057 \times 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right)$$

$A_s = 0.204$

$a = 0.4$

$A_s = 0.194$

$a = 0.38$

$A_s = \underline{0.194}$

$d = 6 - \frac{1}{2} = 5.5$ in. ⟸

3030

$$3030 \times 12 = 0.9 * A_s * 60{,}000 \left(5.5 - \frac{1}{2}\right)$$

$A_s = 0.135$

$a = 0.264$

$A_s = 0.125$

$a = 0.246$

$A_s = \underline{0.125}$ in^2/ft.

$(6 - \frac{1}{2})$

1010

$$1010 \times 12 = 0.9 * A_s * 60{,}000 \left(6 - \frac{1}{2}\right)$$

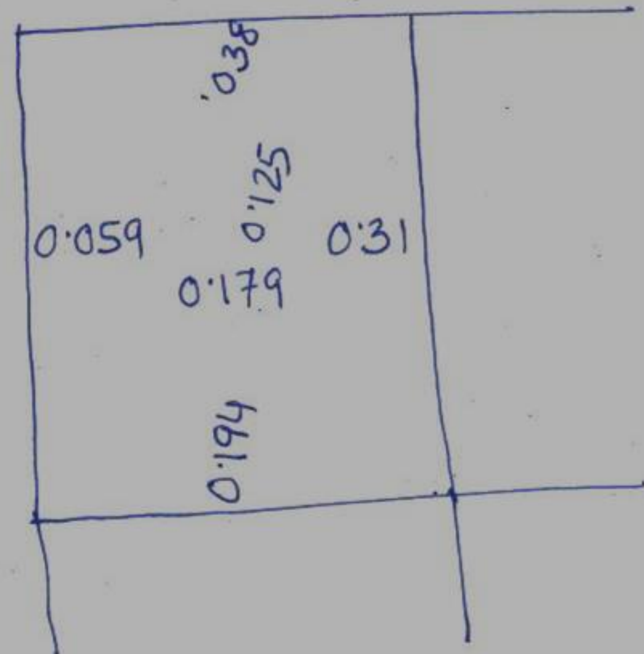
$$A_s = 0.0408$$

$$a = 0.080$$

$$A_s = 0.038$$

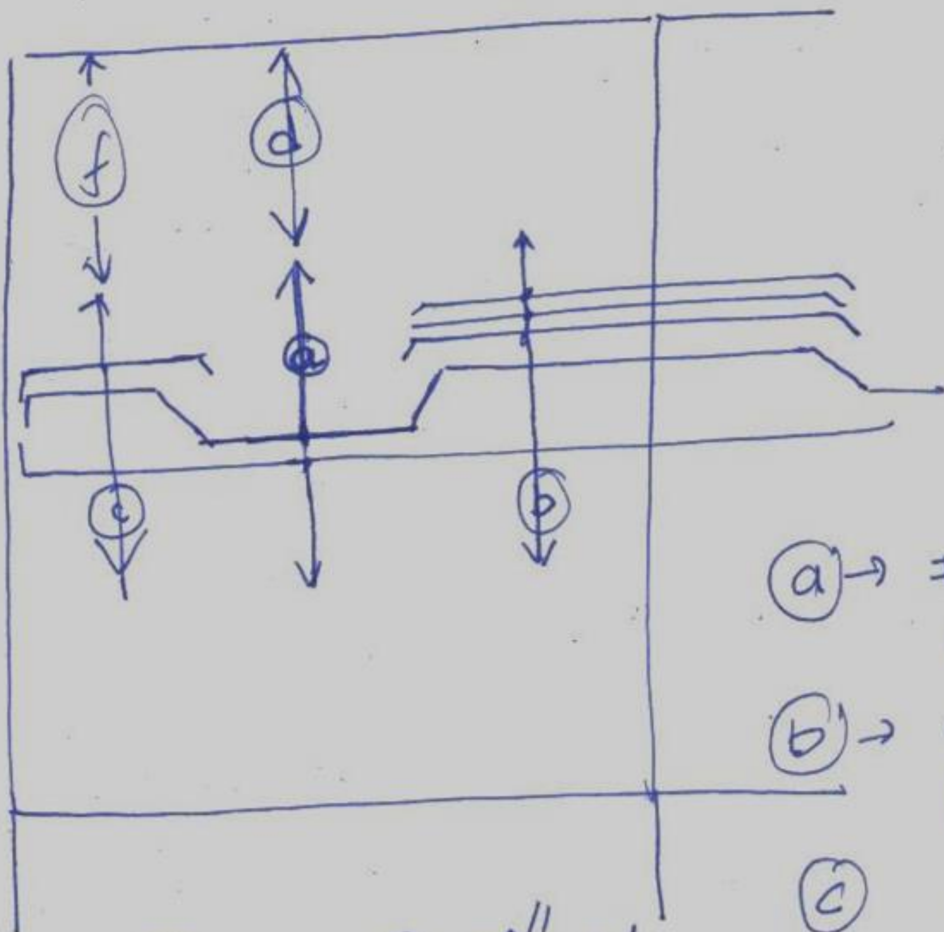
$$a = 0.074$$

$$A_s = \underline{0.0376} \text{ in}^2/\text{ft.}$$



Min $A_s = 0.0018\,bh = 0.0018 \times 12 \times 7 = 0.1512$ in²/ft.

