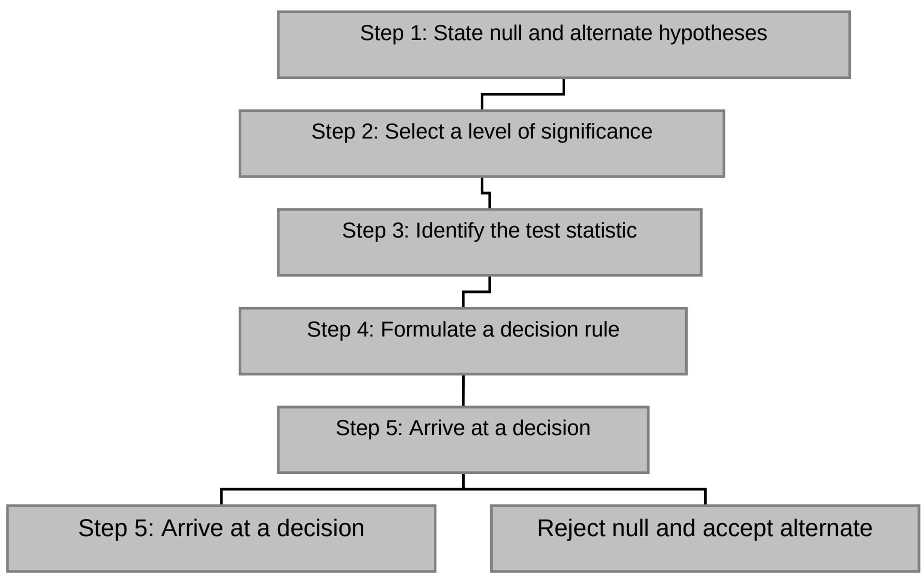
Hypothesis Testing

Hypothesis: A hypothesis is some statement about a population which we want to verify on the basis of information available from a sample.
Example: The mean monthly income for systems analysts is $6,325.

Hypothesis testing: A procedure based on sample evidence and probability theory to determine the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected.

Steps in solving testing of hypothesis: The major steps involved in a hypothesis testing are outlined as follows:



Example: Let us suppose that the bulbs manufactured under some standard manufacturing process have an average life of μ hours and it is proposed to test a new procedure for manufacturing light bulbs. If μ_0 is the average life of the bulbs manufactured by the new process. Thus we have two populations of bulbs, those manufactured by standard process and those manufactured by the new process. In this problem the following three hypothesis may be set up:

1. New process is better than standard process or New process is inferior to standard process
2. There is difference between two process

3. There is no difference between the process

Types of Hypotheses:

There are two types of hypothesis.

1. Null Hypothesis (H_0)
2. Alternative Hypothesis (H_1)

Null Hypothesis (H_0): A statement about the value of a population parameter. That is a null hypothesis is that statement which we want to reject. Null hypothesis denoted by H_0
For the above example the number 3 hypothesis is called hull hypothesis. That is there is no difference between the two processes. Null hypothesis can be written as follows

$$H_0 : \mu = \mu_0$$

Alternative Hypothesis (H_1): A statement that is accepted if the sample data provide evidence that the null hypothesis is false. Alternative hypothesis is usually denoted by H_1 **.**

For the above example the 1 or 2 number hypothesis could be alternative hypothesis. So alternative hypothesis can be written as

$$H_1 : \mu > \mu_0$$

or

$$H_1 : \mu < \mu_0$$

or $H_1 : \mu \neq \mu_0$

One tailed test: The alternate hypothesis H_1, states a direction.

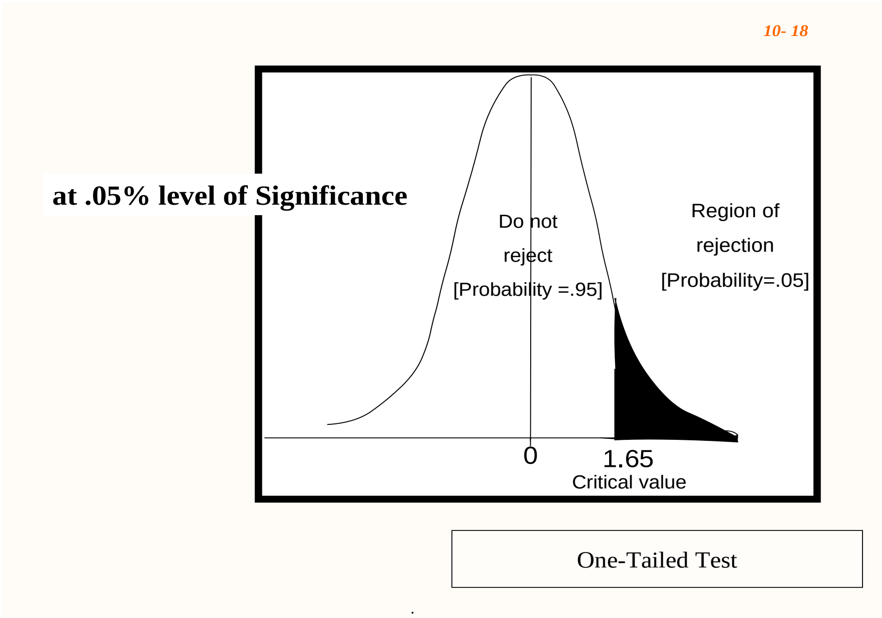
In the above example if the alternative hypothesis is like

$$H_1 : \mu > \mu_0$$

or

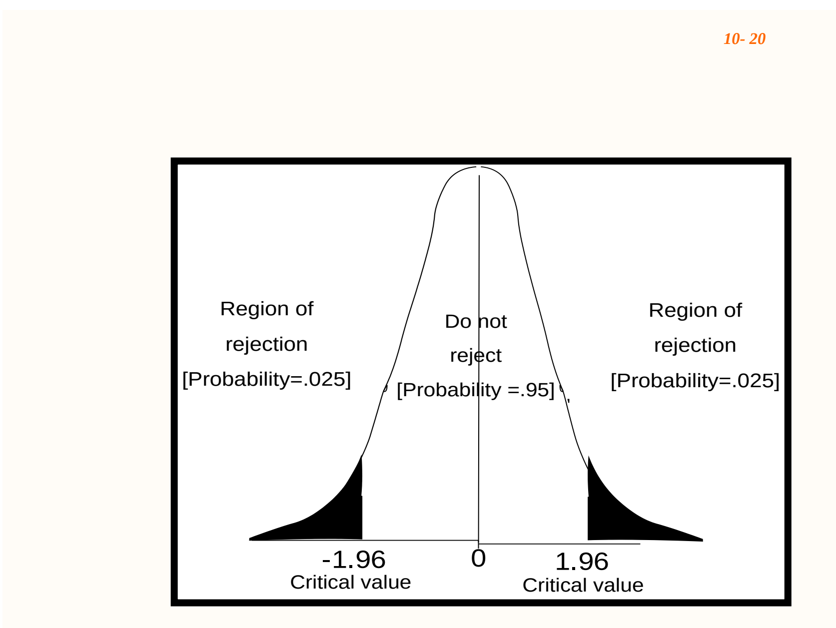
$$H_1 : \mu < \mu_0$$

Then this type of test is called one tailed test.



Two tailed test: No direction is specified in the alternate hypothesis H_1.
In the above example if the alternative hypothesis is like
$H_1 : \mu \neq \mu_0$

Then this type of test is called two tailed test.



Type I Error: Rejecting the null hypothesis when it is actually true.
Example: Suppose a person getting a positive result in a HIV test whether he is not HIV positive.

Type II Error: Accepting the null hypothesis when it is actually false.
Example: Suppose a person getting negative result in HIV test whether he is HIV positive.

Level of Significance: The probability of rejecting the null hypothesis when it is actually true; i.e.the probability of type I error is called level of significance. It is denoted as α.

$$\alpha = \Pr(Type\ \mathrm{I}\ error)$$

Power of the test: The probability of Type II Error is defined as β.
That is β = Pr (Type II Error).Now (1- β) is defined as the power of the test.

The difference between Type I Error and Type II Error given as below:

- Rejecting the null hypothesis when it is actually true is called type I error

  On the other hand, accepting the null hypothesis when it is actually false is called type II error.

- Type I error is known as false positive

  On the other hand, Type II error is known as false negative.

- Type I error is more serious then type II error

  Type II is less serious then type I error.

- The probability of type I error is called level of significance

  On the other hand, if the probability of type II error is subtracted from 1 then that term is called power of the test.

Test statistic: A value, determined from sample information, used to determine whether or not to reject the null hypothesis is called test statistic.
Example: There are many types of test statistic. As like, z test, t test, F test, χ^2 *test* etc.

Decision Rule

If Computed value < critical value *then Null* Hypothesis is accepted.i.e. Alternative hypothesis is rejected

If Computed value > critical value *then Null* Hypothesis is rejected. i.e. Alternative hypothesis is rejected

t-test: Testing for a Population Mean where sample size is small i.e less than 30, and Population Standard Deviation Unknown. The test statistic is the *t* distribution is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

The critical value of t is determined by its degrees of freedom equal to *n*-1.

Assumptions for t-test:

1. When sample size is less than 30, we can use t-test.
2. The sample observation are independent that is sample is random
3. The population standard deviation is unknown.