spacing, $s = \frac{A_{bar}}{A_{req}} \times 12$



(a) → #3 @ 7" c/c alt ckd.

(b) → 3 #3 Extra betn alt ckd.

(c) 1 # 3 Extra bet alt. ckd.

0.179 ⇒ #3 @ 7" c/c

$\underline{\underline{0.31}}$ 12 in reqd ⇒ 0.31

⇒ 14 in reqd ⇒ $\frac{0.31}{12} \times 14 = 0.362 \, in^2/14 in$

Extra reqd = $0.362 - 0.11 = 0.251$

#3 reqd = $\frac{0.251}{0.11} = 3$.

$\underline{\underline{0.059}}$ Alt ckd ⇒ #3 @ 14" c/c (2h is ok)

temp & shrinkage = 0.1512

$0.1512 \times \frac{14}{12} - 0.11 = 0.0664$

1 #3 Extra.

Col strip

temp

(a) ⇒ (d) 7" × 1.5 = 10.5" in c/c. × 8.5" c/c.

(2)

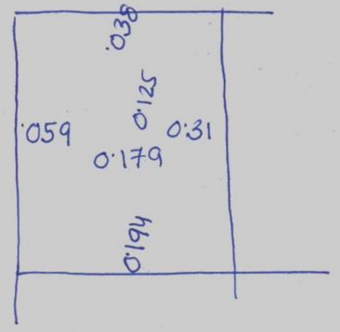
(b) ⇒ (e) ⇒ 3 # 3 Extra top. (e) 2 Extra top

(c) ⇒ (f) = alt ckd. ⇒ 21"