The Steps are included in t-test given as below:

Step 1
State the null and the alternative hypotheses that is we have to state.
$H_0 : \mu = \mu_0$ and $H_1 : \mu > \mu_0$ or

$\mu < \mu_0$ or

$\mu \neq \mu_0$

Step 2
Select the significance level.
Step 3
Calculate test statistic. Here we have to calculate z test.
Step 4
State the decision rule. That is

$\text{Re}\,ject\ H_0$ if $\text{t} > t_0$ or $\text{-t} < -t_0$

Or

$Accept\ H_0$ if $\text{t} < t_0$ or $\text{-t} > -t_0$

Step 5
Make a decision and interpret the results.
Example-1: A drug manufacturer has installed a machine which fills automatically 5 grms in each phile.Randomly 10 sample was taken and found to contain 5.02 grams on an average in a phial. The Standard deviation of the sample was 0.002 grms.Test at 5% levels of significance, if the adjustment in the machine is in order.

Sol:
Here the null and the alternative hypotheses are
$H_0 : \mu = 5$ i.e. the adjustment in the machine is in order
$H_1 : \mu \neq 5$ i.e. the adjustment in the machine is not in order

Now the test statistic is: $t = \dfrac{\bar{X} - \mu}{s/\sqrt{n}}$

Here; $\bar{x} = 5.02, \mu = 5, s = 0.002, n = 10$

$$= \frac{5.02 - 5}{0.002/\sqrt{10}} = 31.62$$

Here (n-1) =9, $\dfrac{\alpha}{2} = \dfrac{5\%}{2} = 0.025$

From the t table, the tabulated value of t is, $t_{tab} = \pm\ 2.262$.

As, the calculated value is greater than the tabulated value, that is,
$t_{cal} = 31.62 > t_{tab} = 2.262$.
So null hypothesis is rejected.
Hence, the adjustment in the machine is not in order.

Example-2: An automobile tyre manufacturer company said that the average life of certain grade of tyre is 25,000 km.A random sample of 15 tyres were tested and mean and standard deviation of 27,000 and 5000 kms respectively, were computed. At 5%level of significance is it reasonable to conclude that the average life of certain grade of tyre is greater than 25,000 km.

Sol: Here the null and the alternative hypotheses are
$H_0 : \mu = 25000$ i.e. the average life of certain grade of tyre is 25,000 km.
$H_1 : \mu > 25000$ i.e. the average life of certain grade of tyre is greater than 25,000 km.

Now the test statistic is: $t = \dfrac{\bar{X} - \mu}{s/\sqrt{n}}$

Here; $\bar{x} = 27000, \mu = 25000, s = 5000, n = 15$

$$= \frac{27000 - 25000}{5000/\sqrt{15}} = 1.549 = 1.55$$

Here (n-1) =14, $\alpha = 5\% = .05$
From the t table, the tabulated value of t is, $t_{tab} = 1.761$.
As, the calculated value is less than the tabulated value, that is,
$t_{cal} = 1.55 < t_{tab} = 1.761$.
So null hypothesis is accepted.
Hence, the average life of certain grade of tyre is 25,000 km

Z-test: Test for the population mean from a large sample ($(n \geq 30)$. The Z statistics is defined as

$$z = \frac{(\overline{X} - \mu)}{\frac{\sigma}{\sqrt{n}}}$$

Here,

$\overline{X}$ is sample mean.

μ is population mean.

σ is population standard deviation.

s is sample standrad deviation.

n sample size.

Assumptions for z-test:

1. When sample size is greater than 30,we can use z-test
2. The population standard deviation is known

Problem: The mean life time of a sample of 100 light tubes produced by a company is found to be 1580 hours and the population standard deviation is 90 hours. Test the hypothesis at the 5% level of significance that the mean of the tubes produced by the company is 1550 hours.

Solution:

Here the null and the alternative hypotheses are

$H_0 : \mu = 1550$

$H_1 : \mu \neq 1550$

Here given that 5 % level of significance. The tabulated value of z_0 is ± 1.96.

Here the test statistic is *z*.

So,

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{1580 - 1550}{90/\sqrt{100}} = 3.33$$

We know,

$Re\ ject\ H_0$ if $z > z_0$

Otherwise accept H_0

Here z = 3.33 and tabulated value z_0 is $\pm$ 1.96.
As 3.33>1.96 that is $z > z_0$,

So null hypothesis is rejected. Hence the mean life time of the tubes produced by the company may not be 1550 hours.

χ^2 **-test:** For testing the population variance σ^2 ,based on sample variance s^2 , χ^2 test is used. The text statistic is given as below:

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2}$$

Here s^2 is sample variance, σ^2 is population variance and degrees of freedom is (n-1).

Assumptions:

1. The sample should contain at least contain 50 observations.
2. The sample observation should be independent of each other
3. It is use for variance test.