temp & shr = $0.1512 + \frac{21}{12} - 0.11 = 0.1546$

$\div 11$

$= 2 \# 3.$



(b) ⇒ $A = \frac{A_{bar}}{s} \times 12 = 0.18857$

$0.31 - .18857/2 = .215$

$s = \frac{.11}{.215} \times 12 = 6.1$

≃ 6" c/c

(c) ~~[struck out]~~

$A_{s\,reqd} = 0.1512 - \frac{.18857}{2} = .0561$

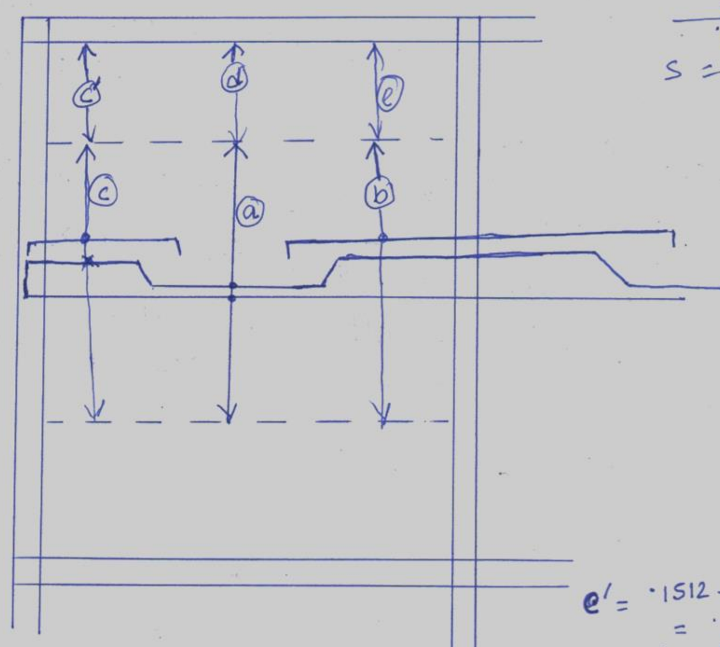
s = 23" c/c

(e) ⇒ $0.31 + \frac{2}{3}$ ~~[struck out]~~ $= 0.207$

$- \frac{.11}{21} \times 12$

$.1438$

s = 9"



$e' = .1512 - \frac{.11}{17} \times 12$

$= .088$

s = 17" c/c.

8.4

(4) a. #3 @ 7" c/c alt ckd

b. #3 @ 6" c/c extra top.

c. #3 @ 23" c/c extra top.

d. #3 @ 10.5" c/c alt ckd. ← #3 @ 8.5" c/c

e. #3 @ 9" c/c extra top ← #3 @ 10" c/c

(b) ⇒ $A = \frac{A_{bar}}{s} \times 12 = 0.18857$

$0.31 - .18857/2 = .215$

$s = \frac{.11}{.2157} \times 12 = 6.1$

$\simeq$ 6" c/c

(c) $A_{s\,reqd} = 0.1512 - \frac{.18857}{2} = .0561$

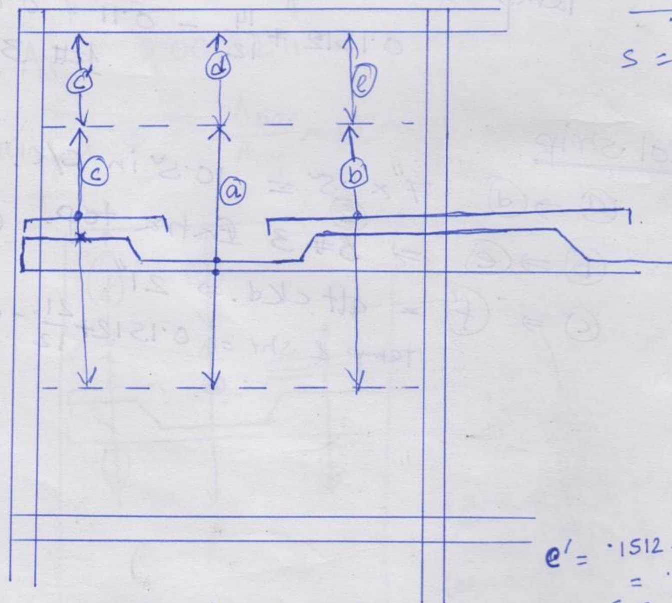
s = 23" c/c

(e) ⇒ $0.31 \times \frac{2}{3} = 0.207$

$- \frac{.11}{21} \times 12$

$.1438$

s = 9"



$e' = .1512 - \frac{.11}{17} \times 12$

$= .088$

s = 17" c/c.

8.4

(4) a. # 3 @ 7" c/c alt ckd

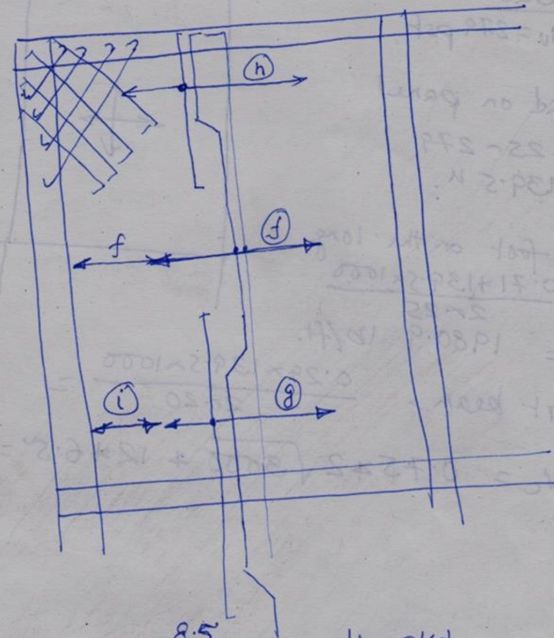
b. # 3 @ 6" c/c extra top.

c. # 3 @ 23" c/c extra top.

d. # 3 @ 10.5" c/c alt ckd. ← # 3 @ 8.5" c/c

e. # 3 @ 9" c/c extra top ← # 3 @ 10" c/c

Corner Reinf. #3@7" c/c both top & bottom



$\cdot 194 - \frac{\cdot 11}{17} \times 12 = 0\cdot 116$

$s = 11''$

f. # 3 @ ~~8~~ 8·5" c/c alt. ckd.

s = 6·6

g. # 3 @ 11" c/c ~~alt.~~ ~~ckd~~ extra top

$0\cdot 1512 - \frac{\cdot 11}{17} \times 12 = \cdot 07$

$s = 17''$ c/c

h. # 3 @ 17" c/c extra top

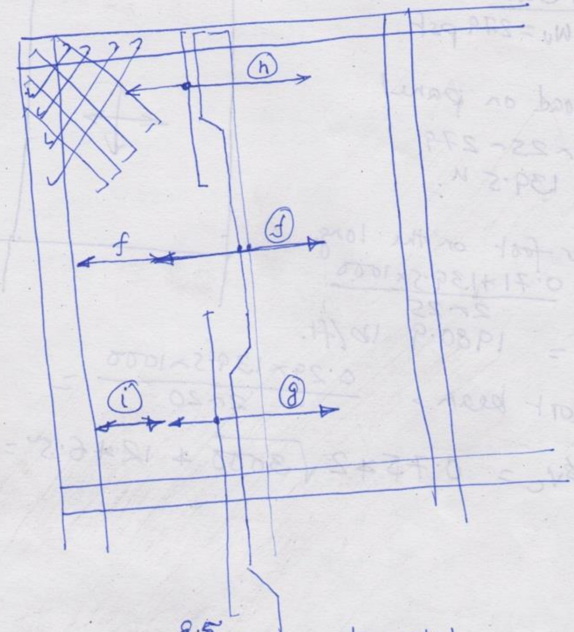
i. # 3 @ 17" c/c " "

g. আয়তন ⇒ 0·194 in²/ft ⇒ $17'' = \frac{\cdot 194}{12} \times 17 = 0\cdot 275$. in²/17"

Provided :, 0·11 Reqr = 0·1648

2 - #3 Extra betn alt. ckd.

Corner Reinf. #3@7" c/c both top & bottom



$\cdot 194 - \frac{\cdot 11}{17} \times 12 = 0.116$
s = 11"

f. #3 @ ~~9~~ 8.5" c/c alt. ckd.
#3 @ 11" c/c ~~alt.~~ ~~ckd~~ extra top

s = 6.6

$0.1512 - \frac{\cdot 11}{17} \times 12 = \cdot 07$
s = 17" c/c

g. #3 @ 17" c/c extra top
h. #3 @ 17" c/c " "
i. #3 @ 17" c/c

g. আবশ্যক ⇒ 0.194 in²/ft ⇒ $17" = \frac{\cdot 194}{12} \times 17 = 0.275$ in²/17"
Provided = 0.11 Reqr = 0.1648
2- #3 Extra bet^n alt. ckd.

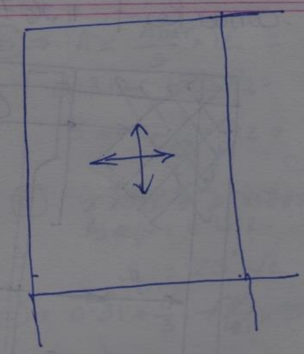
Shear Check

$w_u = 279$ psf.

Total load on panel

$= 20 \times 25 \times 279$

$= 139.5$ k.



Load per foot on the long beam $= \dfrac{0.71 \times 139.5 \times 1000}{2 \times 25}$

$= 1980.9$ lb/ft.

On short beam $= \dfrac{0.29 \times 139.5 \times 1000}{2 \times 20} = 1011.3$ lb/ft.

$\phi V_c = 0.75 \times 2\sqrt{3000} \times 12 \times 6.5 = 6408$ lb/ft.